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# More on meta-stable brane configuration by quartic superpotential for fundamentals

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ABSTRACT

For the case where the gauge theory superpotential has a quartic term as well as the mass term for quarks, the nonsupersymmetric meta-stable brane configuration was found recently. By adding the orientifold 6-planes and the extra fundamental flavors to this brane configuration, we describe the meta-stable nonsupersymmetric vacua of the gauge theory with antisymmetric flavor as well as fundamental flavors in type IIA string theory.



## 1. Introduction

It is known that the dynamical supersymmetry breaking in meta-stable vacua [1,2] occurs in the standard $\mathcal{N}=1$ SQCD with massive fundamental flavors. The extra mass term for quarks in the superpotential implies that some of the F-term equations cannot be satisfied and then the supersymmetry is broken. The corresponding meta-stable brane realizations of type IIA string theory have been found in [3–5]. Very recently Giveon and Kutasov [6,7] have found the type IIA nonsupersymmetric meta-stable brane configuration where an additional quartic term for quarks in the superpotential is present. Geometrically, this extra deformation corresponds to the rotation of D6-branes along the (45)–(89) directions while keeping the other branes described in [3–5] unchanged. Classically there exist only supersymmetric ground states. By adding the orientifold 6-plane to this brane configuration [6], the meta-stable nonsupersymmetric vacua of the supersymmetric unitary gauge theory with symmetric flavor plus fundamental flavors is found [8].

Let us add an orientifold 6-plane and extra eight half D6-branes, located at the NS5′-brane, into the brane configuration of [6] together with an extra NS5-brane and the mirrors for both D4-branes and rotated D6-branes. According to the observation of [9–11], this "fork" brane configuration contains the NS5′-brane embedded in an O6-plane at $x^7=0$. This NS5′-brane divides the O6-plane into two separated regions corresponding to positive $x^7$ and negative $x^7$. Then RR charge of the O6-plane jumps from $-4$ to $+4$. Furthermore, eight semi-infinite D6-branes are present in the positive $x^7$ region. This is necessary for the vanishing of the six-dimensional anomaly. Then the type IIA brane configuration consists of two NS5-branes, one NS5′-brane, D4-branes, rotated D6-branes, an O6-plane and eight half D6-branes. We will see how the corresponding supersymmetric gauge theory, which is a standard $\mathcal{N}=1$ SQCD with massive flavors together with the extra matters, occurs in the context of dynamical supersymmetry breaking in meta-stable vacua.

In this Letter, we study $\mathcal{N}=1$ $SU(N_c)$ gauge theory with an antisymmetric flavor $A$, a conjugate symmetric flavor $\tilde{S}$, $N_f$ fundamental flavors $Q$ and $\tilde{Q}$ and eight fundamental flavors $\hat{Q}$ in the context of dynamical supersymmetric breaking vacua. Now we deform this theory by adding both the mass term and the quartic term for quarks $Q, \tilde{Q}$ in the fundamental representation of the gauge group [6]. Then we turn to the dual magnetic gauge theory [12]. The dual magnetic theory giving rise to the meta-stable vacua is described by $\mathcal{N}=1$ $SU(2N_f-N_c+4)$ gauge theory with dual matter contents. The difference between the brane configuration of [12] and the brane configuration of this Letter is that the D6-branes are rotated in the (45)–(89) directions. By analyzing the magnetic superpotential, along the line of [6,7], we present the behaviors of gauge theory description and string theory description for the meta-stable vacua.

In Section 2, the type IIA brane configuration corresponding to the electric theory based on the $\mathcal{N}=1$ $SU(N_c)$ gauge theory with above matter contents is given. In Section 3, we construct the Seiberg dual magnetic theory which is $\mathcal{N}=1$ $SU(2N_f-N_c+4)$ gauge theory with corresponding dual matters. The rotation of D6-branes is encoded in the mass term for the meson field in the superpotential. In Section 4, the nonsupersymmetric meta-stable minimum is found and the corresponding intersecting brane configuration of type IIA string theory is presented. In Section 5, we comment on the future works.

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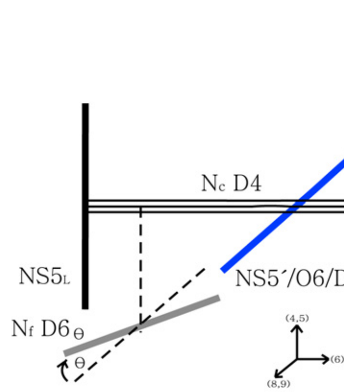


**Fig. 1.** The $\mathcal{N}=1$ supersymmetric electric brane configuration with deformed superpotential (2.1) for the $SU(N_c)$ gauge theory with an antisymmetric flavor $A$, a conjugate symmetric flavor $\tilde{S}$, eight fundamentals $\hat{Q}$, and $N_f$ fundamental massive flavors $Q,\tilde{Q}$. The origin of the coordinates $(x^6, v, w)$ is located at the intersection of $x^6$ and O6-plane. It is evident that the two deformations are characterized by both translation and rotation for D6-branes. As in [12], a combination of a middle NS5′-brane, O6$^+$-plane, O6$^-$-plane and eight half D6-branes is represented by NS5′/O6/D6.

## 2. The $\mathcal{N}=1$ supersymmetric electric brane configuration

The type IIA supersymmetric electric brane configuration [9–12] corresponding to $\mathcal{N}=1$ $SU(N_c)$ gauge theory with an antisymmetric flavor $A$, a conjugate symmetric flavor $\tilde{S}$, eight fundamental flavors $\hat{Q}$ and $N_f$ fundamental flavors $Q,\tilde{Q}$ [13] can be described as follows: one middle NS5′-brane(012389), two NS5-branes(012345) denoted by NS5$_L$-brane and NS5$_R$-brane respectively, $N_c$ D4-branes(01236) between them, $2N_f$ D6-branes(0123789), an orientifold 6 plane(0123789) of positive RR charge, an orientifold 6 plane(0123789) of negative RR charge and eight half D6-branes. The transverse coordinates $(x^4, x^5, x^6)$ transform as $(-x^4, -x^5, -x^6)$ under the orientifold 6-plane(O6-plane) action. Let us introduce two complex coordinates [14]

$$v \equiv x^4 + ix^5, \qquad w \equiv x^8 + ix^9.$$

Then the origin of the coordinates $(x^6, v, w)$ is located at the intersection of $x^6$ coordinate and O6-plane. The left NS5$_L$-brane is located at the left-hand side of O6-plane while the right NS5$_R$-brane is located at the right-hand side of O6-plane. The $N_c$ color D4-branes are suspended between NS5$_L$-brane and NS5$_R$-brane. Moreover the $N_f$ D6-branes are located between the NS5$_L$-brane and the middle NS5′-brane and its mirrors $N_f$ D6-branes are located between the middle NS5′-brane and the NS5$_R$-brane. The antisymmetric and conjugate symmetric flavors $A$ and $\tilde{S}$ are 4–4 strings stretching between D4-branes located at the left-hand side of O6-plane and those at the right-hand side of O6-plane, $N_f$ fundamental flavors $Q$ and $\tilde{Q}$ are strings stretching between $N_f$ D6-branes and $N_c$ color D4-branes and eight fundamental flavors $\hat{Q}$ are strings stretching between eight half D6-branes which are on top of O6$^-$-plane and $N_c$ color D4-branes.

Let us deform this theory which has vanishing superpotential by adding both the mass term and the quartic term for $N_f$ fundamental quarks. The former can be achieved by "translating" the D6-branes along $\pm v$ direction leading to their coordinates $v = \pm v_{D6}$ [14] while the latter can be obtained by "rotating" the D6-branes [6] by an angle $\theta$ in $(w, v)$-plane. We denote them by D6$_\theta$-branes which are at angle $\theta$ with undeformed unrotated D6-branes (0123789). Then their mirrors $N_f$ D6-branes are rotated by an angle $-\theta$ in $(w, v)$-plane according to O6-plane action and we denote them also by D6$_{-\theta}$-branes.[1] Then, in the electric gauge theory, the deformed superpotential is given by

$$W_{\text{elec}} = \frac{\alpha}{2}\,\text{tr}(Q\tilde{Q})^2 - m\,\text{tr}\,Q\tilde{Q} - \frac{1}{2\mu}\big[(A\tilde{S})^2 + Q\tilde{S}A\tilde{Q} + (Q\tilde{Q})^2\big] + \hat{Q}\tilde{S}\hat{Q}, \quad \text{with } \alpha = \frac{\tan\theta}{\Lambda},\ m = \frac{v_{D6}}{2\pi\ell_s^2}, \tag{2.1}$$

where $\Lambda$ is related to the scales of the electric and magnetic theories and $\pm v_{D6}$ is the $v$ coordinate of D6$_{\mp\theta}$-branes. Due to the last term, the flavor symmetry $SU(N_f+8)_L$ is broken to $SU(N_f)_L \times SO(8)_L$. Here the adjoint mass $\mu \equiv \tan(\frac{\pi}{2} - \omega)$ is related to a rotation angle $\omega$ of NS5$_{L,R}$-branes in $(w, v)$-plane. In the limit of $\mu \to \infty$ (or no rotations of NS5-branes $\omega \to 0$), the terms of $\frac{1}{\mu}$ in (2.1) vanish.

Let us summarize the $\mathcal{N}=1$ supersymmetric electric brane configuration with nonvanishing superpotential (2.1) in type IIA string theory as follows and draw it in Fig. 1:

- Two NS5-branes in (012345) directions with $w = 0$.
- One NS5′-brane in (012389) directions with $v = 0 = x^6$.
- $N_c$ color D4-branes in (01236) directions with $v = 0 = w$.
- $N_f$ D6$_{\pm\theta}$-branes in (01237) directions and two other directions in $(v, w)$-plane.
- Eight half D6-branes in (0123789) directions with $x^6 = 0 = v$.
- O6$^\pm$-planes in (0123789) directions with $x^6 = 0 = v$.

By moving the D6$_{\pm\theta}$-branes from Fig. 1 into the outside of NS5$_{L,R}$-branes, there exist $N_f$ flavor D4-branes connecting D6$_{\pm\theta}$-branes and the NS5$_{L,R}$-branes, and the gauge singlet field $N$ appears. At energies much below the mass of $N$, the two brane descriptions coincide with each other. One can think of this new brane configuration as integrating the field $N$ in from Fig. 1 and the superpotential of this

[1] Note that the convention for D6$_\theta$-branes in [12] was such that the angle between unrotated D6-branes and D6$_\theta$-branes was not $\theta$ but $(\frac{\pi}{2} - \theta)$.

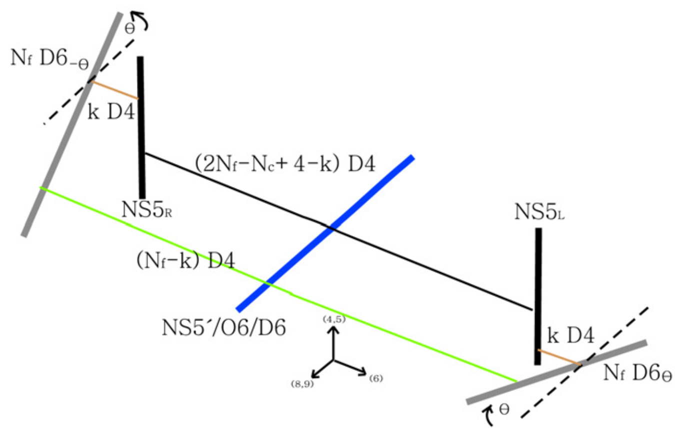


**Fig. 2.** The $\mathcal{N}=1$ supersymmetric magnetic brane configuration for the $SU(2N_f-N_c+4-k)$ gauge theory with an antisymmetric flavor $a$, a conjugate symmetric flavor $\tilde{s}$, $N_f$ fundamental flavors $q, \tilde{q}$ and eight fundamentals $\hat{q}$. The $N_f$ flavor D4-branes connecting between NS5$_L$-brane and D6$_\theta$-branes are related to the dual gauge singlet $M$ and are splitting into $(N_f-k)$ and $k$ D4-branes. The location of intersection between D6$_\theta$-branes and $(N_f-k)$ D4-branes is given by $(v, w)=(0, v_{\mathrm{D6}}\cot\theta)$ while the one between D6$_\theta$-branes and $k$ D4-branes is given by $(v, w)=(-v_{\mathrm{D6}}, 0)$.

electric theory contains the interaction term between $N$ with electric quarks, quadratic term and linear term for $N$ [6]. The classical supersymmetric vacua of this brane configuration are characterized by the parameter $k$ where $k=0,1,\ldots,N_c$ and unbroken gauge symmetry in the $k$th configuration is $SU(N_c-k)$. That is, the $k$ D4-branes among $N_f$ D4-branes (stretched between NS5$_R$-brane and D6$_{-\theta}$-branes) are reconnecting with those number of D4-branes stretched between the middle NS5′-brane and NS5$_R$-brane. Then those resulting $k$ D4-branes are moving to $\pm v$ direction and the remaining $(N_c-k)$ D4-branes are stretching between the middle NS5′-brane and NS5$_R$-brane and $(N_f-k)$ D4-branes are stretched between the NS5$_R$-brane and D6$_{-\theta}$-branes (and their mirrors).

## 3. The $\mathcal{N}=1$ supersymmetric magnetic brane configuration

The magnetic theory is obtained by interchanging the D6$_{\pm\theta}$-branes and NS5$_{L,R}$-branes while the linking number is preserved. After one moves the left D6$_\theta$-branes to the right all the way (and their mirrors, right D6$_{-\theta}$-branes to the left) past the middle NS5′-brane and the right NS5$_R$-brane, the linking number counting [12] implies that one should add $N_f$ D4-branes, corresponding to the meson $M(\equiv Q\tilde{Q})$, to the left side of all the right $N_f$ D6$_\theta$-branes (and their mirrors). Note that when a D6-brane crosses the middle NS5′-brane, due to the parallelness of these, there was no creation of D4-branes. That is, when the D6$_{\pm\theta}$-branes approach the middle NS5′-brane, one should take $\theta=0$ limit (making D6-branes to be parallel to the middle NS5′-brane) and then after they cross the middle NS5′-brane, they return to the original positions given by D6$_{\pm\theta}$-branes as follows:

$$\text{D6}_{\pm\theta}\text{-branes} \quad\rightarrow\quad \text{D6-branes} \quad\rightarrow\quad \text{D6}_{\pm\theta}\text{-branes}. \tag{3.1}$$

Next, let us move the left NS5$_L$-brane to the right all the way past O6-plane (and its mirror, right NS5$_R$-brane to the left), and then the linking number counting [12] leads to the fact that the dual number of colors was $(2N_f-N_c+4)$. There was a creation of D4-branes when the NS5-brane crosses an O6-plane because they are not parallel to each other. From this, the constant term 4 in the dual color above arises, compared with the case of [15]. Now we draw the magnetic brane configuration in Fig. 2 where some of the flavor D4-branes are recombined with those of $(2N_f-N_c+4)$ color D4-branes and those combined resulting D4-branes are moved into $\pm v$ direction. One takes $k$ D4-branes from $N_f$ flavor D4-branes and reconnect them to those from $(2N_f-N_c+4)$ color D4-branes in Fig. 2 such that the resulting branes are connecting from the D6$_\theta$-branes to the NS5$_L$-brane directly. Their coordinates between D6$_\theta$-branes and $k$ D4-branes will be $v=-v_{\mathrm{D6}}$ in order to minimize the energy. This Fig. 2 also can be obtained from the magnetic brane configuration of [6] by adding O6-planes and eight half D6-branes with the right presence of mirrors under the O6-plane action.

Then the low energy dynamics is described by the dual magnetic theory with gauge group $SU(2N_f-N_c+4)$ and this theory is higgsed down to $SU(2N_f-N_c+4-k)$ in the $k$th vacuum by nonzero vacuum expectation value for dual quarks which is determined later. The matters are $N_f$ flavors of fundamentals $q, \tilde{q}$, an antisymmetric flavor $a$, a conjugate symmetric flavor $\tilde{s}$, eight fundamentals $\hat{q}$, gauge singlet $M$ which is magnetic dual of the electric meson field $Q\tilde{Q}$ and other gauge singlet $\tilde{M}$ that is $\hat{Q}\tilde{Q}$. Then the superpotential including the interaction between the meson field $M$ and dual matters with $\mu\rightarrow\infty$ $(\omega\rightarrow 0)$ limit is described by

$$W_{\mathrm{mag}} = \frac{1}{\Lambda} Mq\tilde{s}a\tilde{q} + \frac{\alpha}{2}\operatorname{tr} M^2 - m \operatorname{tr} M + \hat{q}\tilde{s}\hat{q} + \tilde{M}\hat{q}\tilde{q}, \quad M\equiv Q\tilde{Q}, \quad \tilde{M}\equiv\hat{Q}\tilde{Q}, \tag{3.2}$$

where the second and third terms originate from the two deformations (2.1), and the fourth term also comes from electric theory. The $\theta$-dependent coefficient function, $\alpha$, in front of quadratic term of the meson field also occurs in the geometric brane interpretation for the different supersymmetric gauge theory [16]. The $\alpha=0$ limit reduces to the theory given by [12]. The parameters $\alpha$ and $m$ are the same as before in (2.1).[2]

[2] For arbitrary rotation angles of D6$_{\pm\theta}$-branes and NS5$_{\pm\omega}$-branes, there exist also other meson fields containing $A$ or $\tilde{S}$: $M_1\equiv Q\tilde{S}A\tilde{Q}$, $P\equiv Q\tilde{S}Q$ and $\tilde{P}\equiv\tilde{Q}A\tilde{Q}$. They couple to the dual quarks and other flavors in the superpotential via $M_1 q\tilde{q} + Pq\tilde{s}q + \tilde{P}\tilde{q}a\tilde{q}$. As emphasized in [12], the geometric constraint (3.1) at the intersection between D6-branes and the middle NS5′-brane removes the presence of these gauge singlets, $M_1$, $P$ and $\tilde{P}$. That is, when D6$_{\pm\theta}$-branes are crossing the middle NS5′-brane, they do not produce any D4-branes because they are parallel at the origin $x^6=0$. This implies there is no $M_1$ term in the magnetic superpotential. In general, $P$ and $\tilde{P}$-terms arise when D6$_\theta$-branes intersect with its mirrors D6$_{-\theta}$-branes around the origin $x^6=0$. But they are also parallel to each other due to (3.1). This leads to the fact that there are no $P$ or $\tilde{P}$-terms in the magnetic superpotential. Therefore, we are left with (3.2).

From the magnetic superpotential (3.2), the supersymmetric vacua are obtained and the F-term equations are given as follows:

$$\tilde{s}a\tilde{q}M = 0, \qquad a\tilde{q}Mq + \hat{q}\hat{\tilde{q}} = 0, \qquad \tilde{q}Mq\tilde{s} = 0, \qquad Mq\tilde{s}a + \tilde{M}\hat{q} = 0, \qquad \tilde{s}\hat{q} + \tilde{q}\tilde{M} = 0, \qquad \hat{q}\tilde{q} = 0, \qquad \frac{1}{\Lambda}q\tilde{s}a\tilde{q} = m - \alpha M. \tag{3.3}$$

By multiplying $M$ into the last equation with $\hat{q} = 0 = \tilde{M}$, the matrix equation is satisfied $mM = \alpha M^2$. Because the eigenvalues are either 0 or $\frac{m}{\alpha}$, one can take $N_f \times N_f$ matrix with $k$'s eigenvalues 0 and $(N_f - k)$'s eigenvalues $\frac{m}{\alpha}$:

$$M = \begin{pmatrix} 0 & 0 \\ 0 & \frac{m}{\alpha}\mathbf{1}_{N_f-k} \end{pmatrix}, \tag{3.4}$$

where $k = 1, 2, \ldots, N_f$ and $\mathbf{1}_{N_f-k}$ is the $(N_f - k) \times (N_f - k)$ identity matrix [7]. The expectation value of $M$ is represented by the fundamental string between the flavor brane displaced by the $w$ direction and the color brane from Fig. 2. The $k$ of the $N_f$ flavor D4-branes are connected with $k$ of $(2N_f - N_c + 4)$ color D4-branes and the resulting D4-branes stretch from the D6$_\theta$-branes to the NS5$_L$-brane and the coordinate of an intersection point between the $k$ D4-branes and the NS5$_L$-brane is given by $(v, w) = (-v_{D6}, 0)$. This corresponds to exactly the $k$'s eigenvalues 0 of $M$ above. Now the remaining $(N_f - k)$ flavor D4-branes between the D6$_\theta$-branes and the NS5$'$-brane are related to the corresponding eigenvalues of $M$ given by $\frac{m}{\alpha}\mathbf{1}_{N_f-k}$. The coordinate of an intersection point between the $(N_f - k)$ D4-branes and the NS5$'$-brane is given by $(v, w) = (0, v_{D6}\cot\theta)$. Note that using the expressions for $\alpha$ and $m$ from (2.1), one obtains $\frac{m}{\alpha} = \Lambda \frac{v_{D6}\cot\theta}{2\pi\ell_s^2}$ which must be $\Lambda \frac{w}{2\pi\ell_s^2}$. Then $w = v_{D6}\cot\theta$.

Substituting (3.4) into the last equation of (3.3) leads to

$$q\tilde{s}a\tilde{q} = \begin{pmatrix} m\Lambda\mathbf{1}_k & 0 \\ 0 & 0 \end{pmatrix}. \tag{3.5}$$

Since the rank of the left-hand side is at most $2N_f - N_c + 4$, one must have more stringent bound $k \leqslant (2N_f - N_c + 4)$. In the $k$th vacuum the gauge symmetry is broken to $SU(2N_f - N_c + 4 - k)$ and the supersymmetric vacuum drawn in Fig. 2 with $k = 0$ has $q\tilde{s} = a\tilde{q} = 0$ and the gauge group $SU(2N_f - N_c + 4)$ is unbroken. The expectation value of $M$ in this case is given by $M = \frac{m}{\alpha}\mathbf{1}_{N_f} = m\Lambda\cot\theta\mathbf{1}_{N_f}$ explicitly. By moving the D6-branes into the place between the middle NS5$'$-brane and the NS5$_{L,R}$-branes, one obtains other brane configuration. There exist $(N_f - k)$ flavor D4-branes connecting D6$_{\pm\theta}$-branes and the NS5$_{L,R}$-branes after this movement.

Another deformation arises when we rotate the NS5$_{L,R}$-branes by an angle $\pm\theta'$ in the $(v, w)$-plane. This rotation provides an adjoint field of $SU(2N_f - N_c + 4)$ and couples to the magnetic dual matters. Integrating this adjoint field out leads to the fact that there exists a further contribution to the quartic superpotential for $q$ and $\tilde{q}$. In particular, when the rotated NS5$_{\pm\theta'}$-branes are parallel to rotated D6$_{\pm\theta}$-branes, the coupling in front of $M^2$ in the magnetic superpotential vanishes.

## 4. Nonsupersymmetric meta-stable brane configuration

The theory has many nonsupersymmetric meta-stable ground states besides the supersymmetric ones we discussed in previous section. For the IR free region [12], the magnetic theory is the effective low energy description of the asymptotically free electric gauge theory. When we rescale the meson field as $M = h\Lambda\Phi$, then the Kähler potential for $\Phi$ is canonical and the magnetic quarks are canonical near the origin of field space. The higher order corrections of Kähler potential are negligible when the expectation values of the fields $q, \tilde{q}, a, \tilde{s}$ and $\Phi$ are smaller than the scale of magnetic theory. Then the magnetic superpotential can be written in terms of $\Phi$ (or $M$)

$$W_{\text{mag}} = h\Phi q\tilde{s}a\tilde{q} + \frac{h^2\mu_\phi}{2}\operatorname{tr}\Phi^2 - h\mu^2\operatorname{tr}\Phi + \hat{q}\tilde{s}\hat{q} + \tilde{M}\hat{q}\tilde{q}.$$

From this, one can read off the following quantities

$$\mu^2 = m\Lambda, \qquad \mu_\phi = \alpha\Lambda^2, \qquad M = h\Lambda\Phi.$$

The classical supersymmetric vacua given by (3.4) and (3.5) can be described similarly and one decomposes the $(N_f - k) \times (N_f - k)$ block at the lower right corner of $h\Phi$ and $q\tilde{s}a\tilde{q}$ into blocks of size $n$ and $(N_f - k - n)$ as follows:

$$h\Phi = \begin{pmatrix} 0 & 0 & 0 \\ 0 & h\Phi_n & 0 \\ 0 & 0 & \frac{\mu^2}{\mu_\phi}\mathbf{1}_{N_f-k-n} \end{pmatrix}, \qquad q\tilde{s}a\tilde{q} = \begin{pmatrix} \mu^2\mathbf{1}_k & 0 & 0 \\ 0 & \varphi\tilde{\beta}\gamma\tilde{\varphi} & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Here $\varphi$ and $\tilde{\varphi}$ are $n \times (2N_f - N_c + 4 - k)$-dimensional matrices and correspond to $n$ flavors of fundamentals of the gauge group $SU(2N_f - N_c + 4 - k)$ which is unbroken by the nonzero expectation value of $q\tilde{s}$ and $a\tilde{q}$. In Fig. 3, they correspond to fundamental strings connecting the $n$ flavor D4-branes and $(2N_f - N_c + 4 - k)$ color D4-branes. This Fig. 3 can be also obtained from the meta-stable brane configuration of [6] by adding O6-planes and eight half D6-branes with appropriate mirrors under the O6-plane action. The antisymmetric and conjugate symmetric flavors $\gamma$ and $\tilde{\beta}$ are 4–4 strings stretching between $(2N_f - N_c + 4 - k)$ D4-branes located at the left-hand side of O6-plane and those at the right-hand side of O6-plane in Fig. 3. Both $\Phi_n$ and $\varphi\tilde{\beta}\beta\tilde{\varphi}$ are $n \times n$ matrices. The supersymmetric ground state corresponds to the vacuum expectation values given by $h\Phi_n = \frac{\mu^2}{\mu_\phi}\mathbf{1}_n$, $\varphi\tilde{\beta} = 0 = \gamma\tilde{\varphi}$.

Now the full one loop potential for $\Phi_n, \hat{\varphi} \equiv \varphi\tilde{\beta}, \hat{\tilde{\varphi}} \equiv \gamma\tilde{\varphi}$ [12] takes the form

$$\frac{V}{|h|^2} = |\Phi_n\hat{\varphi}|^2 + |\Phi_n\hat{\tilde{\varphi}}|^2 + \left|\hat{\varphi}\hat{\tilde{\varphi}} - \mu^2\mathbf{1}_n + h\mu_\phi\Phi_n\right|^2 + b|h\mu|^2\operatorname{tr}\Phi_n^\dagger\Phi_n,$$

where $b$ is given by $b = \frac{(\ln 4 - 1)}{8\pi^2}(2N_f - N_c + 4)$. Differentiating this potential with respect to $\Phi_n$ and putting $\hat{\varphi} = 0 = \hat{\tilde{\varphi}}$, one obtains

$$h\Phi_n = \frac{\mu^2\mu_\phi^*}{|\mu_\phi|^2 + b|\mu|^2}\mathbf{1}_n \simeq \frac{\mu^2\mu_\phi^*}{b|\mu|^2}\mathbf{1}_n \quad \text{or} \quad M_n \simeq \frac{\alpha\Lambda^3}{(2N_f - N_c + 4)}\mathbf{1}_n \tag{4.1}$$

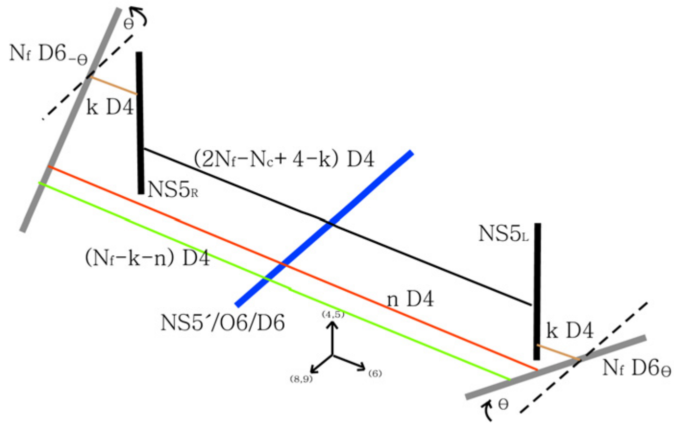


**Fig. 3.** The nonsupersymmetric minimal energy brane configuration for the $SU(2N_f - N_c + 4 - k)$ gauge theory with an antisymmetric flavor $a$, a conjugate symmetric flavor $\tilde{s}$, $N_f$ fundamental flavors $q, \tilde{q}$ and eight fundamentals $\hat{q}$. This brane configuration is obtained by moving $n$ flavor D4-branes, from $(N_f - k)$ flavor D4-branes stretched between the NS5′-brane and the $D6_\theta$-branes in Fig. 2 (and their mirrors).

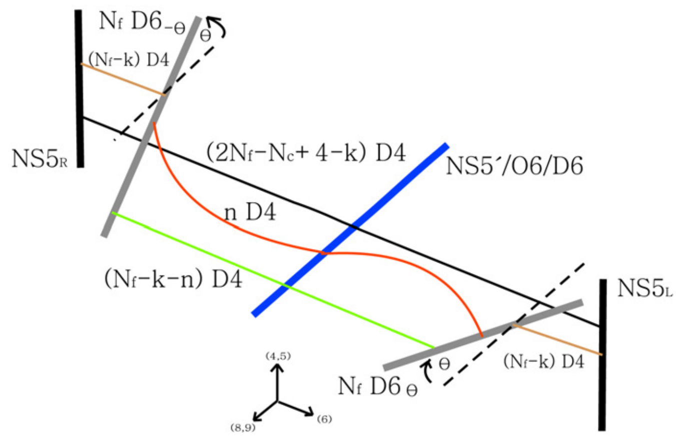


**Fig. 4.** The nonsupersymmetric minimal energy brane configuration for the $SU(2N_f - N_c + 4 - k)$ gauge theory with an antisymmetric flavor $a$, a conjugate symmetric flavor $\tilde{s}$, $N_f$ fundamental flavors $q, \tilde{q}$ and eight fundamentals $\hat{q}$. This brane configuration can be obtained by moving $N_f$ $D6_{\pm\theta}$-branes into the place between the $NS5_{L,R}$-branes and the middle NS5′-brane in Fig. 3 (and their mirrors). Note that there exists a creation of $(N_f - k)$ D4-branes connecting the $D6_\theta$-branes and the $NS5_L$-brane (and their mirrors).

for real $\mu$ and we assume here that $\mu_\phi \ll \mu \ll \Lambda_m$. The vacuum energy $V$ is given by $V \simeq n|h\mu^2|^2$. Expanding around this solution, one obtains the eigenvalues for mass matrix for $\hat{\varphi}$ and $\hat{\tilde{\varphi}}$ will be

$$m_\pm^2 = \frac{|\mu|^4}{(|\mu_\phi|^2 + b|\mu|^2)^2}\big[|\mu_\phi|^2 \pm b|h|^2\big(|\mu_\phi|^2 + b|\mu|^2\big)\big] \simeq \frac{1}{b^2}\big(|\mu_\phi|^2 \pm |bh\mu|^2\big).$$

Then for $|\frac{\mu_\phi}{\mu}|^2 > \frac{|bh|^2}{1-b|h|^2} \simeq |bh|^2$ in order to avoid tachyons the vacuum (4.1) is locally stable.

One can move $n$ D4-branes, from $(N_f - k)$ D4-branes stretched between the NS5′-brane and the $D6_\theta$-branes at $w = v_{D6}\cot\theta$, to the local minimum of the potential and the end points of these $n$ D4-branes are at a nonzero $w$ as in Fig. 3. The remaining $(N_f - k - n)$ flavor D4-branes between the $D6_\theta$-branes and the NS5′-brane are related to the corresponding eigenvalues of $h\Phi$, i.e., $\frac{\mu^2}{\mu_\phi}\mathbf{1}_{N_f-k-n}$. The coordinate of an intersection point between the $(N_f - k - n)$ D4-branes and the NS5′-brane is given by $(v, w) = (0, v_{D6}\cot\theta)$. Finally, the remnant $n$ "curved" flavor D4-branes between the $D6_\theta$-branes and the NS5′-brane are related to the corresponding eigenvalues of $h\Phi_n$ by (4.1). Note that since $\frac{\mu^2\mu_\phi^*}{b|\mu|^2} \ll \frac{\mu^2}{\mu_\phi}$, the $n$ D4-branes are nearer to the $w = 0$ located at the $NS5_L$-brane.

As explained in [6], this local stable vacuum decays to the supersymmetric ground states. The end points of $n$ "curved" flavor D4-branes on the NS5′-brane approach those of the $(2N_f - N_c + 4 - k)$ color D4-branes and two types of branes reconnect each other. For $n \leqslant (2N_f - N_c + 4 - k)$, the final brane configuration is nothing but the supersymmetric vacuum of Fig. 2 with the replacement $k \to (k + n)$. When $n > (2N_f - N_c + 4 - k)$, then the remnant $[n - (2N_f - N_c + 4 - k)]$ flavor D4-branes remain. On the other hand, the $n$ D4-branes can move to larger $w$ and return to the Fig. 2. Also some of the D4-branes approach the intersection point between $D6_\theta$-branes and the NS5′-brane while the remaining D4-branes move to the one between $D6_\theta$-branes and the $NS5_L$-brane.

When $D6_{\pm\theta}$-branes are moved to the place between the NS5′-brane and the $NS5_{L,R}$-branes, one gets the Fig. 4 where the previous $(N_f - k)$ D6-branes in Fig. 3 that were not connected to the $NS5_{L,R}$-branes, through the flavor D4-branes, are now connecting to the $NS5_{L,R}$-branes by the same number of D4-branes while the previous $k$ D6-branes in Fig. 3 that were connected to the $NS5_{L,R}$-branes, through the flavor D4-branes, are now not connecting to the $NS5_{L,R}$-branes. The former corresponds to a creation of D4-branes and the latter corresponds to an annihilation of D4-branes due to the Hanany–Witten transition [14,17].

In the remaining part, we focus on the gravitational potential of the $NS5_L$-brane. Let us remind that the branes are placed as follows:

$$D6_\theta\text{-branes}(01237vw):\quad v=-v_{D6}+w\tan\theta,$$
$$NS5'\text{-brane}(012389):\quad v=0,$$
$$NS5_L\text{-brane}(012345):\quad w=0,$$

where we assume that $D6_\theta$-branes are located at the nonzero

$$y\equiv x^6$$

and the $NS5'$-brane is located at $y=0$ (the origin). The dependence of the distance between the $D6_\theta$-branes and $NS5'$ brane along the $NS5_L$-brane on $w$ can be represented by

$$\Delta x=|-v_{D6}+w\tan\theta|. \tag{4.2}$$

Then the partial differentiation of $\Delta x$ with respect to $w$ leads to an extra contribution for the computation of $\partial_w\theta_{i,w}$ where we define $\theta_{i,w}$ as

$$\theta_{1,w}\equiv\cos^{-1}\left(\frac{y_m}{|w|}\right),\qquad \theta_{2,w}\equiv\cos^{-1}\left(\frac{y_m}{\sqrt{y^2+|w|^2}}\right), \tag{4.3}$$

with $y_m$ that is smallest value of $y$ along the D4-brane. Then the energy density of the D4-brane was computed in [18,19] and is given by

$$E(w)=\frac{\tau_4}{2\ell_s}\sqrt{1+\frac{\ell_s^2}{y_m^2}}\left[|w|^2\sin 2\theta_{1,w}+\left(y^2+|w|^2\right)\sin 2\theta_{2,w}\right] \tag{4.4}$$

corresponding to (3.7) of [6] where $\tau_4$ is a tension of D4-brane in flat spacetime.

It is straightforward to compute the differentiation of $(\frac{\ell_s E(w)}{\tau_4})^2$ with respect to $w$ and it leads to

$$\begin{aligned}\partial_w\left(\frac{\ell_s E(w)}{\tau_4}\right)^2&=\ell_s^2\bar{w}\left(\sin^2\theta_{1,w}+\sin^2\theta_{2,w}+\left[\frac{\sqrt{y^2+|w|^2}}{|w|}+\frac{|w|}{\sqrt{y^2+|w|^2}}\right]\sin\theta_{1,w}\sin\theta_{2,w}\right)+\left[|w|^2\sin 2\theta_{1,w}+\left(y^2+|w|^2\right)\sin 2\theta_{2,w}\right]\\&\quad\times\left[\left(\ell_s^2+|w|^2\cos 2\theta_{1,w}\right)\partial_w\theta_{1,w}+\left(\ell_s^2+\left(y^2+|w|^2\right)\cos 2\theta_{2,w}\right)\partial_w\theta_{2,w}+\frac{\bar{w}}{2}\left(\sin 2\theta_{1,w}+\sin 2\theta_{2,w}\right)\right].\end{aligned} \tag{4.5}$$

In order to simplify this, one uses the partial differentiation of $\Delta x$ (4.2) with respect to $w$ which is equal to $\frac{1}{2}\tan\theta\frac{\bar{w}\bar{\tan}\theta-\bar{v_{D6}}}{|w\tan\theta-v_{D6}|}$. On the other hand, $\Delta x$ was known in [18] and it is

$$\Delta x=\frac{1}{2\ell_s}\left[|w|^2\sin 2\theta_{1,w}+\left(y^2+|w|^2\right)\sin 2\theta_{2,w}\right]+\ell_s(\theta_{1,w}+\theta_{2,w}). \tag{4.6}$$

After differentiating this (4.6) with respect to $w$ and equating it to the previous expression obtained from (4.2), one arrives at

$$\left(\ell_s^2+|w|^2\cos 2\theta_{1,w}\right)\partial_w\theta_{1,w}+\left[\ell_s^2+\left(y^2+|w|^2\right)\cos 2\theta_{2,w}\right]\partial_w\theta_{2,w}+\frac{\bar{w}}{2}(\sin 2\theta_{1,w}+\sin 2\theta_{2,w})=\ell_s\frac{1}{2}\tan\theta\frac{\bar{w}\bar{\tan}\theta-\bar{v_{D6}}}{|w\tan\theta-v_{D6}|} \tag{4.7}$$

corresponding to (3.21) of [6]. Now by putting this relation (4.7) into the second and third lines of (4.5), one gets

$$\begin{aligned}\partial_w\left(\frac{\ell_s E(w)}{\tau_4}\right)^2&=\ell_s^2\bar{w}\left(\sin^2\theta_{1,w}+\sin^2\theta_{2,w}+\left[\frac{\sqrt{y^2+|w|^2}}{|w|}+\frac{|w|}{\sqrt{y^2+|w|^2}}\right]\sin\theta_{1,w}\sin\theta_{2,w}\right)\\&\quad+\left[|w|^2\sin 2\theta_{1,w}+\left(y^2+|w|^2\right)\sin 2\theta_{2,w}\right]\frac{1}{2}\ell_s\tan\theta\frac{\bar{w}\bar{\tan}\theta-\bar{v_{D6}}}{|w\tan\theta-v_{D6}|}.\end{aligned} \tag{4.8}$$

It is easy to see that at $w=v_{D6}\cot\theta$ which is an intersection point between $D6_\theta$-branes and $NS5'$-brane, $\Delta x$ vanishes through (4.2) and this also implies that $\theta_{i,w}$ vanishes from (4.6). Furthermore, the energy $E(w)$ is zero from (4.4). This corresponds to the global minimal energy. For the parallel D6-branes and $NS5'$-brane (i.e., $\tan\theta=0$), then the only stationary point is $w=0$ [12]. If $w\neq 0$, then $\sin\theta_{1,w}=0=\sin\theta_{2,w}$ from the first term of (4.8) but these are not physical solutions.

For real and positive parameters $v_{D6}, w$ and $\tan\theta$, we are looking for the solution with $v_{D6}>w\tan\theta$ and setting the right-hand side of (4.8) to zero, finally one gets with (4.3)

$$\frac{\sin^2\theta_{1,w}+\sin^2\theta_{2,w}+(\frac{\sqrt{y^2+w^2}}{w}+\frac{w}{\sqrt{y^2+w^2}})\sin\theta_{1,w}\sin\theta_{2,w}}{w^2\sin 2\theta_{1,w}+(y^2+w^2)\sin 2\theta_{2,w}}=\frac{\tan\theta}{2\ell_s w}. \tag{4.9}$$

Therefore, the brane configuration of Fig. 4 has a local minimum where the end of D4-brane are located at $w$ given by (4.9). When the $y$ goes to zero ($\theta_{1,w}=\theta_{2,w}\equiv\theta_w$), one can approximate (4.9) and one gets

$$\tan\theta_w\simeq\frac{w\tan\theta}{2\ell_s}.$$

The gauge theory result is valid only when $\theta$ and $\frac{v_{D6}}{\ell_s}$ are much smaller than $g_s$ while the classical brane construction with (4.9) is valid for any angle and the length parameters are of order $\ell_s$ or larger.

## 5. Conclusions and outlook

In this Letter, by adding the orientifold 6-planes and the extra fundamental flavors to the brane configuration [6], we have described the meta-stable nonsupersymmetric vacua of the gauge theory with antisymmetric flavor as well as fundamental flavors, through the Figs. 3 and 4, from type IIA string theory.

It would be interesting to deform the theories given in [19–23] where one of the gauge group factor has the same matter contents as the one of the present Letter and see how the meta-stable ground states appear. Along the lines of [18,19,21,23,24], when the $D6_\theta$-branes are replaced by a single $NS5_\theta$-brane, it would be interesting to see how the present deformation arises in these theories. It is also possible to deform the symplectic or orthogonal gauge group theory with massive flavors [25] by adding an orientifold 4-plane. Similar application to the product gauge group case [26–28] is also possible to study. It is an open problem to see how the type IIB description [29] is related to the present work. To construct a direct gauge mediation [30,31] for the present work is also possible open problem.

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