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A macroscopic node model related to capacity drop

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Abstract

The automobile traffic congestion annually generates an estimated cost of several million euros for such a urban area as a European capital. At the origin of this congestion, the capacity drop is a well-known phenomenon which still remains complex to model [1,2,3,4]. The capacity drop is related to the hysteresis of traffic: for a state of disturbed traffic, the return to the normal of the traffic is delayed when demand decreases [5]. This paper intends to present a macroscopic convergent model to get a better modeling for capacity drop. Considering previous investigations [8], one considers bounded acceleration for the flow.
As the most common case of convergent is the merge of two road lanes, or two motorways, the convergent is modeled as a box with two entry flows and one output flow. A static storage capacity is provided to the box. Vehicles are mainly characterized by their bounded acceleration. The point is to describe the evolution of the convergent considering the number of vehicles stored inside the box.
The process is to consider the convergent box as a cell of network regarding the Godunov scheme [6]. The supply function has a classical fundamental diagram shape, but the demand function is modified regarding the bounded acceleration of vehicles [7]. The partial supply functions for the node cell are calculated accordingly to the importance of the converging roads. Then the model is solved using the Godunov scheme, with an update of the number of stored vehicles for every time step.
The model is to be tested on Paris ring with 40 seconds data.

Keywords: Node model, Convergent, Bounded acceleration; Capacity drop; Godunov scheme

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1. Context

The underlying model to our study is the Lighthill-William-Richards (LWR) model[9][10], which is a first order macroscopic traffic flow model. The basic variables flow Q, density K and speed V, are assumed to be functions of the position x and the time t. The model can be expressed by the following conservation equation:

$$\frac{\partial Q_e(K,x)}{\partial x}+\frac{\partial K(t,x)}{\partial t}=0 \quad (1)$$

The flow Q and the speed V are given by :

$$Q = KV = KV_e(K) = Q_e(K) \quad (2)$$

The representation of the equilibrium flow-density relationship $Q = Q_e(K,x)$ is the so-called fundamental diagram, assumed to be concave, depicted in figure x. This fundamental diagram is resoluble into two distinct equilibrium relationships, the local equilibrium supply , and demand, functions $\Sigma_e(K,x) and\ \Delta_e(K,x)$. These functions, which result from the fundamental diagram, express the greatest possible inflow and outflow at any point x of the studied road.

$$\Sigma_e(\rho,x)=\begin{cases}Q_{max}(x+)\ if\ \rho<\rho_{cr}(x+)\\ Q_e(\rho,x+)\ if\ \rho\geq\rho_{cr}(x+)\end{cases} \quad (3)$$

$$\Delta_e(\rho,x)=\begin{cases}Q_e(\rho,x-)\ if\ \rho\leq\rho_{cr}(x-)\\ Q_{max}(x-)\ if\ \rho\geq\rho_{cr}(x-)\end{cases} \quad (4)$$

And then

$$Q(x,t)=Min\ \Delta_e[(K(x-,t),x),\Sigma_e(K(x+,t),x)] \quad (5)$$

The latter equation is the main principle underlying the numerical resolution of the model.

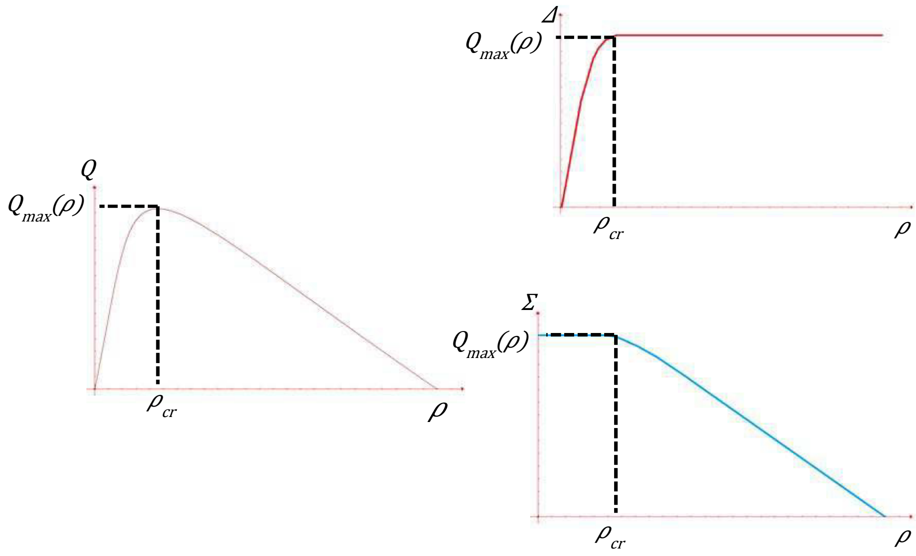

Figure 1: Fundamental diagram(left), demand function (top), supply function (bottom)

The choice was made to consider a bi-parabolic diagram. A fundamental diagram is defined by 5 parameters: The free speed V_x, the critical density K_c which defines the segregation between free and congested traffic, the maximal density K_x which is the maximal number of vehicles a stretch of freeway can store, the maximal flux Q_x also called the capacity of the road, and the maximal shockwave speed W_x, which is the tangent of the diagram for K=K_x.
The fluid part of the diagram is assumed to be given by the following equations:

$$V_e^f = V_x - \frac{K}{K_c}(V_x - V_c), K \leq K_c \quad (6)$$

$$Q_e^f = KV_x - \frac{K^2}{K_c}(V_x - V_c) \quad (7)$$

The congested part of the diagram is mainly defined by W_x , such that :

$$Q_e^c(K_x) = 0 \quad (8)$$

$$Q_e^c(K_c) = Q_{max} \quad (9)$$

$$\left(\frac{\partial Q_e^c}{\partial K}\right)_{K=Kx} = W_x \quad (10)$$

Which finally gives:

$$Q_e^c(K) = (K_x - K)\left[W_x + \left[\frac{Q_x}{(K_x - K_c)^2} - \frac{W_x}{K_x - K_c}\right](K_x - K)\right] \quad (11)$$

2. The Node Model

A node model's goal is to connect upstream and downstream boundary conditions of a merging/diverging section.

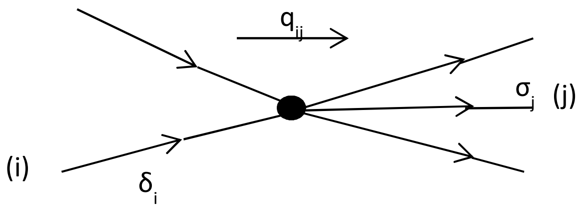

Figure 2: Node illustation

Figure x depicts the classical node case. If one considers quasi stationary traffic conditions, as in a generalized Riemann problem defined by upstream demands δ_i and downstream supplies σ_j, it can be shown that the flux through the node is constrained by:

$$\begin{aligned} \sum_i q_{ij} \leq \delta_i \ \forall i \\ \sum_j q_{ij} \leq \sigma_i \ \forall j \end{aligned} \quad (12)$$

Of course the above system has to be complemented with rules expressing priority rules or conflict resolution in the intersection. In the sequel of the paper one would consider the classic convergent situation, with two incoming fluxes and one out coming flux. The point is to divide linearly the downstream supply between upstream demands. The proportion coefficients β_i represent the fraction of destination traffic lanes available from the origins.[6][11] On the following example one has $\beta_i = 2/3$ because 2 of 3 destination lanes are available for a vehicle whatever its origin is.

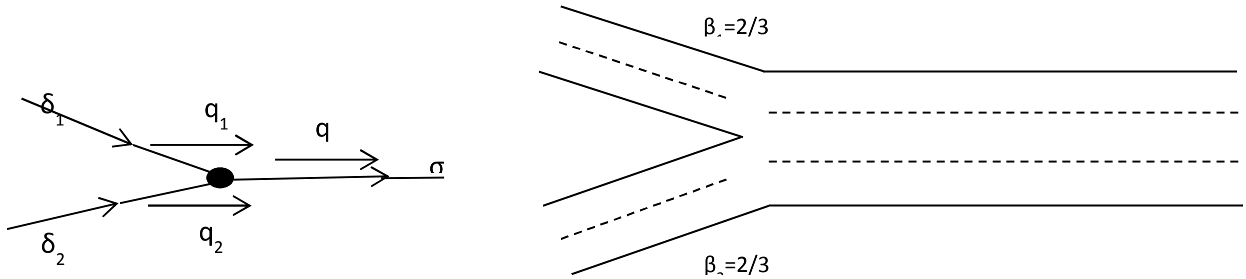

Figure 3: Illustration of a classical convergent

In order to introduce capacity drop in the node model, one would consider that the node can store a limited number of vehicles N_{max}, and that the supply and demand functions are function of the number N of vehicles stored in the node, which is quite equivalent to a density. One would also consider that in congested traffic the demand cannot be maximized because of larger reaction and relaxation times for drivers and vehicles once a queue is formed. This requires building a modified demand function for congested traffic, introducing *the recovery flow of the node*.

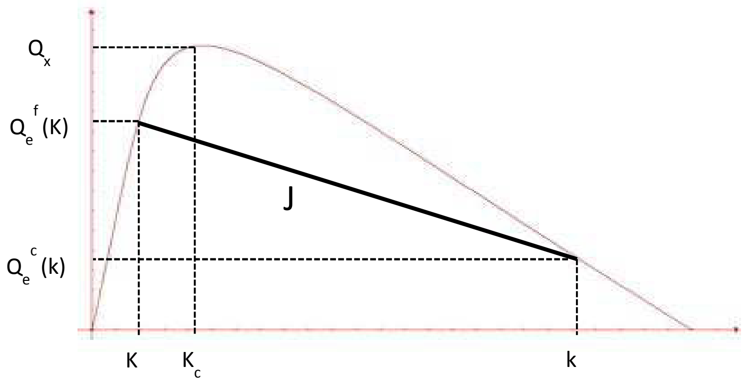

Figure 4: Exemple for a J-line(k)

The slope for the J-line, is expressed by

$$\Theta(k) = V_e^g(k) - \frac{q_0}{k} = \frac{(Q_e^c(k) - q_0)}{k}, q_0 \; constant, homogenous \; to \; a \; flux \qquad (13)$$

The constant q_0 can be interpretated as the idea that in a platoon, vehicles that are about to reaccelerate (a congested->free transition) do it one after the other after a reaction/relaxation time θ (one can also see θ as the time to get out of a wide moving jam) and so the reacceleration of the platoon propagates, with an intervehicular distance close to K, at a relative speed given by:

$$v = -\frac{1}{\theta K} \stackrel{\text{def}}{=} -\frac{q_0}{K} \quad (14)$$

Whose absolute expression is then:

$$\rho(K) \stackrel{\text{def}}{=} V_e(K) - \frac{q_0}{K} \quad (15)$$

One refers to fig.X, the J-line's equation is written

$$Q = Q_e^c + (K - k)\Theta(k) \quad (16)$$

$$Q_e^f(K) = Q_e^c(k) + (K - k)\Theta(k) \quad (17)$$

On deduces

$$Q_e^f(K) = q_0 + \frac{K(Q_e^c(k) - q_0)}{k} \quad (18)$$

Let's define 3 variables:

$$\alpha = V_x \quad (19)$$

$$\beta = \frac{V_x - V_c}{K_c} \quad (20)$$

$$\varphi = \frac{q_0 - Q_e^c(k)}{k} \quad (21)$$

And one can rewrite:

$$\alpha K - \beta K^2 = q_0 - \varphi K \quad (22)$$

Or:

$$\beta K^2 - (\alpha + \varphi)K + q_0 = 0 \quad (23)$$

One solves the equation with:

$$\Delta = \sqrt{(\alpha + \varphi)^2 - 4q_0\beta} \quad (24)$$

$$K_{rec} = \frac{\alpha + \varphi - \sqrt{\Delta}}{2\beta} \quad (25)$$

Because one keeps the smallest solution of K for equation X. Eventually, the recovery flow is given by:

$$Q_{rec} \stackrel{\text{def}}{=} Q_e^f(K_{rec}) = q_0 - K_{rec}\varphi \quad (26)$$

Which gives the following curve for recovery demand:

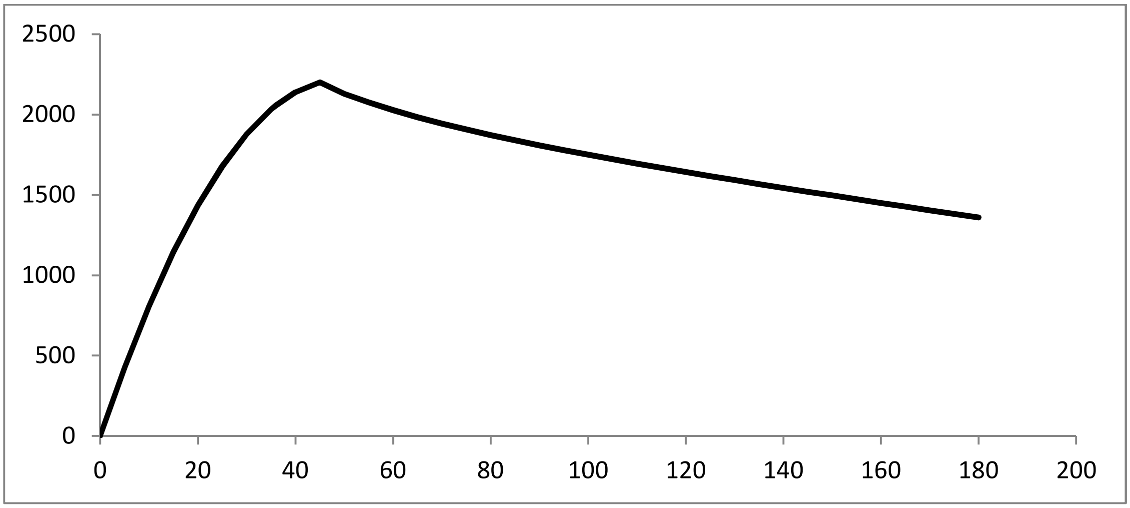

Figure 5: Recovery flux function, flux in veh/h vs density in veh/km

Under the following set of parameters:

Table 1: Parameters set for the recovery demand function

Parameter	Vx	Kx	Kc	Wx	Qx	θ
Value	90 km/h	180 veh/km	45 veh/km	25 km/h	2200 veh/h	1.9s

3. Application to the field test

The developed model is applied to one part of the *Boulevard Périphérique* site in Paris, between the on-ramp "Porte de Brancion" and "Porte de Sèvres". The considered total stretch length is around 2km including 5 measurement stations on the carriage way and one measurement at the on-ramp Brancion (see figure 6).

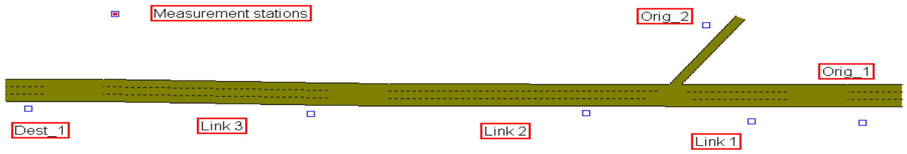

Figure 6: Considered stretch

Several data sets are considered for the calibration and validation process (18 to 22 March 1991). The available data set time aggregation is 40 seconds. After data screening, 2 days are selected for the parameter calibration and validation results. The BOX algorithm [12] was used for the calibration

Parameter	V_x	K_{max}	K_c	lamda	Q_x
Link 1	82.36 km/h	139 veh/km	30.3 veh/km	0.703	2359 veh/h/l
Link 2	82.49 km/h	158 veh/km	26.6 veh/km	0.840	2370 veh/h/l
Link 3	96.82 km/h	164 veh/km	37.5 veh/km	0.867	2245 veh/h/l

Table 1: Optimal parameters found by BOX algorithm

Table 2 includes the optimal model parameters found using box algorithm for the first day (19 March 1991). In particular, the free speed (V_x), the critical density (K_c), the maximum density (K_{max}), the lamda parameter for the capacity drop, and the capacity of the link (Q_x).

The second day (03/22/1991) is used for validation.

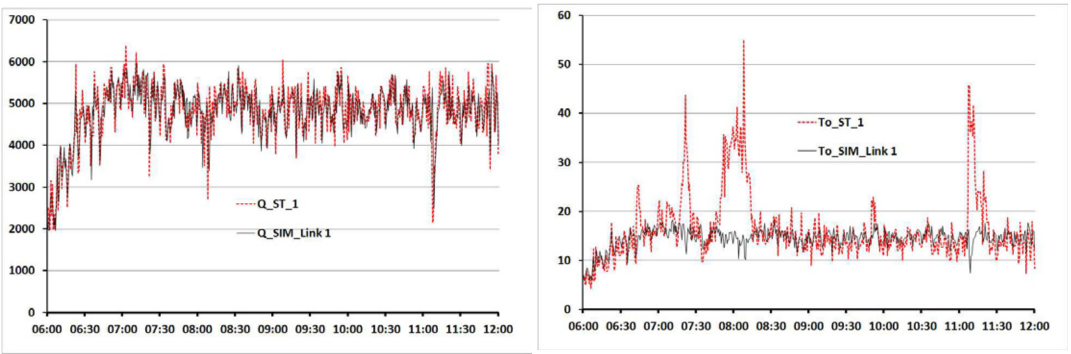

Figure 7: Time evolution of measured and simulated flows (left) and occupation rates (right) (ST1)

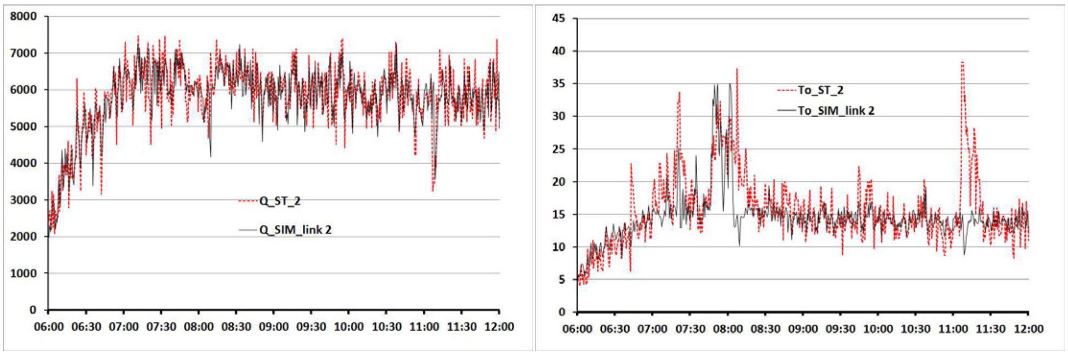

Figure 8: Time evolution of measured and simulated flows (left) and occupation rates (right) (ST2)

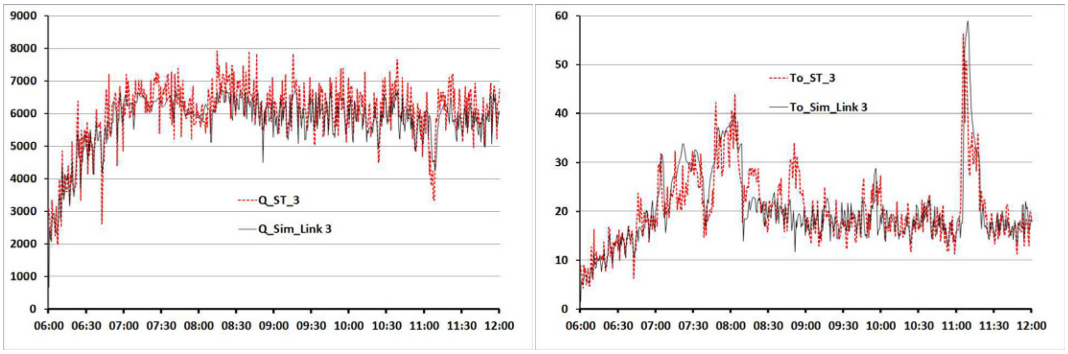

Figure 9: Time evolution of measured and simulated flows (left) and occupation rates (right) (ST3)

Parameter	Av. Q	MSD Q	Av. To	MSD To
ST1	4827	446	14.42	5.55
ST2	5814	562	14.22	3.75
ST3	6005	659	19.28	6.39

Table 2: Average and the Mean Square Deviation (MSD) for traffic volumes and occupancy rates.

As reported in figures 7, 8 and 9, we can observe that the accuracy of developed model is satisfactory. As a matter of fact, time evolution of flow follows the real measurements. With respect to the occupancy rates, congestions are reproduced at same location and the time as the measurements on Link-2 and Link-3 (downstream of the node). This means that capacity drop is well modelled. However, for the link-1 (upstream of the node), the congestion apparition is very limited compared to the measurements. This is probably due to the measurement station location. The total length of the link-1 is equal to 200 m and the associated number of segment is equal to only 1 segment. Consequently, the density and the occupancy rate are much smoothed. Table 3 includes the Mean and the Mean Square Deviation (MSD) of both variables: traffic volumes and occupancy rates.

4. Conclusions

The developed model has been successfully applied to a limited part of the ringway of Paris test site. Model parameters have been identified on one set of the real data (18/03/91). The validation has been made on the 22/03/91 real data set with the same identified parameters found for the 18/03/91 day. The output results concerning the output trajectories of both traffic variables (volume, occupancy rates) indicate that the Limit Acceleration model follows with sufficient accuracy the time evolution of the traffic conditions in the considered stretch and in particular at the node location. This means that the developed model is able to cope with different traffic conditions (fluid, dense and congested) at an acceptable level of accuracy. In more quantitative terms, calculation of the mean square deviation between the measured and the simulated variables was used as an indicator of accuracy.

As a final conclusion, the suggested node model is able to cope with the capacity drop phenomena. In particular, the parameter "Lamda" of the 3 considered links are different. The capacity drop parameter of the in link-1 at the node indicates that the capacity drop in this link is higher compared to the other links. This is due to the on-ramp Brancion which generates conflict at the node. Investigations on node model are underway aiming at improving congestion spillback at the upstream links of the node.

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