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On Barnette's conjecture

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ABSTRACT

Barnette's conjecture is the statement that every cubic 3-connected bipartite planar graph is Hamiltonian. We show that if such a graph has a 2-factor $\mathcal{F}$ which consists only of facial 4-cycles, then the following properties are satisfied:

(1) If an edge is chosen on a face and this edge is in $\mathcal{F}$, there is a Hamilton cycle containing all other edges of this face.
(2) If any face is chosen, there is a Hamilton cycle which avoids all edges of this face which are not in $\mathcal{F}$.
(3) If any two edges are chosen on the same face, there is a Hamilton cycle through one and avoiding the other.
(4) If any two edges are chosen which are an even distance apart on the same face, there is a Hamilton cycle which avoids both.

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1. Introduction

All graphs considered in this paper are finite and simple. We use [1] as reference for undefined terms.

Let $\mathcal{P}$ be the family of cubic 3-connected bipartite planar graphs. Barnette ([12], Problem 5) conjectured that every graph in $\mathcal{P}$ is Hamiltonian. In [4], Goodey proved that if a graph in $\mathcal{P}$ has only faces with 4 or 6 sides, then it is Hamiltonian. It is well known ([13], p. 38) that the faces of any graph in $\mathcal{P}$ can be colored with 3 colors (with adjacent faces having different colors). Feder and Subi [2] generalized Goodey's result by showing that when the faces of a graph in $\mathcal{P}$ are 3-colored, if two of the three color classes contain only faces with either 4 or 6 sides the conjecture holds. Kelmans [8] proved that Barnette's conjecture holds if and only if for any graph in $\mathcal{P}$ and for any two edges belonging to the same face of this graph, there is a Hamilton cycle through one and avoiding the other (see property (3) of the Abstract). Holton, Manvel and McKay [7] used computer search to confirm Barnette's conjecture for graphs up to 64 vertices, and also to confirm properties (3) and (4) of the Abstract for graphs up to 40 vertices. The problem whether a cubic bipartite planar graph has a Hamilton cycle (without the assumption of 3-connectivity) is NP-complete, as shown by Takanori, Takao, and Nobuji [11].

We know ([1], p. 96) that the dual graph P^* of a graph $P \in \mathcal{P}$ is unique. Let $\mathcal{E} = \mathcal{P}^* = \{P^* : P \in \mathcal{P}\}$. $\mathcal{E}$ is the family of all maximal Eulerian (i.e. every vertex is of even degree) planar graphs. Stein [10] (see also [6,9]) proved that $P \in \mathcal{P}$ is Hamiltonian if and only if P^* contains two disjoint induced trees which together cover all vertices of P^*. This yields the following dual form of Barnette's conjecture.

(B_1) *Every graph $G \in \mathcal{E}$ contains two disjoint induced trees which together cover all vertices of G.*

It is well known ([1], p. 101) that every maximal planar graph with at least four vertices is 3-connected. Hence, we may talk about the set $F(G)$ of faces of a graph $G \in \mathcal{E}$ without referring to any embedding of G ([1], p. 89). Note that every face in G is a 3-cycle. Let $V(G)$ be the vertex set of G. A set $U \subseteq V(G)$ is called a *cover of faces in G* if every face in G is incident with a vertex of U. It is easy to see that the following condition is equivalent to the condition (B_1).

(B_2) *Every graph $G \in \mathcal{E}$ contains an induced tree the vertex set of which is a cover of faces in G.*

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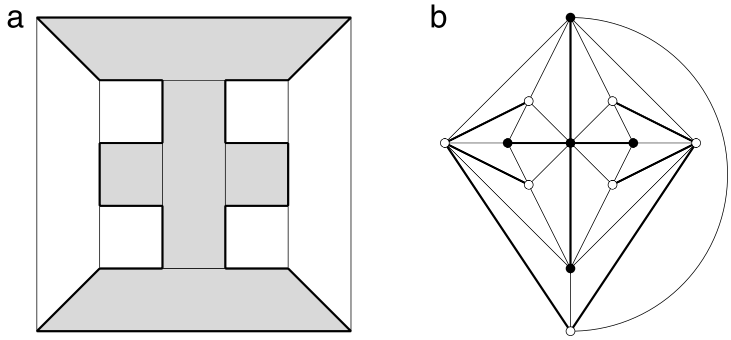

Fig. 1. (a) The graph P_0. (b) The graph P_0^*.

Fig. 1(a) shows an example of a Hamilton cycle in a graph $P_0 \in \mathcal{P}$ ([7], p. 296) as a boundary cycle of a tree of dark faces in the graph. Let E be the set of edges of the Hamilton cycle. Fig. 1(b) shows the dual graph $P_0^* \in \mathcal{E}$ in which E^* is a minimal cut ([1], p. 104) such that both two components of a subgraph $P_0^* - E^*$, obtained by deleting all the edges in E^*, are induced trees of P_0^*. The vertex set in either of the trees is a cover of faces in P_0^*. A similar approach of searching for induced trees in a sort of "dual graphs" instead of Hamilton cycles in graphs can also be used if one is searching for Hamilton cycles in graphs embedded into higher genus surfaces (see [3]).

Let $\mathcal{P}(4) \subset \mathcal{P}$ ($\mathcal{P}'(4) \subset \mathcal{P}$) be the family of graphs having a 2-factor which contains only facial cycles with 4 sides (one of size $2n$ and all others of size 4, respectively). In other words, when the faces of a graph in $\mathcal{P}(4)$ are 3-colored, then one of the three color classes contains only faces with 4 sides. Let $\mathcal{E}(4) = (\mathcal{P}(4))^*$ and $\mathcal{E}'(4) = (\mathcal{P}'(4))^*$. Hence $\mathcal{E}(4)$ is the family of all maximal Eulerian planar graphs with an independent set of vertices (i.e. no two of its elements are adjacent) which is a cover of faces and contains only vertices of degree 4. We prove (in the dual form) that every graph in $\mathcal{P}'(4)$ is Hamiltonian (see Theorem 3.2 for graphs in $\mathcal{E}'(4)$). Moreover, we prove (in the dual form) that every graph in $\mathcal{P}(4)$ satisfies properties (1) through (4) of the Abstract (see Theorems 3.1 and 3.3, as well as Corollaries 3.1 and 3.2 for graphs in $\mathcal{E}(4)$).

2. Construction of graphs of the family $\mathcal{E}(4)$

Assume that $G \in \mathcal{E}$ and let $V_4(G)$ denote the set of all vertices of degree 4 in G. A cycle C in G is called a *separating cycle*, if a subgraph $G - C$, obtained by deleting all the vertices in G and their incident edges, is disconnected.

Lemma 2.1. *Let $G \in \mathcal{E}'(4)$ and suppose that U is an independent vertex cover of $F(G)$ such that $U \setminus \{b\} \subseteq V_4(G)$ for some $b \in U$. If G is not 4-connected then $\deg b \geq 6$ and b is a vertex of a separating 3-cycle.*

Proof. Suppose that U is an independent vertex cover of $F(G)$ such that $U \setminus \{b\} \subseteq V_4(G)$ for some $b \in U$. It is well known that the vertices of G are 3-colored (with adjacent vertices having different colors). Therefore, U is one of the three color classes. If G is not 4-connected then there is a separating 3-cycle, say D, and one of its vertices, say u, belongs to U. If $\deg u = 4$ then there exists a vertex (adjacent to u) which is adjacent to every vertex of D. But this is impossible because the vertices of G are 3-colored. Hence $\deg u \geq 6$ and $u = b$. ■

Corollary 2.1. *Every graph $G \in \mathcal{E}(4)$ is 4-connected.*

For $U \subset V(G)$, the neighbors in $V \setminus U$ of vertices in U are called *neighbors* of U. If the induced graph $G[U]$ is connected, then G/U is a graph obtained from G by contracting the set U to a single vertex v_U, which becomes adjacent to all the former neighbors of U ([1], p. 19).

Suppose that $G \in \mathcal{E}(4)$ and that an independent set $U \subseteq V_4(G)$ is a cover of $F(G)$. For a path s, v, t, which is disjoint with U, we define an *operation* $\alpha = \alpha(s, v, t)$ in the following way : let $v_1, \ldots, v_n, v_1$ be a cycle induced by neighbors of v ($s = v_1$ and $t = v_k$). Replace the vertex v with a path x, u, y and join the vertices of this path with the former neighbors of v – provided that x is adjacent to $v_1, \ldots, v_k$, and u is adjacent to v_1 and v_k, and y is adjacent to $v_k, \ldots, v_n, v_1$. Let $\alpha(G)$ be the graph constructed from G by α and $\alpha(U) := U \cup \{u\}$. Since the set $\alpha(U) \subseteq V_4(\alpha(G))$ is independent and is a cover of $F(\alpha(G))$ we have $\alpha(G) \in \mathcal{E}(4)$. Hence, we have

(i) *The family $\mathcal{E}(4)$ is closed under the operation α.*

Suppose that $H \in \mathcal{E}(4)$ and that an independent set $U \subseteq V_4(H)$ is a cover of $F(H)$. Let $u \in U$ and s, x, t, y, s be a 4-cycle induced by neighbors of u. If u belongs to two different separating 4-cycles u, s, w, t, u and u, x, z, y, u, for some vertices w and z, then $w = z$. Hence, by Corollary 2.1, H is the octahedron. Otherwise we can assume that there is no separating 4-cycle containing the path x, u, y. By contracting the set $\{x, u, y\}$ to a single vertex v we obtain a graph $H/\{x, u, y\} \in \mathcal{E}(4)$ with an independent vertex cover $U \setminus \{u\}$. Notice that if $\alpha = \alpha(s, v, t)$ then $H = \alpha(H/\{x, u, y\})$ and $U = \alpha(U \setminus \{u\})$. Hence we have

(ii) *Every graph $G \in \mathcal{E}(4)$ can be constructed (up to isomorphism) from the octahedron by iterating the operation α.*

3. Main results

Recall that $\mathcal{E}(4) \subset \mathcal{E}'(4) \subset \mathcal{E}$ have been defined in Introduction. Note that the octahedron is the only graph in $\mathcal{E}$ with six vertices. If $G \in \mathcal{E}$ and if C is a separating induced cycle in G then $G - C$ consists of two components of G, say C^+ and C^-. Notice that any separating 4-cycle in a 4-connected graph is induced.

It is well known ([5], p. 272) the following result (in the dual form, that is, for cubic connected planar graphs).

(*) *There are no maximal planar graphs with two adjacent vertices (one vertex) of odd degree and all other of even degree.*

Lemma 3.1. *Let $G \in \mathcal{E}'(4)$, with $|V(G)| > 6$, be 4-connected and suppose that U is an independent vertex cover of $F(G)$ such that $U \setminus \{b\} \subseteq V_4(G)$ for some $b \in U$. If there is a separating 4-cycle containing a path a, b, c, then there is a separating 4-cycle $C = a, b, c, d, a$ with $d \notin U$. Moreover,*

(a) $G^+ := (G - C^+) + bd \in \mathcal{E}$ *and* $G^- := (G - C^-) + bd \in \mathcal{E}$,
(b) $U \setminus V(C^+) \subseteq V_4(G^+) \cup \{b\}$ *is an independent cover of* $F(G^+)$,
$U \setminus V(C^-) \subseteq V_4(G^-) \cup \{b\}$ *is an independent cover of* $F(G^-)$.

Proof. Let $C = a, b, c, d, a$ be a separating cycle. If $d \in U$ then $\deg d = 4$. Since C is induced there exist vertices $d_0 \in V(C^+) \setminus U$ and $d_1 \in V(C^-) \setminus U$ which are adjacent to c, d and a. Hence, a, b, c, d_0, a or a, b, c, d_1, a is a separating 4-cycle, because G is 4-connected and $|V(G)| > 6$. Let now $d \notin U$. Since $b \in U$, by condition (*), in the graph $G - C^+$ and $G - C^-$ the vertices a, b, c, d have even, odd, even, odd degrees, respectively. Hence $(G - C^+) + bd \in \mathcal{E}$ and $(G - C^-) + bd \in \mathcal{E}$. Since $b \in U$ and $d \notin U$ the condition (b) holds. ■

Theorem 3.1. *Let $G \in \mathcal{E}(4)$ and suppose that $U \subseteq V_4(G)$ is an independent vertex cover of $F(G)$. For every edge ab, $b \in U$, there is a set $X \subset V(G)$ which is a cover of $F(G)$, and the induced graph $G[X]$ is a tree containing both a and b and avoiding all other neighbors of a.*

Proof. We may assume that $|V(G)| > 6$. Fix an edge ab with $b \in U$. Let us consider the following cases:

(i) there is a separating 4-cycle containing the edge ab,
(ii) there is no separating 4-cycle containing the edge ab.

Case (i). By Corollary 2.1 and Lemma 3.1 there is a separating cycle $C = a, b, c, d, a$ such that $G^+ = (G - C^+) + bd \in \mathcal{E}(4)$ and $G^- = (G - C^-) + bd \in \mathcal{E}(4)$. Hence, by induction, there is a set $T^+ \subset V(G^+)$ ($T^- \subset V(G^-)$) which is a cover of $F(G^+)$ ($F(G^-)$, respectively) and the induced graph $G^+[T^+]$ ($G^-[T^-]$, respectively) is a tree containing a and b and avoiding all other neighbors of a. Notice that $c \in T^+ \cap T^-$. Since $a, b, c \in T^+ \cap T^-$ the set $X = T^+ \cup T^- \subset V(G)$ satisfies the condition of Theorem 3.1 for the graph G.
Case (ii). Let a, b, c, b_0 be an induced path in G with $b_0 \in U$. By contracting the set $\{a, b, c\}$ into a new vertex a_0 we obtain a graph $G_0 = G/\{a, b, c\} \in \mathcal{E}$. Since an independent set $U \setminus \{b\} \subseteq V_4(G_0)$ is a cover of $F(G_0)$ we have $G_0 \in \mathcal{E}(4)$. By induction there is a set $X_0 \subset V(G_0)$ which is a cover of $F(G_0)$ and the induced graph $G_0[X_0]$ is a tree containing a_0 and b_0 and avoiding all other neighbors of a_0. Hence, the set $X_0 \cup \{a, b, c\} \setminus \{a_0\} \subset V(G)$ satisfies the condition of Theorem 3.1 for the graph G. ■

Theorem 3.2. *Let $G \in \mathcal{E}'(4)$ and suppose that U is an independent vertex cover of $F(G)$ such that $U \setminus \{b\} \subseteq V_4(G)$ for some $b \in U$. If $W \subset V(G)$ is a maximum independent set of neighbors of b, there is a set $X \subset V(G)$ which is a cover of $F(G)$ such that the induced graph $G[X]$ is a tree containing the set $\{b\} \cup W$.*

Proof. If G is not 4-connected then by Lemma 2.1 there exists a separating (induced) 3-cycle D containing the vertex b. By condition (*), $G - D^+ \in \mathcal{E}'(4)$ and $G - D^- \in \mathcal{E}'(4)$. Hence, by induction, for graphs $G - D^+$ and $G - D^-$ the result follows. This yields Theorem 3.2 for the graph G.

Suppose that G is 4-connected. We may assume that $|V(G)| > 6$. Let W and Z be independent sets such that $W \cup Z$ is the set of all neighbors of b. Let us consider the following cases

(i) there is a separating 4-cycle containing an edge ab with $a \in W$,
(ii) there is no separating 4-cycle satisfying condition (i).

Case (i). By Lemma 3.1 there is a separating cycle $C = a, b, c, d, a$ such that $c \in W$, $G^+ = (G - C^+) + bd \in \mathcal{E}'(4)$ and $G^- = (G - C^-) + bd \in \mathcal{E}'(4)$. Hence, by induction, there is a set $T^+ \subset V(G^+)$ ($T^- \subset V(H^-)$) which is a cover of $F(G^+)$ ($F(G^-)$) and the induced graph $G^+[T^+]$ ($G^-[T^-]$) is a tree containing the set $\{b\} \cup W \setminus V(C^+)$ ($\{b\} \cup W \setminus V(C^-)$). Since $a, b, c \in T^+ \cap T^-$ the set $X = T^+ \cup T^- \subset V(G)$ satisfies the condition of Theorem 3.2 for the graph G.
Case (ii). By contracting the set $W \cup \{b\}$ into a new vertex b_0 we obtain a graph $G_0 = G/(W \cup \{b\}) \in \mathcal{E}$ (because G is 4-connected). Since an independent set $U \setminus \{b\} \subseteq V_4(G_0)$ is a cover of $F(G_0)$ we have $G_0 \in \mathcal{E}(4)$. By Theorem 3.1 there is a set $X_0 \subset V(G_0)$ which is a cover of $F(G_0)$ and the induced graph $G_0[X_0]$ is a tree containing b_0 and avoiding all vertices of the set Z. Hence, the set $X_0 \cup W \cup \{b\} \setminus \{b_0\} \subset V(G)$ satisfies the condition of Theorem 3.2 for the graph G. ■

Theorem 3.3. *Let $G \in \mathcal{E}(4)$ and suppose that $U \subseteq V_4(G)$ is an independent vertex cover of $F(G)$. For every vertex $a \notin U$, there is a set $X \subset V(G)$ which is a cover of $F(G)$ and the induced graph $G[X]$ is a tree containing a and all of its neighbors not belonging to U.*

Proof. We may assume that $|V(G)| > 6$. Fix a vertex $a \notin U$. Let us consider the following cases:

(i) there is a separating 4-cycle containing an edge ab with $b \in U$,
(ii) there is no separating 4-cycle satisfying condition (i).

Case (i). By Corollary 2.1 and Lemma 3.1 there is a separating cycle $C = a, b, c, d, a$ such that $d \notin U$, $G^+ = (G - C^+) + bd \in \mathcal{E}(4)$ and $G^- = (G - C^-) + bd \in \mathcal{E}(4)$. Hence, by induction, there is a set $T^+ \subset V(G^+)$ ($T^- \subset V(G^-)$) which is a cover of $F(G^+)$ ($F(G^-)$) and the induced graph $G^+[T^+]$ ($G^-[T^-]$) is a tree containing a and all neighbors of a not belonging to U. Notice that $c \notin T^+ \cup T^-$ because three other neighbors of b in G^+ (G^-) belong to T^+ (T^-). Since $a, d \in T^+ \cap T^-$ and $b, c \notin T^+ \cup T^-$ the set $X = T^+ \cup T^- \subset V(G)$ satisfies the condition of Theorem 3.3 for the graph G.
Case (ii). Let $N(a)$ be the set of all neighbors of a and $U(a) := N(a) \cap U$. By contracting the set $U(a) \cup \{a\}$ into a new vertex a_0 we obtain a graph $G_0 = G/(U(a) \cup \{a\}) \in \mathcal{E}$ (because G is 4-connected). Since an independent set $U \cup \{a_0\} \setminus U(a) \subset V(G_0)$ is a cover of $F(G_0)$ and $U \setminus U(a) \subseteq V_4(G_0)$ we have $G_0 \in \mathcal{E}'(4)$. By Theorem 3.2 there is a set $X_0 \subset V(G_0)$ which is a cover of $F(G_0)$ and the induced graph $G_0[X_0]$ is a tree containing the set $\{a_0\} \cup N(a) \setminus U(a)$. Hence, the set $X_0 \cup \{a\} \setminus \{a_0\} \subset V(G)$ satisfies the condition of Theorem 3.3 for the graph G. ■

Remark 3.1. Let $G = P_0^* \in \mathcal{E}(4)$ be the graph of Fig. 1(b). Assume that $U \subseteq V_4(G)$ is an independent vertex cover of $F(G)$, and let a denote the only vertex of degree 8 in the graph. Holton, Manvel, and McKay [7] noticed (in the dual form) that there is no set $X \subset V(G)$ which is a cover of $F(G)$ and the induced graph $G[X]$ is a tree containing a and all of its neighbors belonging to U.

Corollary 3.1. *Let $G \in \mathcal{E}(4)$. If a vertex $v \in V(G)$ and any two of its neighbors s, t are chosen then there is a set $X \subset V(G)$ which is a cover of $F(G)$ such that the induced graph $G[X]$ is a tree containing the vertices s, v and avoiding the vertex t.*

Proof. Let $U \subseteq V_4(G)$ be an independent vertex cover of $F(G)$. Suppose that $v \notin U$ and that $s \notin U$, $t \notin U$ are its two neighbors. By the operation $\alpha = \alpha(s, v, t)$ we obtain the graph $\alpha(G) \in \mathcal{E}(4)$, without the vertex v but with the new vertices x, u, y such that the independent set $U \cup \{u\} \subseteq V_4(\alpha(G))$ is a cover of $F(\alpha(G))$. By Theorem 3.3 there is a set $X_0 \subset V(\alpha(G))$ which is a cover of $F(\alpha(G))$ and the induced graph $\alpha(G)[X_0]$ is a tree containing the vertices x, s, y and avoiding the vertices u, t. Hence, $X = X_0 \cup \{v\} \setminus \{x, y\}$ is a cover of $F(G)$ and the induced graph $G[X]$ is a tree containing the vertices s, v and avoiding the vertex t.

All other cases follow from Theorem 3.1 or Theorem 3.3. ■

Theorem 3.4. *Let $G \in \mathcal{E}(4)$ and suppose that $U \subseteq V_4(G)$ is an independent vertex cover of $F(G)$. If any two neighbors $x, y \in U$ of a vertex $v \notin U$ are chosen then there is a set $X \subset V(G)$ which is a cover of $F(G)$ such that the induced graph $G[X]$ is a tree containing the vertices v, x, y and avoiding all neighbors of v not belonging to U.*

Proof. We may assume that $|V(G)| > 6$. Fix $v \notin U$ and two of its neighbors $x, y \in U$. If $\deg v = 4$ then the result follows from Theorem 3.1. Suppose that $\deg v > 4$. Let $b \in U \setminus \{x, y\}$ be a neighbor of the vertex v and v, a, w, c, v be a 4-cycle induced by neighbors of the vertex b. Let us consider the following cases:

(i) there is a separating 4-cycle containing the vertices a, b, c,
(ii) there is no separating 4-cycle satisfying condition (i).

Case (i). By Corollary 2.1 and Lemma 3.1 there is a separating cycle $C = a, b, c, d, a$ such that $G^+ = (G - C^+) + bd \in \mathcal{E}(4)$ and $G^- = (G - C^-) + bd \in \mathcal{E}(4)$. Without loss of generality we may assume that $\{x, v, y\} \subset V(G^+)$. By induction, there is a set $T^+ \subset V(G^+)$ which is a cover of $F(G^+)$ and the induced graph $G^+[T^+]$ is a tree containing the vertices x, v, y and avoiding all neighbors of v not belonging to U. Hence, $a, c \notin T^+$. By Theorems 3.2 and 3.3 there is a set $T^- \subset V(G^-)$ which is a cover of $F(G^-)$ and the induced graph $G^-[T^-]$ is a tree satisfying one of the following conditions:

(a) $a, d, c \notin T^-$ for $d \notin T^+$,
(b) $a, b, c \notin T^-$ for $b \notin T^+$,
(c) $a, b, c \notin T^-$ for $b, d \in T^+$.

Notice that $a, d, c \notin T^+ \cup T^-$ in the case (a), and $a, b, c \notin T^+ \cup T^-$ in the case (b). Notice also that in the case (c): $d \in T^-$ and $T^- \cup \{b\}$ induces a tree in the graph $G - C^-$. Hence, the set $X = T^+ \cup T^- \subset V(G)$ satisfies the condition of Theorem 3.4 for the graph G.
Case (ii). By contracting the set $\{a, b, c\}$ we obtain a graph $G_0 = G/\{a, b, c\} \in \mathcal{E}$. Since an independent set $U \setminus \{b\} \subseteq V_4(G_0)$ is a cover of $F(G_0)$ we have $G_0 \in \mathcal{E}(4)$. By induction there is a set $X_0 \subset V(G_0)$ which is a cover of $F(G_0)$ and the induced graph $G_0[X_0]$ is a tree containing the vertices v, x, y and avoiding all neighbors (in G_0) of v not belonging to U. Let us define:

$$X_1 = \begin{cases} X_0 \cup \{b\} & \text{for } w \notin X_0, \\ X_0 & \text{for } w \in X_0. \end{cases}$$

It is easy to see that the set $X_1 \subset V(G)$ satisfies the condition of Theorem 3.4 for the graph G. ■

With the help of Theorems 3.2–3.4 it is easy to see that the following corollary holds.

Corollary 3.2. *Let $G \in \mathcal{E}(4)$. If any two neighbors x, y of a vertex v are chosen which are an even distance apart on the circle induced by the set of neighbors of v then there is a set $X \subset V(G)$ which is a cover of $F(G)$ such that the induced graph $G[X]$ is a tree containing the vertices v, x, y.*

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