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Special issue in honour of Ernest Borisovich Vinberg

It is a great privilege for us to be the editors of the special issue of the *Journal of Algebra* dedicated to Professor Ernest Borisovich Vinberg on the occasion of his 70th birthday. Throughout his distinguished career, Vinberg has made fundamental and profound contributions to Algebra and Geometry. His work concerns Kähler manifolds, homogeneous convex cones, real and

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complex Lie groups, Lie algebras and their generalisations, Invariant Theory, algebraic groups, K3 surfaces, discrete reflection groups, geometry of Lobachevsky spaces, spherical varieties, coisotropic actions, and commutative homogeneous spaces—with seminal contributions in each subject. Not only did he settle a number of important problems, but he also opened new directions of research which have inspired his many students and other mathematicians throughout the world. Among such areas of research are the theory of homogeneous convex cones and T-algebras, invariant theory of θ-groups (linear groups associated with periodic automorphisms of reductive Lie algebras), the complexity of reductive groups actions, spherical subgroups and spherical varieties, theory of crystallographic reflection groups in Lobachevsky spaces, foundations of the theory of commutative homogeneous spaces. His vast and encyclopaedic knowledge of mathematics has been (and is) a great source of inspiration for generations of his students and colleagues.

The impact and influence of Vinberg's work on the development of Invariant Theory cannot be overstated. His classification of locally transitive representation of simple Lie groups (1960), and the subsequent classification of simple irreducible linear algebraic groups all of whose orbits have a non-trivial Lie algebra stabiliser (1967) and those with a free algebra of invariants (1976) opened a period of intensive investigations centered around the determination of classes of linear representations of reductive algebraic groups with good properties. In the 70s, Vinberg developed the general theory of θ-groups. This theory includes the adjoint representation and the isotropy representation of a symmetric space as a special case. In his seminal paper "The Weyl group of a graded Lie algebra" (1976), Vinberg showed that, for θ-groups, there is a natural analogue of a Cartan subspace, Weyl group, and the Jordan decomposition. He proved that this Weyl group is generated by complex reflections and that θ-groups have a free algebra of invariants. He also obtained a general method of classification of orbits for θ-groups. In 1986, Vinberg considered the general concept of the complexity, $c(G, X)$, of the action of a connected reductive group G on an irreducible variety X. He proved that $c(G, X)$ equals the modality of the action on X of a Borel subgroup of G. Vinberg also made significant contributions to the theory of spherical homogeneous spaces, prehomogeneous varieties, and reductive algebraic semigroups. In the last decade, his interest in spherical varieties naturally led him to profound study of commutative homogeneous spaces and coisotropic actions of Lie groups on Poisson manifolds.

Vinberg was the first to begin a systematic study of hyperbolic reflection groups. His fundamental article "Discrete groups generated by reflections in Lobachevsky spaces" (1967) laid the foundations of the whole theory. In this article, he developed an elegant method based on weighted graphs in order to describe the combinatorial structure of acute angled polytopes and therefore the fundamental polytope of a hyperbolic reflection group. He also obtained an arithmeticity criterion of a hyperbolic reflection group and proved that an arithmetic hyperbolic group is commensurable with the group of units of a suitable hyperbolic lattice. In 1971, Vinberg posed the classification problem of hyperbolic lattices over $\mathbb{Z}$ whose group of units contains a subgroup of finite index generated by reflections. Such lattices are called reflective. He developed the main tools for studying groups of units of reflective lattices: a key lemma on the inheritance of reflectivity and a general algorithm for constructing a fundamental region for a discrete reflection group in Λ^n. In 1983, Vinberg proved that there are no hyperbolic reflection groups of compact type in Lobachevsky spaces of dimension $\geqslant 30$.

Ernest Borisovich has been connected with the Faculty of Mechanics and Mathematics (Mekhmat) of the Moscow State University his entire career. He graduated from MSU at 1959 and completed his Ph.D. thesis under the supervision of E.B. Dynkin and I.I. Piatetski-Shapiro in 1962. Since then, he has been working at the Chair of Algebra of Mekhmat. Vinberg (to-

gether with A.L. Onishchik) has run the seminar on Lie groups and Invariant Theory in MSU for more than 40 years, producing the book "Lie groups and algebraic groups," a by-product of the early years. This book provides one of the best and unconventional introductions to the subject. A master textbook writer, Vinberg has published "Linear representations of groups," "A course in Algebra," and numerous survey articles for the "Encyclopaedia of Mathematical Sciences" (green series in Russian and yellow series in the Springer translation), all of which are widely known and often cited. An excellent lecturer and one of the most popular professors of the Mekhmat Faculty, he is famous for his well-prepared fascinating presentation combined with a careful selection of problems, displayed in his clear neat writing on the blackboard. Over the years Vinberg has attracted a large flow of students, with many (sometimes about 10 a year) asking him to be their scientific advisor.

Vinberg has created strong scientific schools in such different fields as Invariant theory, discrete reflection groups in Lobachevsky spaces, and coisotropic actions on Poisson varieties. Remarkably, Vinberg has had a profound impact through his students on several areas of mathematics (and even physics) which are distant from the mainstream of his research, such as algebraic groups over non-closed fields, Riemannian geometry, infinite-dimensional Lie algebras, and supersymmetry.

This issue is dedicated to Ernest Borisovitch as a symbol of the scientific excellence that he represents to all his friends, colleagues, and students, the level of excellence to which we would all strive to achieve. We hope that the variety of articles presented in this volume reflects to some extent the variety of his scientific interests.

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