Investigation of Estimation Methods for Time-varying Residual Magnetic Moment

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Abstract

Generally, the dominant attitude disturbance source for the low Earth orbit small satellites is the residual magnetic moment (RMM). The RMM should be estimated and compensated in orbit to increase the attitude estimation and control accuracy. Although the estimator is usually built with the assumption that these parameters are constant, the RMM changes with sudden shifts caused by the variations in the onboard electrical current. The estimator should quickly track these unobserved parameters in case of change and perform accurate estimation for the rest of the procedure. In this paper, we investigate applicability of the existing estimators, such as the particle filter, for the RMM estimation. We present the initial results for an extended Kalman particle filter (EKPF) based method which may be useful for improving the estimation accuracy and tracking capability.

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1. Introduction

The main attitude disturbance sources for Earth orbiting satellites are usually categorized in four as the gravity gradient, sun pressure, aerodynamic drag and residual magnetic moment (RMM). Specifically for low Earth orbit (LEO) small satellites, the RMM becomes dominant amongst these disturbance sources and the effects of the rest decrease because of the small size of the satellite [1, 2]. However, the magnetic disturbance is mainly caused by the onboard electric current loop, small permanent magnet in some devices or some special material on the satellite, and...
does not strongly depend on the satellite size [3]. Moreover other disturbance sources can be modeled accurately with onground simulations and further minimized during the design process. On contrary, pre-launch testing for onboard RMM characteristics is not easy because of the difficulties for providing a magnetically clean environment and these characteristics differ after the launch [4].

The effects of the RMM on the attitude determination and control accuracy and the necessity for the RMM compensation are well discussed in [1-3]. In [1] an observer is designed for the RMM estimation and then the disturbance effect is cancelled out using a feed-forward technique. In [2] Inamori et al. mainly propose a Kalman Filter (KF) based method and compare the RMM estimation accuracy of the Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF). In these studies the RMM components are assumed to be constant in time. However, in practice, these parameters may change with sudden shifts because of the instantaneous variations in the onboard electrical current. Such instantaneous variations in the current may be caused by switching on/off of the onboard electronic devices or going into/out of eclipse. In such cases, the estimator cannot catch the new value of the parameter quickly if it is designed for high steady state estimation accuracy. Among the mentioned studies only [3] refers to the possibility of the sudden changes in the RMM terms but the authors state that such changes are estimated with an off-line method and a clear description for the estimation process is not given.

In our previous researches we used an UKF algorithm for in-orbit estimation of time-varying RMM [5]. Since the traditional UKF (same as other variations of the KF) lacks an accurate model for the RMM terms with the assumption that these parameters are constant all the time, there must be a tradeoff between the estimation accuracy and tracking agility while designing the filter and the researcher/engineer must sacrifice one of them in practice. Therefore in [5,6] we proposed a simple filter adaptation method for increasing filter’s tracking agility in case of abrupt changes in the estimated RMM terms and assured both accurate estimation and good tracking performance.

In this study we extend our survey on this topic and investigate applicability of the existing estimators such as the particle filter for the RMM estimation. We present the initial results for an extended Kalman particle filter (EKPF) based method which may be useful for improving the estimation accuracy and tracking capability. The EKPF runs several EKFs with different models for the process noise covariance and a sampling is performed using the measurements. In terms of running with different models, the given method is similar with the Multiple Model Adaptive Estimation (MMAE) technique and differs only if a resampling step added. For the sampling procedure we propose different concepts and choose the most efficient one. We compare the algorithm with the regular EKF algorithms in the sense of RMM estimation accuracy and computational load.

2. The RMM Estimation using EKPF

2.1. The RMM Estimation

For the specific problem, the estimated state vector is composed of the body angular rates with respect to the inertial frame and RMM terms as given with

\[
\mathbf{x} = \begin{bmatrix} \boldsymbol{\Omega}_{BR} \\ \mathbf{M} \end{bmatrix},
\]

where, \( \boldsymbol{\Omega}_{BR} = [\omega_x \ \omega_y \ \omega_z]^T \) is the angular rate vector and \( \mathbf{M} = [M_x \ M_y \ M_z]^T \) is the RMM vector. The system model for the filtering is given in discrete-time by:

\[
\begin{align}
\mathbf{x}_{k+1} &= f(\mathbf{x}_k, k) + \mathbf{w}_k \\
\mathbf{y}_k &= h(\mathbf{x}_k, k) + \mathbf{v}_k
\end{align}
\]

Here \( \mathbf{x}_k \) is the state vector and \( \mathbf{y}_k \) is the measurement vector; \( f(\cdot) \) is the nonlinear process function and \( h(\cdot) \) is the nonlinear measurement function. Moreover \( \mathbf{w}_k \) and \( \mathbf{v}_k \) are the process and measurement error noises, which are assumed to be Gaussian white noise processes with the covariances of \( \mathbf{Q}_k \) and \( \mathbf{R}_k \) respectively.

The nonlinear process model is obtained by discrete-time integration of the dynamics equation [5,6]. Nevertheless, since the onboard gyros directly supply \( \boldsymbol{\Omega}_{BR} \) information, the measurement model may be represented
with linear equation as
\[
\dot{y}_k = [I_{3\times3} \ 0_{3\times3}]x_k + v_k ,
\]  
(3)

where \( I_{3\times3} \) and \( 0_{3\times3} \) are \( 3\times3 \) identity and null matrices, respectively. Note that for our case the satellite has magnetometers and gyros as the attitude sensors and we assume that the sensors are already calibrated using one of the existing techniques [7, 8].

In this study, the RMM terms are modeled as constant but with unexpected abrupt changes as discussed in the introduction. Assuming the RMM as piecewise constant is valid in general since the high frequency time-varying components of the RMM are negligibly small compared to the constant components and magnitude of the changes caused by instantaneous variations in the onboard electrical current [3, 6]. High frequency time-variation in the RMM should be suppressed in the design process of the satellite. Hence, the hypothesis for the RMM model is

\[
M(t) = \begin{cases}
M_0 & t_0 \leq t < t_1 \\
M_1 & t_1 \leq t < t_2 \\
\vdots & \vdots \\
M_M & t_M \leq t < t_{orb} \\
M_M & t_{orb} \leq t < t_0
\end{cases}
\]  
(4a)

and

\[
\Delta M_j = \|M_j - M_{j-1}\| \text{are the magnitude of the changes in the RMM that occur at } t_j \text{ for } j = 1...\alpha .
\]  
(4b)

Here \( t_j \) are the unknown time instances that a change occurs within one orbit period \( t_{orb} \), \( M_j \) are constant RMM vectors and \( \Delta M_j \) are the magnitude of the changes in the RMM that occur at \( t_j \) for \( j = 1...\alpha \).

2.2. Extended Kalman Particle Filter

Particle filters comprise a very broad class of suboptimal nonlinear filters based on sequential Monte Carlo (MC) simulations, and there are numerous different particle filtering techniques in literature [8, 9]. The key idea is to represent the required posterior density function by a set of randomly generated samples that have associated weights and to perform the estimation based on these samples and weights. Several different types, improving strategies and applications of the PF can be found in [9].

When we estimate parameters, classic approaches to the PF such as the Sequential Importance Resampling (SIR) PF cannot be used and alternative approaches are necessary [10, 11]. One of these methods is the EKPF. The essence is using a bank of EKFs (one for each particle) for state propagation and measurement update and then sampling the particles using the measurements. The details may be seen in [12].

In this study we use a EKPF where all the EKFs run with different models for the process noise covariance matrix, \( Q_k \). The state estimate is obtained by the sum of each filter’s estimate weighted by the defined likelihood function. The idea is similar with the MMAE and differs only if a resampling step is added after the sampling and the filter estimate is obtained as the mean of the resampled states [13]. We examine two different cases for the sampling procedure:

a) Likelihood function that is used for weighing each filter’s estimate is defined using the filter residuals as:

\[
q_i = \frac{1}{(2\pi)^m | P_i |^{1/2}} \exp \left( -\frac{1}{2} \left[ y_i - \hat{y}_{i,k} \right]^T R_i^{-1} \left[ y_i - \hat{y}_{i,k} \right] \right).
\]  
(5)

Here \( m \) is the number of measurements, \( y_i \) is the measurement vector and \( \hat{y}_{i,k} \) is the estimated measurements vector that is build using the estimated states for each EKF, \( \hat{x}_{i,k} \). This is the classical approach for defining the likelihood function for the EKPF [12] likewise the MMAE [13].

b) We propose defining the likelihood function as

\[
q_i = \exp \left( -\frac{|g_{i,k}|}{\beta} \right),
\]  
(6)

where
\begin{align}
g_{k,i} &= \tilde{x}_{k,i} + (1-\lambda)S_{k,i}, \quad (7a) \\
S_{k,i} &= \frac{1}{\sqrt{m}} \{ H P_{k-1,i} H^T + R \}^{-1/2} \{ y_k - \hat{y}_{k,i} \}. \quad (7b)
\end{align}

Here $P_{k-1,i}$ is the predicted covariance matrix for each EKF, $H$ is the measurement matrix given as $H = \begin{bmatrix} I_{3 \times 3} & 0_{3 \times 3} \end{bmatrix}$, $1_m$ is unit vector with $m$ unit elements, $\lambda$ is the forgetting factor as $0 \leq \lambda < 1$ and $\beta$ is a positive scalar used for tuning the input of the exponential function. $S_{k,i}$ given here is the sum of the normalized innovation sequence for each filter and $g_{k,i}$ is the geometric moving average (GMA) of $S_{k,i}$.

When a change occurs in the estimated parameters $g_{k,i}$, which is a zero mean value in the normal case, increases depending on the $Q_{k,i}$ [5,6]. For the EKFs with high noise this change will be low and vice versa. Hence in case of a change, estimates of the EKFs with high noise will be weighted higher while the estimates for the all EKFs will be almost equally weighted in the steady state regime.

Note that for both approaches the calculated weights are normalized before being used.

3. Numerical Example

We tested the proposed EKPF algorithms for the RMM estimation. Four different EKFs run simultaneously (so there are 4 particles) and the different levels of the process noise covariance that are selected for each filter are,

\begin{align}
Q_{k,i} &= \begin{bmatrix} (1E-20) I_{3 \times 3} & 0_{3 \times 3} \\
0_{3 \times 3} & \Lambda_i \times (1E-10) I_{3 \times 3} \end{bmatrix} 
\end{align}

where $\Lambda_i$ is 0.1, 1, 10 and 100 for $i = 1, 2, 3, 4$ respectively. Simulations are run for 4000 seconds. The real values for the RMM terms change as

\begin{align}
M &= \begin{bmatrix} 0.1 & 0.02 & -0.05 \end{bmatrix} Am^2 \quad t < 2000 \text{sec} \\
&= \begin{bmatrix} 0.25 & 0.1 & -0.15 \end{bmatrix} Am^2 \quad t \geq 2000 \text{sec}. 
\end{align}

The hypothetical satellite, for which the algorithm is tested, is same as the one described in [5,6]. For the GMA based likelihood function $1$ and $0.997$.

The estimation result for the EKPFs with two different definitions of the likelihood function is given in Fig.1. As seen the EKPF with the classical residual based likelihood function gives noisy estimation results in case of change in the estimated parameters and it takes almost 1000 seconds for the filter to settle to the true values. Moreover, even after 1000 seconds, the steady state accuracy for the estimation is not high because of the remaining noise that lasts until the end of the simulation. On the other hand, the EKPF with the proposed GMA based likelihood function converges quickly to the new value of the $M$, and there is no noise in the estimations like in case “a”. The Root Mean Squared Error (RMSE) for the estimated RMM terms in case “b” are $11.653 \times 10^{-4} Am^2$, $6.927 \times 10^{-4} Am^2$ and $7.943 \times 10^{-4} Am^2$, respectively for $M_x$, $M_y$ and $M_z$ estimations in between 1000th and 2000th seconds and this is sufficiently accurate regarding the overall attitude determination and control requirements.

In Table 1 the GMA based likelihood function for 4 different EKFs is tabulated at 3 sampling times. As clearly seen when there is no change in the estimated parameters, estimations of each EKF are almost equally weighted. After the change occurs in the RMM terms, the EKFs with higher noise become to be more weighted whereas at $t = 2300$ sec. weight of the EKF with $\Lambda = 100$ reaches to 0.5912 and this filter’s outputs dominate the estimation result.
Fig. 1. Estimation of the RMM in y axis in case of sudden change: a) EKPF with classical residual based likelihood function, b) EKPF with the proposed GMA based likelihood function. Upper parts of the figures compare the estimation results with the actual values, and lower parts present the estimation error.

Table 1. Normalized values of the GMA based likelihood function for each EKF at three different sampling times.

<table>
<thead>
<tr>
<th></th>
<th>EKF with $\Lambda = 0.1$</th>
<th>EKF with $\Lambda = 1$</th>
<th>EKF with $\Lambda = 10$</th>
<th>EKF with $\Lambda = 100$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t = 1500$ sec.</td>
<td>0.2497</td>
<td>0.2499</td>
<td>0.2514</td>
<td>0.2490</td>
</tr>
<tr>
<td>$t = 2200$ sec.</td>
<td>0.1009</td>
<td>0.1516</td>
<td>0.2630</td>
<td>0.4845</td>
</tr>
<tr>
<td>$t = 2300$ sec.</td>
<td>0.0419</td>
<td>0.0995</td>
<td>0.2673</td>
<td>0.5912</td>
</tr>
</tbody>
</table>

Finally in Fig. 2 we compare the estimation results of the EKPF with the results obtained separately from EKFs with $\Lambda = 1$ and $\Lambda = 100$. As expected estimation performance of the EKPF in the sense of both accuracy and tracking capability is in between the performance of these two EKFs. It is more accurate than the EKF with $\Lambda = 100$ and more agile than the EKF with $\Lambda = 1$.

The only drawback of the proposed method is inherently the computational load. In our case 4 separate EKFs running simultaneously and the extra process required for the weighting calculations of the PF make the EKPF almost 6 times computationally heavier than a single EKF algorithm used for the RMM estimation. Especially for the small satellite applications, if the algorithm will be used onboard in real time, this issue should be carefully considered. Computational load might be reduced by using only two EKFs, one with low and the other with high noise but in result the performance of the filter will relatively degrade.

In future researches we will aim at defining a more useful likelihood function which makes the filter more efficient especially in the steady state regime by weighting the EKFs with low noise more than the rest rather than equally weighting all the filters. Then a comparison with the adaptive UKF method given in [5,6] will be interesting.

4. Conclusion

Previous researches proved that the residual magnetic moment (RMM) is the dominant disturbance source for small satellite missions. When the RMM is time-varying, it becomes difficult to obtain good estimation performance by the traditional filtering approaches such as the Extended Kalman Filter. In this paper we presented the initial results for an extended Kalman particle filter (EKPF) based method that is used for estimating the time-varying RMM. The EKPF runs several EKFs with different models for the process noise covariance and a sampling performed using the measurements. We demonstrated the proposed algorithm for a small satellite. The results show that the EKPF is both accurate in steady state regime and agile for catching the new values of the RMM when there is a change.
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References