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Studies of Compression Corner Flowfields Using THREE Turbulent Models

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Abstract

This paper presents the numerical investigations on shock-wave/turbulent boundary-layer interaction flow over a compression corner using SA, SST, and WJ EASM $k-\omega$ turbulence models, and compared with experiment data. The applicability of the three different turbulence models has been estimated comprehensively for this kind of the complex flow. The results indicate that the wall pressure distribution, the friction coefficients and the positions of separation and attachment with the three models agree well with the experimental data.

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1. Introduction

Shockwave turbulent boundary layer interaction induced by two-dimensional compression corner in the supersonic flows is classical benchmark for evaluating turbulence model performances. Configurations same as the compression corner, such as aircraft’s inlet, transonic compressor and control surface between the airfoils and fuselages produce a large number shockwave structures. Separation flows resulted in shockwave turbulent boundary layer interaction give rise to turbulence complexity and affect aircraft configuration design, aerodynamic analysis, propulsive efficiency and thermal load computations. Experimental and numerical studies on the compression corner in supersonic or hypersonic flow have been implemented since the middle of the last century. Settles et. al.[1-3] carried out a series of experiments in supersonic wind tunnel in Princeton University. Attachment flow, incipient separation flow and large separation flow induced by interaction between shockwave and turbulent boundary layer were studied at high Reynolds numbers. Elfstrom, G.M. et. Al [4] implemented experiment of compression corner flow at the Mach number of 9.22 in the No.2 supersonic wind tunnel in Imperial College. The effects of wall temperature and attack angle were investigated in their research works. Fernholz et. Al [5] provided numerous experimental data which can supplement theoretical study on the shockwave turbulent boundary layer interaction flow. Experiments cost enormous time and money, and can not solve extremely complex turbulent flows in reality. With the developments of the computer science and Computational Fluid Dynamics, numerical study of the shockwave turbulent boundary layer interaction becomes easier to implement. Furthermore, numerical simulation has lower cost compared with the experiment study. Dolling et. Al [6] summarized conditions of development in compression corner flow based on the Reynolds averaged equations, and drew a conclusion that most of turbulence models can not give rise to accurate results comprehensively. Yan et. Al [7] did not predict separation region precisely in the numerical simulation of two-dimensional compression corner flow using several eddy viscosity models. This work calculated the two-dimensional compression corner flow using WJ EASM \( k-\omega \) model, Spalart-Allmaras [9] model and Shear Stress Transport [10] model.)The performances of these three turbulent models in the calculation are compared and analyzed in the present research work.

2. Governing Equations and Turbulence Models

2.1. Governing Equations

The governing equations are the unsteady compressible Faver-filtered Navier-Stokes equations, written in nondimensional variables and convention forms as follow:

\[
\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho u_i) = 0
\]

(1)

\[
\frac{\partial}{\partial t}(\rho u_i) + \frac{\partial}{\partial x_j}(\rho u_i u_j + \rho \delta_{ij}) = \frac{\partial}{\partial x_j}(\tau_{ij} + \tau_2)
\]

(2)

\[
\frac{\partial}{\partial t}((\rho E + p)u_j) + \frac{\partial}{\partial x_j}((\rho E + p)u_j) = \frac{\partial}{\partial x_j}((\tau_{ij} + \tau_2)u_i + q_i + q_j)
\]

(3)

\[
\tau_{ij} = \mu(u_{i,j} + u_{j,i} - \frac{2}{3} u_{k,k} \delta_{ij})
\]

(4)

\[
\tau_2 = \mu_k \left(u_{i,j} + u_{j,i} - \frac{2}{3} u_{k,k} \delta_{ij}\right) - \frac{2}{3} \rho k \delta_{ij}
\]

(5)
Where, \( \tau_{ij} \) is the Reynolds stress tensor and \( \mu_t \) is determined by different modeling method in eddy viscosity models.

### 2.2. Turbulent Models

The RANS (Reynolds-averaged Navier-Stokes equations) is the main method for numerical simulating turbulent flow at present because of lower request for computer hardware, easier achievement and better robustness. Eddy viscosity model and Reynolds stress model are created on the basis of different treatments with the Reynolds stress in the Navier-Stocks equations. Linear viscosity model based on the Boussinesq theory is easy to be carried out in procedure and possesses good robustness. This model is suitable for shear flow in which shear stress is dominant, but is inappropriate in the separation flow, large reverse pressure gradient flow and secondary flow. The SA and SST models used in the present work belong to linear eddy viscosity model. On the other hand, Reynolds stress model built transport equation for Reynolds stress, which describe complex turbulent flow more precisely and more actually, however, it is more expensive and need rigid numerical scheme in order to avoiding divergence. Therefore, there are fewer applications in the numerical study of engineering flows.

Actually researchers focus on merits containing easy-to-use in linear eddy viscosity model and better simulating capability in Reynolds stress model. Explicit algebraic stress model in which lesser differential terms in the Reynolds stress transport equations is ignored and local equilibrium hypothesis is applied made a success to a certain extent. This model reserves pressure-strain ratio correlation term same as secondary moment transport equations, and constructs coefficients based on averaged strain rate tensor and rotation velocity invariant. Furthermore, second-order term describes anisotropic turbulence features. WJ EASM \( k-\omega \) model used in present research work is one of this type models which is reasonable to employ in turbulent flow involving large reverse pressure gradient and anisotropy. This paper presents the performances of WJ EASM \( k-\omega \) model, SA model and SST model in the two-dimensional compression corner flow.

### 2.3. Numerical Method

We use finite-volume approach to discretize the time-dependent governing equations which are written as a strong conservation form. The computational algorithm is the explicit, upwind-difference scheme with the convection terms discretized by Roe’s flux-difference-splitting technique [13], and with diffusion terms evaluated by the second-order central-difference scheme. ‘M.U.S.C.L.’ (Monotonic Upstream Scheme for Conversation Law) extrapolation technique is employed to maintain accuracy, monotonicity and robustness in cases of shockwave discontinuities. We determine the steady state solution of the differential equations for each computational cell by using five-stage Runge-Kutta marching algorithm, in which the Runge-Kutta coefficients are set equal to 1/4, 1/6, 3/8, 1/2, and 1/1, respectively.

### 3. Computational Results and Analysis

#### 3.1. Computational model
Present numerical simulation is based on an experiment of two-dimensional 24° compression corner carried out by Settles et. al in the supersonic tunnel at Princeton University. This experiment contains abundant and complex flow behaviors in which characteristic parameters are summarized as follows: incoming flow Mach number $M_\infty=2.84$, total pressure $P_0=6.89\times10^5\text{Pa}$, total temperature $T_0=262k$, wall temperature $T_w=276k$, characteristic boundary layer thickness $\delta=23\text{mm}$, and unit Reynolds number $Re=6.3\times10^7$. A $180\times110$ nonuniform Cartesian grid generated algebraically is employed in the calculations. The first grid interval perpendicular to the wall surface is $2.0\times10^{-7}\text{m}$ which guarantees $y^+<1.0$. Compression waves caused by supersonic incoming air currents impact to the compression ramp and assemble to shockwaves. Boundary layer develops much thicker, and means velocity is reduced behind the shockwave. Furthermore, adverse pressure gradient becomes much greater resulted in shockwave effect. Distinct separation flow behind the shockwave is appearing due to the large attack angle of 24°. Flow features can not spread to upperstram of supersonic flow field. On the contrary, large adverse pressure transfers flow information to upperstram in subsonic region of boundary layer. In this way, pressure gradient and boundary layer thickness become much greater, and shockwave located a new place to maintain equilibrium. Air flow backwards and form new compression region after reattachment. Fig.1 is a flow sketch map of two-dimensional compression corner flow under the supersonic incoming parameters. In addition, experimental values come from reference [4].

![Fig.1 The flow field of shock wave/turbulence boundary layer interaction](image)

### 3.2. Computational Results and Analysis

Some flow features such as separation, reattachment, shockwave location and wall surface friction are mostly concerned in the research work of shockwave turbulent boundary layer interaction. Thus, comparison between the computational results of three turbulence models and experimental values are shown as follows. By this means, the performances of turbulence models simulating shockwave turbulent boundary layer interaction will be illustrated.

#### 3.2.1. Flow Separation Region

Three models can obtain separation region in the present numerical simulation which illustrated these models have capability to predict shockwave turbulent boundary layer interaction flow in a certain extent. The locations of separation and reattachment are shown in the table 1.
Table 1 Separation locations and length

<table>
<thead>
<tr>
<th>Separation location</th>
<th>Reattachment location</th>
<th>Separation length</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exp.</td>
<td>-1.47</td>
<td>0.48</td>
</tr>
<tr>
<td>SA</td>
<td>-1.92</td>
<td>1.97</td>
</tr>
<tr>
<td>SST</td>
<td>-2.71</td>
<td>1.77</td>
</tr>
<tr>
<td>WJ</td>
<td>-2.10</td>
<td>0.98</td>
</tr>
</tbody>
</table>

Fig. 2 shows separation region around the corner. Three turbulence models represent common features of predicting separation point too early and predicting reattachment fall late. Comparatively speaking, SST is the worst to calculate separation point in three models and SA gives rise to computational result according with experimental value. For reattachment point, WJ is better than other two model. This three turbulence models are widely used in present fluid engineering field, but their performances for describing separation flow induced by shockwave turbulent boundary layer interaction are not satisfactory. The main reason is that the anisotropic turbulence viscosity and Reynolds stresses are not taken account. In a word, WJ is the best one of three turbulence models at predicting the locations of separation, reattachment and length of separation region.

3.2.2. Surface Pressure Distribution

The disagreement of surface pressure distribution calculated by SST and experimental values is most visible compared with two other models. Pressure rising location illustrates this appearance in Fig. 3. WJ predicts pressure influence location in according with experiment exactly. Simulation results of WJ and SA behind apex of corner are very similar and better than data coming from SST. Frankly speaking, pressure rising values of three turbulence model at ramp surface are less than experimental data given in reference [4]. Comprehensively analyzed, WJ and SA obtain the relative accurate shockwave location compared with SST. The reason is that advanced turbulence model such as SST (in comparison with \( k-\varepsilon \), \( k-\omega \)) always underestimates turbulent viscosity and causes pressure recovery later.

![Fig.2 Streamlines around separated area](image-url)
3.2.3. Wall Surface Friction Coefficients Distribution

Fig. 4 shows the comparison of wall surface friction resulted in three turbulence models and experimental values. SA presents better performance in simulating separation region before the apex of corner than the others. WJ predicts variation trends of the wall friction coefficients exactly although the values are different from experiment. The performance of SST in calculating wall friction coefficients is the worst among present three models. Researchers did not explain wall friction coefficients around the backflow region in their experiment, so the comparison between the numerical simulation and experiment can not be carried out in this region. It is certain that calculated results of wall friction based on these three models are very similar in the separation region before the apex of corner. WJ gives rise to minimum value of friction coefficient and makes friction variation trend go upward rapidly, which is according with experimental data proximately behind the reattachment point. The computational results of wall friction coefficients based on SA and SST are not satisfactory especially behind the reattachment point.

4. Conclusions

We studied shockwave turbulent boundary layer interaction flow induced by two-dimensional compression corner, which contains complex turbulent separation flow, large adverse pressure gradient and strong anisotropic turbulent viscosity in present paper. Three turbulence models exhibit different performances in predicting this type flow. Conclusions are summarized as follows:

(1) Separation point obtained by SA is according with one of WJ. But, SST predicts separation location too early. Three models did not calculate reattachment point exactly. Comparatively speaking, WJ gives rise to the best result among these turbulence models, SST the second, and SA the third.

(2) Shockwave locations predicted by SA and WJ satisfy experimental value perfectly. SST captures shockwave location shifting to upstream excessively.

(3) Before the separation point, wall friction coefficients given by SA are slightly greater than experimental data. Calculated results by using WJ and SST are according with experiment. Behind the reattachment point, we got the closest wall friction coefficients to experimental values using WJ, but SA shows the worst performance.
Three turbulence models used in present paper have capability to simulate complex separation flow induced by shockwave turbulent boundary layer interaction to a certain extent. SA is more suitable for separation region before apex of corner, but, displays very bad performance behind the reattachment point. WJ describes flow features well either separation or reattachment flow. SST did not obtain satisfactory results neither separation nor reattachment flow. We can draw a conclusion based on present research that WJ is prefer to SA and SST in calculating separation flow resulted in shockwave turbulent boundary layer interaction. For attachment flow, we can get good numerical simulation results using SA model.

References


