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Energy analysis and criteria for structural failure of rocks

Heping Xie[1,2*], Liyun Li[1,2], Ruidong Peng[2], Yang Ju[2]

[1] *Sichuan University, Chengdu, 610065, China*

[2] *State Key Laboratory of Coal Resources and Safe Mining, China University of Mining and Technology, Beijing, 100083, China*

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Abstract: The intrinsic relationships between energy dissipation, energy release, strength and abrupt structural failure are key to understanding the evolution of deformational processes in rocks. Theoretical and experimental studies confirm that energy plays an important role in rock deformation and failure. Dissipated energy from external forces produces damage and irreversible deformation within rock and decreases rock strength over time. Structural failure of rocks is caused by an abrupt release of strain energy that manifests as a catastrophic breakdown of the rock under certain conditions. The strain energy released in the rock volume plays a pivotal role in generating this abrupt structural failure in the rocks. In this paper, we propose criteria governing (1) the deterioration of rock strength based on energy dissipation and (2) the abrupt structural failure of rocks based on energy release. The critical stresses at the time of abrupt structural failure under various stress states can be determined by these criteria. As an example, the criteria have been used to analyze the failure conditions of surrounding rock of a circular tunnel.

Key words: energy dissipation; energy release; strength deterioration; structural failure; breakage size

1 Introduction

Descriptions of rock strength and deformational behavior are fundamental to evaluate the stability of engineering structures located in rock. For a long time, strength criteria based on classical elastoplastic theory have been used to judge whether rock-mounted engineering structures would fail [1–5]. However, it is difficult to assess strength criteria that depend only on judgments of stress-strain behavior. This is because stress-strain relationships of rocks are generally nonlinear and scale-dependent due to the fact that the interior structures of rocks may be extremely inhomogeneous, as well as the exterior load of rocks may be complex. The stress-strain behavior of a rock, describing it specific mechanical state, is only one aspect of a rock's thermodynamic state. The deformation and failure of rock are irreversible process involving energy dissipation. During the process, the application of external forces changes the stress and strain distribution within the rock, while at the same time some of the dissipated energy may produce damage in the rock [6, 7]. The damage state of the rock, in turn, has an effect on the stress and strain in the rock. Locally high stress and strain can cause strength deteriorations but not structural failures. It is only when strain energy is released completely and suddenly that a rock will rupture (i.e. structurally fail). The nature of such a structural failure is quite different under static or dynamic loading. From a thermodynamic viewpoint, energy conversion is an essential physical process, and it can be inferred that the rupture of the rock is the final result of an energy-driven destabilization process. An improved understanding of the rules of energy conversion and the relationship between strength deterioration and structural failure is needed. Recent literatures support this work and use energy analysis to investigate the deformation and failure of rocks [8–25].

Our work aims to analyze the energy conversion that occurs during the deformation and failure of rock elements. Furthermore, we investigate the essential physical relationship between energy dissipation and strength deterioration as well as the relationship between energy release and structural failure. We will show that rock damage is caused by energy dissipation, which results in strength deterioration, and structural failure is caused by the release of elastic energy stored in rocks. By means of (1) the definition of a damage variable based on energy, (2) a strength deterioration criterion based on energy dissipation, and (3) a

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*Corresponding author. Tel: +86-10-62331253; E-mail: xiehp@scu.edu.cn

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structure failure criterion based on energy release, we will determine the critical stresses present at the time of abrupt structural failure of rock under various stress states. These energy criteria are applied to the practical example of a circular tunnel and to the analysis of the failure conditions of the surrounding rock of tunnel. This paper establishes a framework to facilitate the analysis of strength and structural failure of rock based on energy dissipation and energy release principles.

2 Experimental studies of energy release during rock failure

2.1 Rock samples and test scheme

To investigate the relationship between rock failure and related energy conversion, a series of uniaxial compression tests were performed on rock specimens at Beijing Key Laboratory of Fracture and Damage Mechanics of Rocks and Concrete. The Shimadzu EHF-EG 200 kN Full-Digital Servohydraulic Testing System was used. It features loading control and data acquisition that are both executed automatically with real-time displays of measurement load and displacement (the stroke of the actuator) and the corresponding load-displacement. Granite, limestone and sandstone samples were collected from an open-cast mine at Pingshuo in Shanxi Province. To maintain test precision, all samples were carefully prepared from cylinder drill cores with two parallel end surfaces perpendicular to the core sample axes. The samples for each type of rocks have different sizes: ϕ 25 mm × 50 mm for granite, ϕ 50 mm × 50 mm for limestone, and ϕ 50 mm × 100 mm for sandstone. At the beginning of each test, the initial load is 1.5 kN, which results in full contact between the sample and the compression plate. Compression was maintained until the sample broke. The load was controlled in velocity-controlled mode and the loading speed is 0.001 mm/s. The data sampling interval was 1 s.

2.2 Experimental results and discussion

Figure 1 shows stress-strain curves for rock samples under uniaxial compression, in which stress was calculated according to the axial load F divided by the cross-sectional area A of the sample and strain was determined by the displacement L divided by the height h of the sample. Figures 2–4 show the failure modes of rock samples. The three-digit sample notation is as follows. The first digit of the rock sample number indicates rock type: 1 for granite, 2 for limestone and 3 for sandstone. The second digit indicates the dimension of the rock sample: L for 25 mm × 50 mm, M for 50 mm × 50 mm and S for 50 mm× 100 mm. The third digit indicates the serial number of the sample with the same

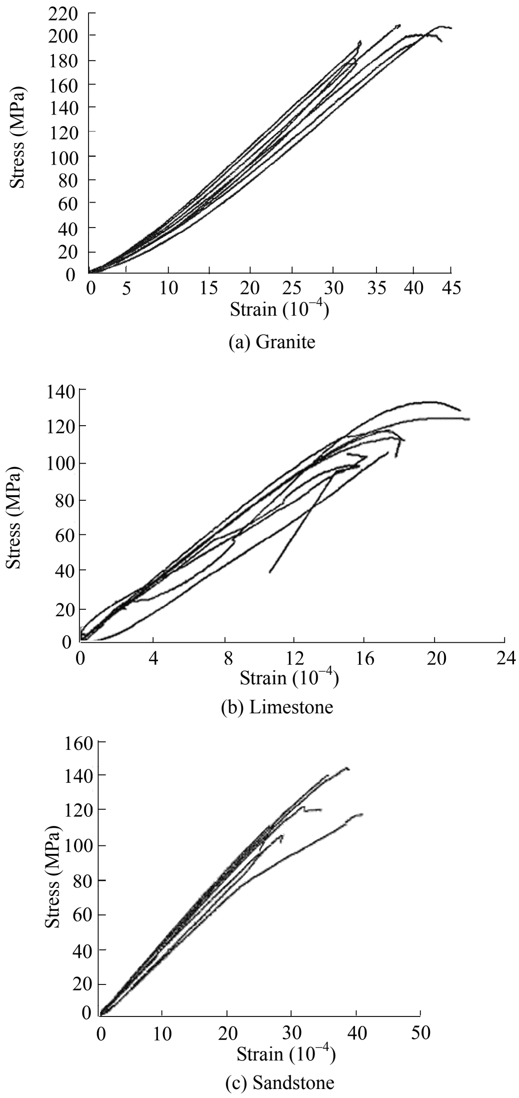


Fig.1 Stress-strain curves of rock samples under uniaxial compression.

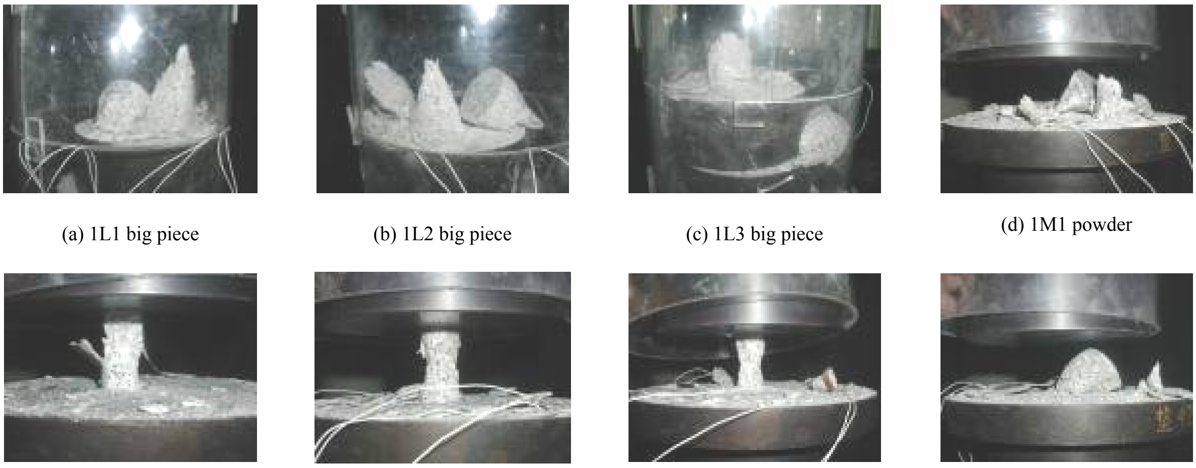
rock type and dimension. Table 1 lists the test results for all samples of different rock types and dimensions. The total work of the external load W is calculated by the integral of the load-displacement curve, which is

$$W = \int_l F \mathrm{d}L \tag{1}$$

The total work of the external load is converted to the elastic energy E_s accumulated in the testing machine system and the energy U absorbed by the rock sample due to deformation and failure [26]. Assuming that the rock samples are intact and without joints, the absorbed energy of a unit-volume rock sample e is given by

$$e = \frac{U}{V} = \frac{W - E_\mathrm{s}}{V} = \frac{\int_l F\,\mathrm{d}L - E_\mathrm{s}}{\frac{1}{4}\pi d^2 h} \tag{2}$$

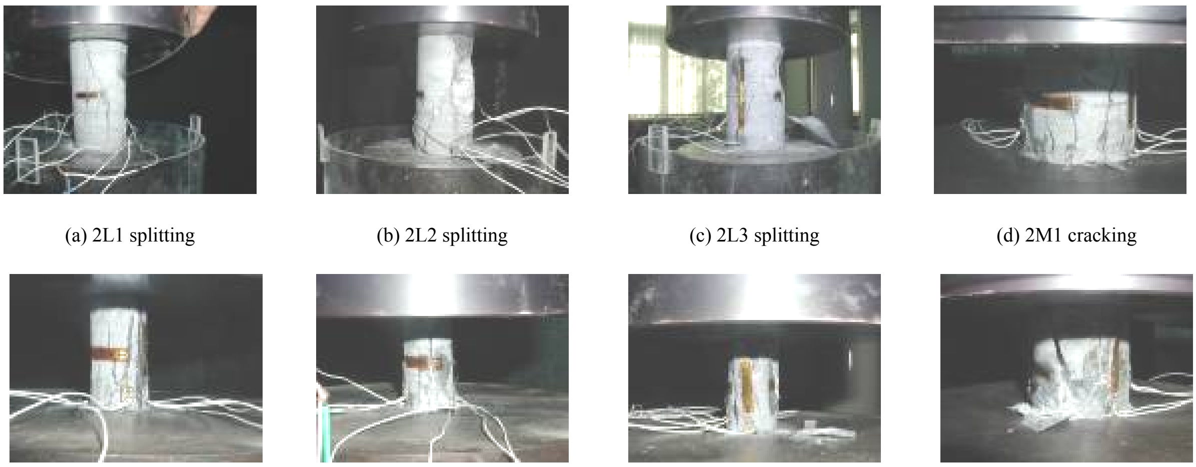
where V is the volume of the rock sample, and d is the diameter of rock sample. E_s can be calculated according to the rigidity curve of the testing machine system [26].



(a) 1L1 big piece (b) 1L2 big piece (c) 1L3 big piece (d) 1M1 powder

(e) 1S1 small piece (f) 1S2 small piece (g) 1S3 small piece (h) 1M2 powder

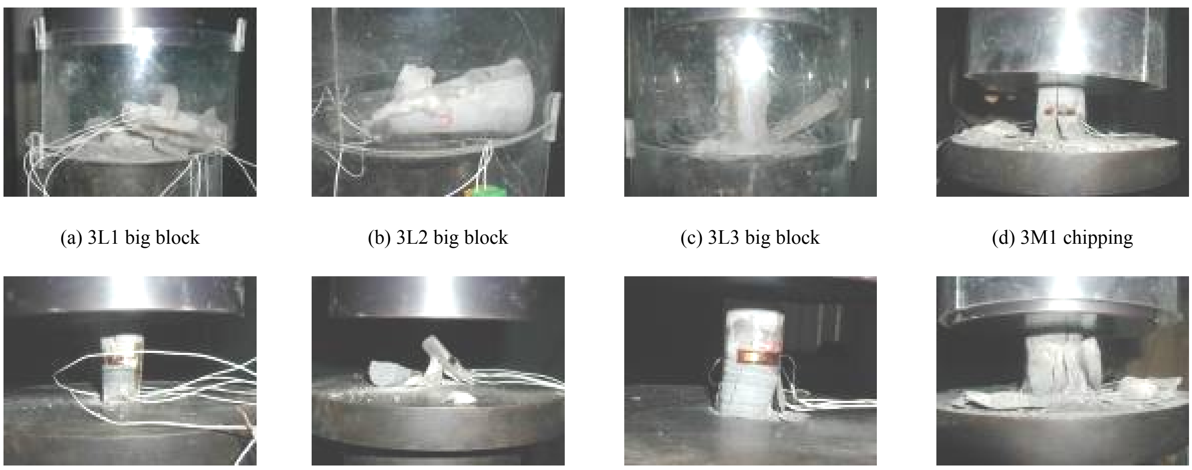
Fig.2 Failure modes of granite samples under uniaxial compression.



(a) 2L1 splitting (b) 2L2 splitting (c) 2L3 splitting (d) 2M1 cracking

(e) 2S1 cracking (f) 2S2 splitting (g) 2S3 cracking (h) 2M2 cracking

Fig.3 Failure modes of limestone samples under uniaxial compression.



(a) 3L1 big block (b) 3L2 big block (c) 3L3 big block (d) 3M1 chipping

(e) 3S1 cracking (f) 3S2 small block (g) 3S3 cracking (h) 3M2 chipping

Fig.4 Failure modes of sandstone samples under uniaxial compression.

Table 1 Energy and failure modes of rock samples under uniaxial compression.

Rock samples	Compressive strength σ_c (MPa)	Failure strain ε_c (10^{-6})	Total work W/(J)	Absorbed energy of unit-volume rock sample e (mJ/mm^3)	Failure modes: Types	Failure modes: Descriptions
1M2	199	2 019	189.74	0.86	Powder	Most of broken samples are powders
1M1	187	3 317	167.79	0.68	Powder	
1S2	201	4 357	23.36	0.61	Small piece	Broken samples are mainly small pieces and lots of powders
1S3	210	3 807	23.03	0.56	Small piece	
1S1	193	4 061	21.39	0.51	Small piece	
1L2	182	3 310	169.55	0.49	Big piece	Broken samples are mainly large pieces and lots of powders
1L3	209	4 486	216.56	0.46	Big piece	
1L1	195	3 380	192.23	0.45	Big piece	
3M1	120	2 124	89.11	0.49	Chipping	Broken samples are still upstanding with lots of powders
3M2	118	4 088	76.11	0.41	Chipping	
3S2	144	3 908	15.10	0.39	Small block	Broken samples are mainly a few small blocks
3L1	140	3 583	115.45	0.30	Big block	Broken samples are mainly a few big blocks
3L3	123	3 478	94.75	0.29	Big block	
3L2	122	1 526	96.06	0.28	Big block	
2M2	110	1 520	71.58	0.34	Cracking	Broken sample is still upstanding with a few blocks
2M1	113	1 753	63.89	0.24	Cracking	
2S1	117	1 823	9.29	0.25	Cracking	
2S3	133	2 168	10.34	0.25	Cracking	
3S1	106	2 874	9.50	0.23	Cracking	
3S3	79	1 831	6.60	0.19	Cracking	
2L2	118	1 764	76.62	0.19	Splitting	Broken samples are mainly split into big blocks
2L3	124	2 219	79.45	0.18	Splitting	
2S2	99	1 594	6.76	0.17	Splitting	
2L1	113	1 855	65.91	0.13	Splitting	

It can be seen (Figs.1–4) that the stress-strain curves under uniaxial compressions for all the rock samples are almost uniform, but their failure modes are very different. This implies that rock failure cannot be described well by stress and strain. For different sample dimensions (Figs.2–4), the absorbed energy of each unit-volume rock sample is distinct, which suggests a unique relationship between the rock failure mode and the absorbed energy for each group of rock samples. Table 1 lists the compressive strength and the absorbed energy of unit-volumes of the rock samples grouped by failure mode. Note that when more energy was absorbed by a unit-volume rock sample, the sample was broken into more pieces. This is because more energy needs to be dissipated or released, resulting in the development of more cracks and fragments. Because no certain relationship between compressive strength and rock failure mode can be defined, rock failure can only be described adequately from the viewpoint of energy.

3 Damage strength criteria for rocks based on energy dissipation

The deformation of a rock element under an external load can be considered a closed system assuming (1) that there is no heat conversion from mechanical work and (2) that the total energy U produced due to the work done by the external load can be calculated according to the first law of thermodynamics as

$$U = U^{\mathrm{d}} + U^{\mathrm{e}} \tag{3}$$

where U^{d} is dissipated energy, and U^{e} is releasable elastic strain energy. U^{d} results in internal damage and irreversible plastic deformation in the rock. U^{e} is related directly to the elastic modulus and Poisson's ratio during unloading. Figure 5 illustrates a typical stress-strain curve of rock. It can be spread into a 3D stress situation, in which the lighter dotted area under the stress-strain curve represents the dissipated energy, and the darker spotted area represents the releasable elastic strain energy stored in rocks. Energy dissipation is unidirectional and irreversible, whereas energy release is bidirectional and reversible under certain condition.

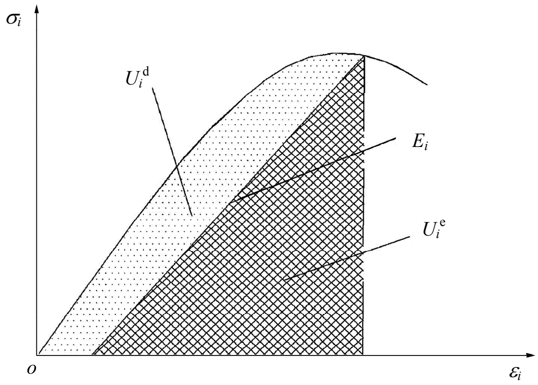


Fig.5 Relationship between dissipated energy and releasable strain energy of a rock element.

Energy dissipation continually damages the internal microscopic structure of rock, which leads to a deterioration in rock strength and eventually complete structural failure. In this paper, the deterioration in the strength of a rock element is defined as the cohesion it has lost after a certain energy dissipation when the damage extent reaches its critical value.

Under general stress conditions, the energy of rock elements meets the following relation:

$$U^{\mathrm{d}} = U - U^{\mathrm{e}} \tag{4}$$

Each part of the rock element energy in the principal stress space [27] can be calculated as

$$U = \int_0^{\varepsilon_1} \sigma_1 \mathrm{d}\varepsilon_1 + \int_0^{\varepsilon_2} \sigma_2 \mathrm{d}\varepsilon_2 + \int_0^{\varepsilon_3} \sigma_3 \mathrm{d}\varepsilon_3 \tag{5}$$

$$U^{\mathrm{e}} = \frac{1}{2}\sigma_1\varepsilon_1^{\mathrm{e}} + \frac{1}{2}\sigma_2\varepsilon_2^{\mathrm{e}} + \frac{1}{2}\sigma_3\varepsilon_3^{\mathrm{e}} \tag{6}$$

$$\varepsilon_i^{\mathrm{e}} = \frac{1}{E_i}[\sigma_i - \nu_i(\sigma_j + \sigma_k)] \tag{7}$$

where ε_i (i = 1, 2, 3) is the total strain in the three principal stress directions, $\varepsilon_i^{\mathrm{e}}$ is the related elastic strain, E_i is the unloaded elastic modulus, and ν_i is the unloaded Poisson's ratio. Then the damage variable based on energy of each rock element D can be defined as

$$D = \frac{U^{\mathrm{d}}}{U^{\mathrm{c}}} \tag{8}$$

where U^{c} is the critical energy dissipation corresponding to the strength deterioration. U^{c} is a material curve dependent on the stress situation. It can be approximately determined by uniaxial tension, uniaxial compression and pure shear tests. In practice, when dissipated energy U^{d} in a rock element reaches its critical value U^{c}, that is:

$$D = \frac{U^{\mathrm{d}}}{U^{\mathrm{c}}} = 1 \tag{9}$$

strength deterioration must occur. Substituting Eq.(9) into Eq.(4), recalling Eqs.(5) and (6), and using the Einstein summation convention, Eq.(9) can then be expressed as

$$U^{\mathrm{c}} = \int_0^{\varepsilon_i} \sigma_i \mathrm{d}\varepsilon_i - \frac{1}{2}\sigma_i\varepsilon_i^{\mathrm{e}} \tag{10}$$

Equation (10) is the criterion of strength deterioration for rock element based on energy dissipation.

In engineering studies, a rock mass or structure is usually discretized into many rock elements and then the energy dissipation of each rock element can be calculated by a nonlinear finite element method. Doing this, a forecast of the distribution of rock element damage can be made based on Eq.(10).

It should be emphasized that the strength deterioration of a rock element is different from the structural failure of the rock element. Damage must result in the strength deterioration, but does not necessarily lead to structural failure. For example, the strength deterioration of rock can occur under conditions of both uniform tension and uniform compression. However, a rock element under uniform tension may more easily experience structural failure, whereas a rock element under uniform compression may not fail as easily due to the effect of confining pressure. A complete structural failure of a rock element under uniform compression can only occur when the confining pressure in at least one direction is unloaded.

4 Structural failure criteria for rocks based on energy release

According to Eqs.(6) and (7), the releasable elastic strain energy U^{e} stored in rock elements is related to the elastic modulus E_i and Poisson's ratio ν_i of damaged rock elements during unloading. First, it was assumed that the damage of rock elements is orthogonally anisotropic. The releasable elastic energy of rock elements can be given as

$$U^{\mathrm{e}} = \frac{1}{2}\sigma_i\varepsilon_i^{\mathrm{e}} = \frac{1}{2}\left\{\left(\frac{\sigma_1^2}{E_1} + \frac{\sigma_2^2}{E_2} + \frac{\sigma_3^2}{E_3}\right) - \left[\left(\frac{1}{E_1} + \frac{1}{E_2}\right)\sigma_1\sigma_2 + \left(\frac{1}{E_2} + \frac{1}{E_3}\right)\sigma_2\sigma_3 + \left(\frac{1}{E_1} + \frac{1}{E_3}\right)\sigma_1\sigma_3\right]\right\} \tag{11}$$

To express the damage effect, a set of general damage variables ω_i is used on the unloaded elastic moduli E_i of the rock element:

$$E_i = (1 - \omega_i)E_0 \tag{12}$$

where E_0 is the initial elastic modulus of undamaged rock elements. Assuming that Poisson's ratio ν is not affected by damage, substituting E_i into Eq.(11) leads to

$$U^{\mathrm{e}} = \frac{1}{2}\sigma_i\varepsilon_i^{\mathrm{e}} = \frac{1}{2E_0}\left\{\frac{\sigma_1^2}{1-\omega_1} + \frac{\sigma_2^2}{1-\omega_2} + \frac{\sigma_3^2}{1-\omega_3} - \nu\left[\left(\frac{1}{1-\omega_1} + \frac{1}{1-\omega_2}\right)\sigma_1\sigma_2 + \left(\frac{1}{1-\omega_2} + \frac{1}{1-\omega_3}\right)\sigma_2\sigma_3 + \left(\frac{1}{1-\omega_1} + \frac{1}{1-\omega_3}\right)\sigma_1\sigma_3\right]\right\} \tag{13}$$

In engineering practice, rocks can be considered as pseudo-isotropic materials. This enables a simplification of Eq.(13). If the average effect of damage along the three principal stresses directions is considered, the E_i can be defined as one:

$$\overline{E} = E_0(1-\overline{\omega}), \quad \omega_i = \overline{\omega} \quad (i = 1,\ 2,\ 3)$$

where the average value $\overline{E}$ can be determined by a cyclic uniaxial compressive loading and unloading test. Then the releasable elastic strain energy U^{e} can be written as

$$U^{\mathrm{e}} = \frac{1}{2\overline{E}}[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3)] \tag{14}$$

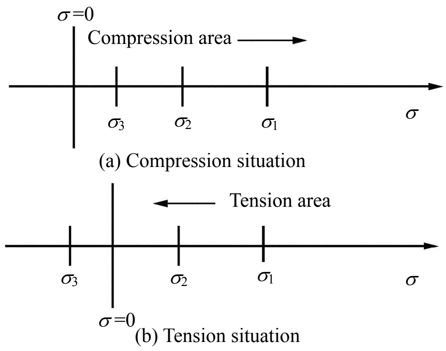
Equation (14) is convenient for engineering applications. Furthermore, if the initial elastic modulus E_0 is used as an approximation, then Eq.(14) becomes

$$U^{\mathrm{e}} = \frac{1}{2E_0}[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3)] \tag{15}$$

It should be noted that Eq.(15) indicates a linear unloading process for nonlinear rock elements. It is different from the energy calculation in linear elastic mechanics, although they have the same formula style. The releasable strain energy of an entire rock mass or structure can be calculated by the summation of U_i^{e} in each rock element as $\sum U_i^{\mathrm{e}}$. With the onset of rock damage, strength deterioration of some rock elements becomes more severe. If the releasable strain energy U_i^{e} in a certain rock element reaches a critical value, the rock element will break. If a sufficient number of rock elements break at the same time, the structural failure of the entire rock mass will occur.

Based on the ideas above, a process for the structural failure of rock elements is suggested as follows. Some fraction of the external force is translated into dissipated energy U^{d}, which results in a progressive deterioration in strength, while another fraction of the external force is translated into the releasable strain energy that is stored in the rock. When the stored strain energy U^{e} reaches its critical value, the strain energy U^{e} is released, resulting in the failure of the rock element.

In rock mechanics, compression stress is considered positive and tension stress is considered negative. The theoretical criteria for the structural failure of rock elements can be expressed further: U^{e} is difficult to release along the direction of the maximum principal stress σ_1, but easy to release along the direction of minimum pressure stress σ_3. The construction of the criteria is discussed in detail from the aspects of compression and tension (Fig.6).



(a) Compression situation

(b) Tension situation

Fig.6 Loading cases.

4.1 Compression state (σ_1>σ_2>σ_3≥0, the minimum stress is zero)

In most engineering cases, rock masses are generally in a state of three-dimensional compression (Fig.6(a)). The rate of energy release G_i is defined as

$$G_i = K_{\mathrm{c}}(\sigma_1 - \sigma_i)U^{\mathrm{e}} \quad (i = 1,\ 2,\ 3) \tag{16}$$

where K_{c} is the material parameter in the case of compression.

It is inferred that the maximum energy release is in the direction of the minimum principal stress σ_3, i.e. $G_3 = K_{\mathrm{c}}(\sigma_1 - \sigma_3)U^{\mathrm{e}}$. This indicates that hydrostatic pressure could not result in the structural failure while also causing maximal shear stress. As soon as the maximum energy release rate G_3 reaches the critical value G_{c}, the strain energy stored in rock element would be released along this direction and result in the structural failure of the rock element.

According to the above analysis, the structural failure of the rock element would occur when it meets:

$$G_3 = K_{\mathrm{c}}(\sigma_1 - \sigma_3)U^{\mathrm{e}} = G_{\mathrm{c}} \tag{17}$$

K_{c} and G_{c} can be determined by the uniaxial compression test. U^{e} can be calculated by Eq.(15).

If $\sigma_1 = \sigma_{\mathrm{c}}$, σ_{c} is uniaxial compression of rocks, $\sigma_2 = \sigma_3 = 0$, it can be obtained from Eq.(15) by

$$U^{\mathrm{e}} = \frac{\sigma_{\mathrm{c}}^2}{2E_0} \tag{18a}$$

Substituting Eq.(18a) into Eq.(17) obtains

$$G_{\mathrm{c}} = K_{\mathrm{c}}\frac{\sigma_{\mathrm{c}}^3}{2E_0} \tag{18b}$$

Eliminating K_c from Eq.(17) gives

$$(\sigma_1-\sigma_3)U^e=\frac{\sigma_c^3}{2E_0} \tag{19a}$$

Considering Eq.(15), the criterion for the structural failure of rock elements in a compressional state can be given as:

$$(\sigma_1-\sigma_3)[\sigma_1^2+\sigma_2^2+\sigma_3^2-2\nu(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_1\sigma_3)]=\sigma_c^3 \tag{19b}$$

Table 2 lists some critical stresses for rock elements where structural failure occurs. These have been calculated assuming that the compressive stress is positive and Poisson's ratio meets $0\le\nu\le 0.3$. It can be inferred that: (1) with the increase of confining pressures within a rock element, energy release becomes more difficult and the critical stress corresponding to structural failure rises from σ_c to ∞, and (2) if the initial Poisson's ratio ν or the average Poisson's ratio $\bar{\nu}$ is small (i.e. rocks are more brittle), the critical stress is smaller, whereas, the more ductile a rock is, the larger the critical stress will be. These inferences agree well with actual failure conditions in rock engineering.

4.2 Tension state (σ_3<0, the minimum stress is less than zero)

Underground engineering sometimes involves rock masses in a state of tension. Structural failure can easily occur in such situations. If at least one of the principal stresses is tensile (Fig.6(b)), the rate of energy release G_i within a rock elements in the direction of σ_i is defined as

$$G_i=K_t\sigma_iU^e\quad(i=1,\ 2,\ 3) \tag{20a}$$

This means that the maximum energy release occurs in the direction of the maximum tensile stress σ_3. The structural failure of the rock element would occur, when it meets:

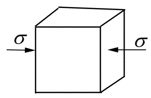
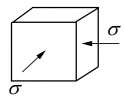
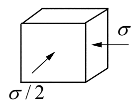
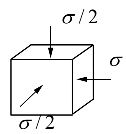
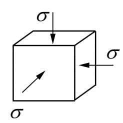
$$G_3=K_t\sigma_3U^e=G_t \tag{20b}$$

where K_t and G_t are the material parameters in a tension setting. This can be determined by the uniaxial tension test. U^e can be calculated by Eq.(15).

If $\sigma_3=\sigma_t, \sigma_2=\sigma_1=0$, then according to Eq.(15), it can be given as

$$U^e=\frac{\sigma_t^2}{2E_0} \tag{21a}$$

Table 2 Critical stresses for rock element failure in some special compressional cases.

No.	The relations of three principal stress	Stress state	The stress for the structural failure (σ)	Specific values of σ		Sketch of element and its loading
				ν =0	ν =0.3	
1	$\sigma_1=\sigma$, $\sigma_2=\sigma_3=0$	Uniaxial compression	σ_c	σ_c	σ_c	
2	$\sigma_1=\sigma_2=\sigma$, $\sigma_3=0$	Uniform bidirectional compression	$\dfrac{\sigma_c}{\sqrt[3]{2(1-\nu)}}$	0.794σ_c	0.894σ_c	
3	$\sigma_1=2\sigma_2=\sigma$, $\sigma_3=0$	Nonuniform bidirectional compression	$\sigma_c\sqrt[3]{\dfrac{4}{5-4\nu}}$	0.928σ_c	1.017σ_c	
4	$\sigma_1=2\sigma_2=2\sigma_3=\sigma$	Nonuniform three-directional compression	$\sigma_c\sqrt[3]{\dfrac{4}{3-5\nu}}$	1.10σ_c	1.39σ_c	
5	$\sigma_1=\sigma_2=\sigma_3=\sigma$	Uniform three-directional compression	∞	∞	∞	

Introducing Eq.(21a) into Eq.(20b), it can be obtained:

$$G_{\rm t} = K_{\rm t} \frac{\sigma_{\rm t}^3}{2E_0} \tag{21b}$$

Eliminating $K_{\rm t}$ from Eq.(20b) gives

$$\sigma_3 U^{\rm e} = \frac{\sigma_{\rm t}^3}{2E_0} \tag{21c}$$

Therefore, considering Eq.(15), the criterion of structural failure for rock element in a state of tension is

$$\sigma_3[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3)] = \sigma_{\rm t}^3 \tag{22}$$

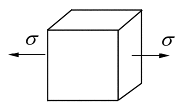
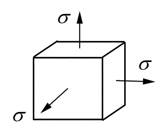
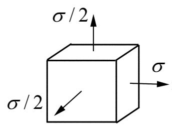
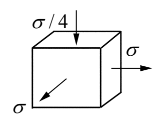
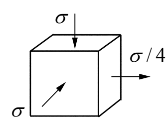
Table 3 lists some critical stresses for rock elements under tension where structural failure occurs. These have been calculated assuming that the compressive stress is positive and Poisson's ratio meets $0 \le \nu \le 0.3$. It can be inferred that with an increase in tension, a rock element expands more. Therefore, energy release becomes easier and the critical stress corresponding to structural failure becomes smaller. In a situation where tension is three-dimensional or bidirectional, the critical stress will either be smaller than the uniaxial tensile strength $\sigma_{\rm t}$ or approach it. The critical stress under uniform three-dimensional tension is the minimum. With an increase in the extent of compression within rock elements in the other one or two directions, energy release becomes more difficult and the critical stress corresponding to structural failure becomes larger (as much as several times its uniaxial tensile strength). If the initial Poisson's ratio ν or the average Poisson's ratio $\bar{\nu}$ is smaller, rocks become more brittle and the critical stress is smaller. Whereas, the more ductile the rocks become, the larger the critical stress is. These points agree well with actual failure situations in rock engineering.

From the above analysis of critical stress corresponding to structural failure under compression and tension, we conclude that the criteria for the structural failure of rocks based on the releasable strain energy have clear and reliable physical meanings. We also conclude that the concept of critical stress, with its simple formulation, is in agreement with observations of actual rock engineering situations. Further research shows that the failure points obtained by this method in principal stress space all lie in a monotonic partitioned curved face that is similar to the enveloping surface given by the twin-shear unified strength theory [28]. Such a curved face is symmetric about the hydrostatic axis $\sigma_1 = \sigma_2 = \sigma_3$ and intersects the symmetric axis at only one crossing point, i.e.

$$\sigma_1 = \sigma_2 = \sigma_3 = -\frac{\sigma_{\rm t}}{\sqrt[3]{3(1-2\nu)}} \tag{23}$$

which means that the uniform compression point is never on the curved face except the state of uniform tension. Therefore, hydrostatic pressure could not result in structural failure; instead, hydrostatic pressure can only result in strength deterioration, as mentioned above.

Table 3 Critical stresses for rock element failure in some special tension cases.

No.	The relation of three principal stresses	Stress state	The stress for the whole structural failure (σ)	Specific values of σ		Sketch of element and its loading
				$\nu = 0$	$\nu = 0.3$	
1	$\sigma_3 = -\sigma$, $\sigma_2 = \sigma_1 = 0$	Uniaxial tension	$-\sigma_{\rm t}$	$-\sigma_{\rm t}$	$-\sigma_{\rm t}$	
2	$\sigma_3 = \sigma_2 = \sigma_1 = -\sigma$	Uniform three-dimensional tension	$-\frac{\sigma_{\rm t}}{\sqrt[3]{3(1-2\nu)}}$	$-0.69\sigma_{\rm t}$	$-0.94\sigma_{\rm t}$	
3	$\sigma_3 = 2\sigma_2 = 2\sigma_1 = -\sigma$	Nonuniform three-dimensional tension	$-\sigma_{\rm t}\sqrt[3]{\frac{2}{3-5\nu}}$	$-0.88\sigma_{\rm t}$	$-1.10\sigma_{\rm t}$	
4	$-4\sigma_3 = \sigma_2 = \sigma_1 = -\sigma$	High tension and low compression	$-\sigma_{\rm t}\sqrt[3]{\frac{16}{33-16\nu}}$	$-0.79\sigma_{\rm t}$	$-0.83\sigma_{\rm t}$	
5	$-\sigma_3 = -\sigma_2 = 4\sigma_1 = -\sigma$	Low tension and high compression	$-\sigma_{\rm t}\sqrt[3]{\frac{64}{33-16\nu}}$	$-1.25\sigma_{\rm t}$	$-1.31\sigma_{\rm t}$	

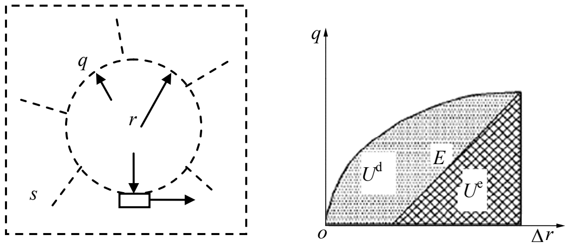
5 Application example

The criteria governing strength deterioration and structural failure of rocks discussed above can be used to analyze structural problem found in rock engineering. For example, considering a tunnel through a model that can be framed in terms of a circular hole in an infinite block with radius r and the uniform inner pressure q (Fig.7(a)), structural failure in such an example is discussed in the following three ways.

(1) The loading process of the tunnel can be represented by the q-Δr curve (see Fig.7(b)), which can be obtained by in situ measurements. The total releasable energy can be calculated as follow:

$$U^{\mathrm{e}}_{\mathrm{Tot}} = 2\pi r L' U^{\mathrm{e}} \tag{24}$$

where L' is the length of the tunnel, and r is the initial radius of the tunnel.



(a) Mode size (b) q-Δr curves

Fig.7 Tunnel in rock and curves of q-Δr.

$U^{\mathrm{d}} + U^{\mathrm{e}}$ represents the generalized work done by the inner pressure q in the direction of radius r, which is transformed into the dissipated energy and the releasable strain energy stored in the rock. If the weight is neglected and the deformation is small, for this axisymmetric problem with uniform stress on the circumference, the stress can be calculated according to the equilibrium condition as

$$\sigma_r = -\sigma_\theta = q \tag{25}$$

$$\sigma_z = \nu(\sigma_r + \sigma_\theta) = 0 \tag{26}$$

This stress situation is in a state of tension, so Eq.(22) is adopted and the critical inner pressure corresponding to structural failure can be obtained as

$$q = q_{\mathrm{t}} = \sigma_{\mathrm{t}} / \sqrt[3]{1+\nu} \le \sigma_{\mathrm{t}} \tag{27}$$

The stress on the circumference of tunnel is tensile in one direction and compressive in another. Therefore, the strain energy stored in the rock is easy to release along the direction of tension more than in the state of uniaxial tension, and the critical stress corresponding to structural failure is $\sigma_r = -\sigma_\theta = q \le \sigma_{\mathrm{t}}$, which is lower than the uniaxial strength σ_{t}. Note that this just illustrates the first step in the failure of the inner surface of the tunnel.

(2) If the inner pressure q and the total energy $U_{\exp}$ are caused by an explosion, it is evident that $U^{\mathrm{e}}_{\mathrm{Tot}} \le U_{\exp}$ and the greatest part of the energy $U_{\exp}$ results in rock damage and plastic deformation. In this case, Eq.(27) can be applied under the conditions that q is altered to the equivalent dynamic inner pressure q_{eq} and the tensile strength of the rock σ_{t} is altered to the dynamic uniaxial tensile strength $\sigma_{\mathrm{t,\,d}}$.

(3) Under dynamic loading, the releasable strain energy U^{e} stored in rock is usually larger than the dissipated energy U^{d} that results in the abruption of the rock, i.e. $U^{\mathrm{e}} > U^{\mathrm{d}}$. Therefore, the difference ΔU is transformed into the kinetic energy of the rock blocks. If the rotation of failed and free rock blocks is neglected, and M is defined as the total mass of free rock blocks, the average velocity $\bar{V}$ of the blocks of rock can be calculated according to $\Delta U = \dfrac{1}{2} M \bar{V}^2$.

6 Conclusions

Mechanically, rocks can be defined as complex heterogeneous material. Their distinct anisotropic and nonlinear responses complicate the theoretical study of structural failure. The nature of energy dissipation and release that occur during the deformation and failure of rocks has been discussed, and a new method for studying the failure of rocks has been proposed, incorporating rock failure theory based on the analysis of energy dissipation and energy release. Our main conclusions are listed here.

(1) Laboratory studies demonstrate that rock failure cannot be completely described by only stress and strain. Various types of rock sample failure are observed to differ from each other. Even in situations where the stress-strain curves are similar, the quantity of energy released from rock samples differs. The more energy is absorbed by a rock sample, the more fragments will be produced from the sample. Consequently, the rock deformation and fracture process can be well described from the viewpoint of energy.

(2) Essentially, the history of deformation and failure in a rock sample is an integration of energy dissipation and energy release in the rock over time. The energy, corresponding to the work done by external forces, dissipates to produce irreversible deformation inside the rock that deteriorates the material and eventually decreases its strength. The strain energy released in the rock volume plays a pivotal role in the abrupt structural failure of the rock. The physical concepts of energy dissipation, energy release, strength deterioration and abrupt structural failure in rock have been defined and discussed in terms of relevant expressions. Importantly, the deterioration of rock strength is different from the structural failure of rock. Energy criteria can be applied to explain the differences between the static and the dynamic failure.

(3) Criteria for the deterioration of rock strength based on energy dissipation and the abrupt structural failure of rocks based on energy release have been proposed. The critical stress at the time of abrupt structural failure in rock under various compressive and tensile stress states can be determined by the latter criterion. In the compressional case, with an increase in confining pressures on rock elements, the critical stress corresponding to structural failure rises from σ_c under uniaxial compression to ∞ under uniform three-dimensional compression. In the case of tension, the critical stress for uniform three-dimensional tension is the minimum, which is either smaller than the uniaxial tensile strength σ_t or approaches it. In the case of a change from compression to tension, i.e. with an increase in the extent of tension, the critical stress corresponding to structural failure becomes smaller from time of the uniaxial tensile strength σ_t to smaller than the uniaxial tensile strength σ_t. Hydrostatic pressure could not result directly in structural failure, but it could result in strength deterioration.

(4) The failure points in the principal stress space determined by our criteria all lie on the enveloping surface of the monotonic stress function. This function is related to the Poisson's ratio and could indicate the effect of damage in the rock. It can be used for all sorts of rock materials.

The present mechanical theory is mainly related to energy dissipation, which emphasizes the function of energy dissipation and corresponding local failure. However, the abrupt structural failure of rocks is an integrated process related to energy dissipation, and to energy release. Present research based on the analysis of energy dissipation and energy release establishes a new system of findings by means of theoretical and experimental studies. Applications of these results will improve our investigations of mechanical problems in rock engineering.

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