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# Least Cost Tolerance Allocation for Systems with time-variant Deviations

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**Abstract**

This paper presents a methodology for the least cost tolerance allocation of systems with time-variant deviations, such as deformation, thermal expansion, mobility of parts and wear. By means of the approach, the product developer can identify the system's optimal tolerance design against the backdrop of the diverging requirements: wide tolerances to reduce costs vs. narrow tolerances to ensure functionality. Therefore, Particle Swarm Optimization – a global statistical optimization technique – is applied to the time-variant tolerance-optimization problem to determine the tolerance range as well as a certain mean shift for each non-ideal dimension.

The practical use of the methodology is illustrated for a modified tolerance stack-up problem. We provide a comprehensive walkthrough of the methodology's application to the stack-up problem. Therefore, the essential mathematical background, the required functional dependencies and the considered parameters are detailed in a step-by-step procedure. So, we aim to establish the application of the statistical tolerance-cost-optimization in academia and industry and to motivate students and young graduates to apply this methodology by themselves.



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*Keywords:* tolerance allocation; time-dependent mechanisms; deformation; thermal expansion; particle swarm optimization

## 1. Introduction

Tolerance Engineering [1] still remains a cumbersome activity within product design. Engineers merely see a difference between ideal CAD-parts and the resulting non-ideal real parts. This phenomenon is all the more serious, since the deviations of parts and their effects on products and processes are omnipresent during manufacturing, assembly and use [2] and can cause loss in product quality and thus significant economic problems in industry [3, 4].

Hence, the consideration of deviations and their corresponding geometric and dimensional tolerances (GD&T) is an essential and helpful step during product development. Usually, statistical tolerance analyses are performed to evaluate the effects of appearing deviations on the relevant functional key characteristics (FKCs) of products. According to their effects on FKCs (Figure 1), these deviations can be classified into [5]:

- Random deviations causing a variation of the FKC (such as a manufacturing-caused variation of the diameter of a drilled hole)
- Systematic deviations resulting in a mean shift of the FKC's probability distribution (such as the deterministic deformation of a beam due to appearing bending forces).

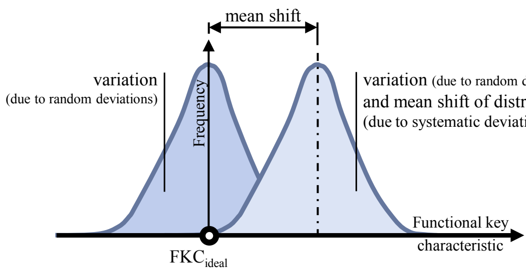


Fig. 1. Variation of FKC (due to random deviations) and mean shift of the FKC's probability distribution (due to systematic deviations)

The product developer has to face these deviations and their different effects by specifying an "optimal" tolerance design that both causes a low scrap rate and results in low manufacturing costs. Against the backdrop of these two


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diverging requirements, in consequence, the product developer has to adjust i) the tolerance range and ii) the mean of the tolerance range for each appearing random dimensional deviation of the product in the early phases of product development. However, the definition of the "optimal" tolerance design is usually a complicated and thus, time- and money-consuming procedure, which should be in the hands of well-qualified and experienced product developers as well as tolerance experts. The commonly used iterative process of tolerance specification, tolerance analysis and (if required) tolerance synthesis (according to [6]) may be an appropriate procedure for the tolerance design of simple products and processes. However, with an increasing number of dimensional and geometrical deviations as well as the move towards more complex systems (such as non-linear mechanisms), the question on the "best (which usually means "cheapest") but still fully functional tolerance design" remains unanswered, even by professional tolerance experts [7].

This paper presents a methodology for the statistical tolerance-cost-optimization of mechanisms with time-variant deviations. Therefore, current and existing research activities on tolerance analysis and tolerance synthesis of mechanisms are discussed in the upcoming Section 2. The methodology is detailed in Section 3. Afterwards, a demonstrator system, whose functionality is essentially affected by several time-variant deviations, is presented in Section 4. The necessity of an appropriate methodological support for the definition of the "optimal tolerance design" is discussed in Section 5, based on a statistical tolerance analysis of the demonstrator system's initial tolerance design. The practical use of the presented methodology for the statistical tolerance-cost-optimization of the demonstrator system is shown in Section 6. The paper closes with a brief summary and a critical discussion of the methodology's benefits and limitations.

The authors' contribution to theory is seen in the discussion of the importance of the effects of time-variant deviations on the motion accuracy of mechanisms and thus on quality and costs. Furthermore, the presented methodology for the statistical tolerance-cost-optimization of mechanisms with time-variant systematic deviations henceforth supports product developers in specifying "optimal tolerance designs" for mechanisms.

## 2. State of the Art

Tolerance Engineering involves three main activities during engineering design [6]: First, initial tolerances of the parts are specified by the product developer (*tolerance specification*). Usually these are based on previous projects, existing drawings and, in particular, the product developer's experience. However, also general tolerances (according to ISO 2768) are commonly used. Secondly, the effects of deviations (limited by the previously defined tolerances) on the product's FKCs are investigated in a *tolerance analysis* [8]. Finally, the specified tolerances can be modified to achieve a better tolerance design, for instance, concerning costs or quality (*tolerance synthesis*). Nevertheless, this procedure is usually iterative and thus may cause high computational and financial expense. As a consequence, approaches to optimal allocation of tolerances based on both manual allocation schemes and mathematical optimization algorithms have been of significant interest in tolerance-related research for almost 50 years [9, 10, 11].

Subsections 2.1 and 2.2 detail the relevant contributions to tolerance analysis and tolerance synthesis of time-dependent mechanisms.

### *2.1. Statistical Tolerance Analysis of time-dependent Mechanisms*

The mobility of a mechanism is essentially affected by the dimensional and geometric deviations of its components. Since the motion behavior is usually highly time-dependent, the application of statistical tolerance analysis methods was extended towards mechanisms. Several publications detail the consideration of random manufacturing-caused deviations for mechanisms with both higher [12] and lower kinematic joints [13, 14, 15].

However, systematic deviations may also affect a mechanism. These systematic deviations are mostly force induced (such as deformation) as well as time-dependent and may appear in all stages of product development, however, especially during the product's use [2]. The deformation of flexible components is considered in the tolerance analyses of mechanisms in [16] and [17], while [15] takes into account the systematic displacement among parts of mechanisms with joint clearance. Further work details the integration of both random manufacturing-caused as well as systematic operation-dependent deviations in statistical tolerance analyses [18]. However, the time dependence of these deviations is merely considered. An "integrated tolerance analysis of systems in motion"-approach that enables the product developer to perform tolerance analyses of mechanisms with time-dependent random and systematic deviations is presented in [2] and [19]. Furthermore, interactions between appearing deviations can be taken into account [5].

### *2.2. Statistical Tolerance Synthesis of time-dependent Mechanisms*

In 2011, CAMPATELLI stated that tolerance synthesis is "currently one of the most proficient ways to reduce the cost of machined parts" [20]. Tolerance synthesis is used to allocate (according to an allocation scheme) the accepted variations of the FKCs among random deviations of the system's components [10]. In contrast, tolerance optimization uses mathematical optimization algorithms to determine the optimal tolerance allocation corresponding to the diverging requirements of the optimization's objectives and constraints.

Publications on tolerance optimization detail a large variety of different numerical methods and optimization algorithms used to allocate the single parts' tolerances. For instance, [21] uses Simulated Annealing to optimize the tolerance design in terms of the resulting manufacturing costs. The optimal worst-case tolerance design of a one-way-clutch is determined by genetic algorithms in [22]. Another commonly used algorithm for tolerance allocation is Particle

Swarm Optimization [23]. This technique can be used for the tolerance design of systems [24] with both continuous [25] and multi-stage tolerance-cost-relations [26]. Further publications on tolerance optimization take into account the variation as well as the appearing mean shifts of the tolerances and their effects on the functionality [22, 27, 28, 29, 30].

The tolerance allocation of time-dependent systems, however, is rarely the subject of research. Into the bargain, the effects of different kinds of deviations are hardly taken into account in these works. The practical use of a sequential tolerance synthesis process for a non-ideal crank mechanism is illustrated in [31]. The optimization considers one full rotation sequence of the mechanism. However, the functional relation is not a time-dependent function. Moreover, only random manufacturing-caused deviations are taken into account. In contrast to [31], [32] integrates the systematic and time-dependent displacement in lubricated joints of mechanisms in tolerance optimizations, while time-dependent functional relations between the FKCs and appearing deviations are established in [18] and [33].

In conclusion, the brief discussion of the state of the art reveals a lack of appropriate methods for the statistical tolerance optimization of time-dependent mechanisms whose FKCs are affected by variation (due to random deviations) as well as time-variant mean shifts (due to systematic time-variant deviations such as thermal expansion or deformation).

## 3. Methodology for Tolerance-Cost-Optimization

The essential basis of each tolerance investigation during engineering design is sufficient information about the mechanism and its components as well as the identification of all appearing random and systematic deviations. Then, the four steps of the statistical tolerance-cost-optimization can be applied (Figure 2):

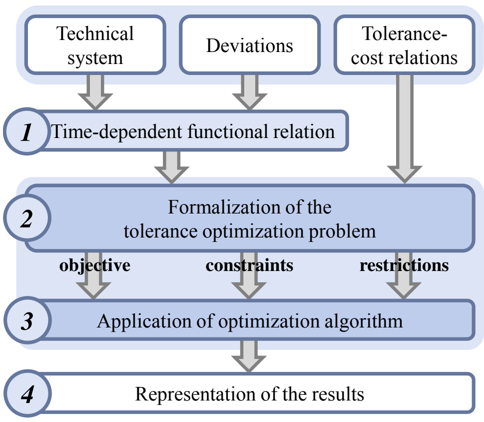


Fig. 2. Methodology: Tolerance-cost optimization of mechanisms [7]

First – analogous to tolerance analysis – the functional relation between the FKC and the deviations which appear during manufacturing, assembly and use is established. Therefore, several techniques are available, such as vector-chain-based approaches [8], Deviation Domains [34], T-Maps® [35] or the Skin Model [36, 37]. In order to apply tolerance optimization to mechanisms the functional relation has to be time-dependent and include time-dependent terms for each deviation $Dev_i$.

$$FKC(t) = f(Dev_1(t), Dev_2(t), \ldots\ldots, Dev_i(t), t) \quad (1)$$

The second step involves the formulation of the optimization problem. Therefore, three components are required: The *objective* is to minimize the manufacturing costs $K_{total}$ that result from the specification of the tolerances $T_i$ and their mean shifts $MS(T_i)$:

$$min(K_{total}) \quad (2)$$

However, since mean shifts of tolerances do not cause additional costs in manufacture, $MS(T_i)$ is weighted by the factor zero in Equation (3). Nevertheless, the product developer can adjust the mean shifts and thus these must be included as parameters in the optimization's objective function.

$$K_{total} = \sum_i [K(T_i) + 0 \cdot MS(T_i)] \quad (3)$$

Furthermore, the *constraints* are defined. Constraints are usually inequalities, such as the requirement of the FKC to be kept within the (still to be defined) lower and upper specification limits (LSL and USL).

$$LSL(t) < FKC(t) < USL(t) \quad (4)$$

A complete definition of constraints requires *restrictions*. These restrictions are limits for certain values of the optimization. Examples are LSL and USL, as well as the lower bound of tolerances, according to:

$$T_i > 0 \quad (5)$$

The best compromise between the two diverging requirements (wide tolerances to reduce costs vs. narrow tolerances to ensure functionality) can be found by means of optimization algorithms. The remaining step is similar to the final step of a tolerance analysis and includes the result representation [38] of the final tolerances $T_i$, their corresponding mean shifts $MS(T_i)$ and the manufacturing costs $K_{total}$.

## 4. Demonstrator – Mechanism with time-variant Deviations

The practical use of tolerance-cost-optimization is illustrated for a demonstrator system. This system is based on a simple tolerance stack-up problem (Figure 3). However, the parts are subject to random deviations (see Table 1) as well as time-variant deviations (deformation, thermal expansion, mobility of parts and wear).

The functional relevant key characteristic (FKC) is the remaining gap between the stack-up and the cross-beam, which should not exceed the lower and upper specification limits of LSL = 0 mm and USL = 0.2 mm. The appearing random deviations are detailed in Table 1.

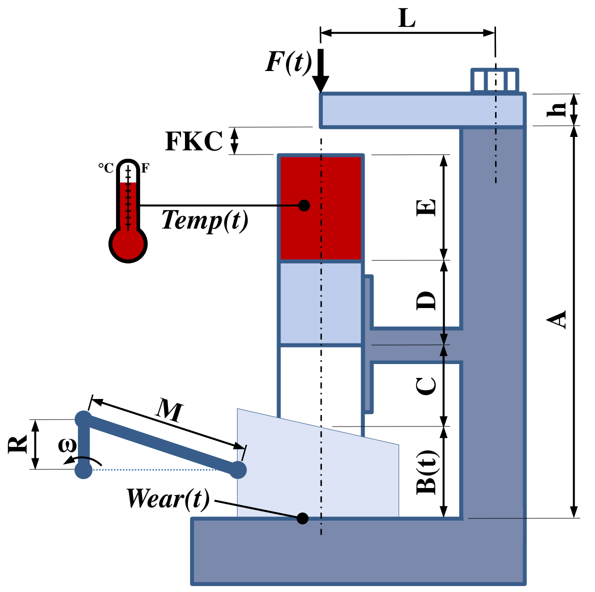


Fig. 3. Stack-up problem with time-variant deviations at t = 0s.

Table 1. Initial tolerance specification

| Parameter | Mean | Tolerance | Distribution |
|---|---|---|---|
| A (height of support) | 30 mm | ±0.04 mm | uniform |
| C (height of white part) | 7 mm | ±0.03 mm | normal (±3σ) |
| D (height of blue part) | 7.85 mm | ±0.03 mm | normal (±3σ) |
| R (crank radius) | 4 mm | ±0.05 mm | uniform |
| h (height; cross-beam) | 12 mm | ±0.01 mm | trapezoid |
| L (force distance) | 50 mm | ±0.5 mm | triangle |

## 5. Tolerance Analysis of initial Tolerance Design

The effects of the initial tolerance specification on the FKC are investigated in a statistical tolerance analysis. The tolerance analysis covers one motion sequence consisting of one rotation of the crank within $T_{total}$ = 10 s.

### *5.1. Formulation of the Functional Relation*

A closed vector-chain is used to establish the functional relation between the FKC and all appearing random and systematic deviations:

$$FKC(t) = A - [B(t) - Wear(t)] - C - D - [E + \Delta ETempt - Def(t) \quad (6)$$

with t = [0; $T_{total}$]. The random deviations are already fully defined. However, the time-variant systematic deviations are required to set up a Monte-Carlo-Simulation (Section 5.2). The following subsections detail the determination of the systematic deviations.

#### *5.1.1. Deformation Def(t)*

The upper cross-beam (made from short fiber reinforced polymer Crastin® LW9020 NC01 [39] underlies a time-variant force F(t):

$$F(t) = 10N - 5N \cdot cos\left(\frac{2 \cdot \pi}{T_{total}} \cdot t + \frac{\pi}{3}\right) \quad (7)$$

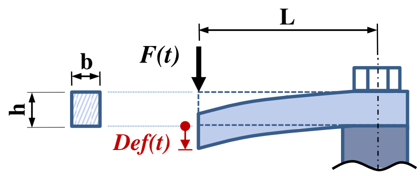


Fig. 4. Deformation Def(t) of cross-beam

The distance between the point of force application and the rigid support (screw) is L = 50 mm, whereas the cross-section of the beam in bending has the height h (see Table 1) and the width b = 10 mm (Figure 4). According to [40] the deformation of the cross-beam is

$$Def(t) = \frac{F(t) \cdot L^3}{3 \cdot E_{polymer,isotropic} \cdot I} \quad (8)$$

whereas the corresponding isotropic Young's modulus of the polymer ($E_{polymer,isotropic}$ = 4200 MPa) [39, 41] and the beam's moment of inertia I (rectangular cross-section) are required [40].

$$I = \frac{b \cdot h^3}{12} \quad (9)$$

The resulting deformation of the cross-beam during the considered time of $T_{total}$ = 10s is shown in Figure 6.

#### *5.1.2. B(t) due to Mobility of Crank Mechanism*

The rotation of the crank with the rotational speed

$$\omega = \frac{2 \cdot \pi}{T_{total}} = 0.2 \cdot \pi \frac{1}{s} \quad (10)$$

starting at

$$\varphi_0(t = 0s) = \frac{\pi}{2} \quad (11)$$

causes a translational motion of the lowest part of the stack-up (Figure 3). Due to a skewed upper surface of this part, the dimension B in the functional relation (6) underlies a time-variant systematic deviation. The kinematic crank mechanism is detailed in Figure 5.

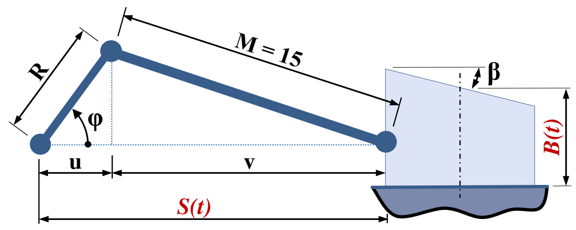


Fig. 5. Crank mechanism: Dimensions and resulting height B(t)

The translational mobility S(t) is the sum of its horizontal components u(t) and v(t) for $\varphi = [\pi/2; 5\pi/2]$:

$$u(t) = R \cdot cos(\varphi) = R \cdot cos\left(\omega \cdot t + \frac{\pi}{2}\right) \quad (12)$$

$$v(t) = \sqrt{M^2 - [R \cdot sin(\varphi)]^2} \quad (13)$$

$$S(t) = R \cdot cos(\varphi) + \sqrt{M^2 - [R \cdot sin(\varphi)]^2} \quad (14)$$

The difference between S(t) and the initial position

$$S_0\left(\varphi = \frac{\pi}{2}\right) = \sqrt{M^2 - R^2} \quad (15)$$

leads to a deviation of the skewed part's height B(t) from its initial value $B_0(t = 0s) = 5$ mm due to the skewness of $\beta = 0.5°$. The resulting time-variant height

$$B(t) = B_0 + [S(t) - S_0] \cdot tan(\beta) \quad (16)$$

and the deformation Def(t) of the cross-beam (Section 5.1.1) of the nominal demonstrator system (no deviations appear) are shown in Figure 6.

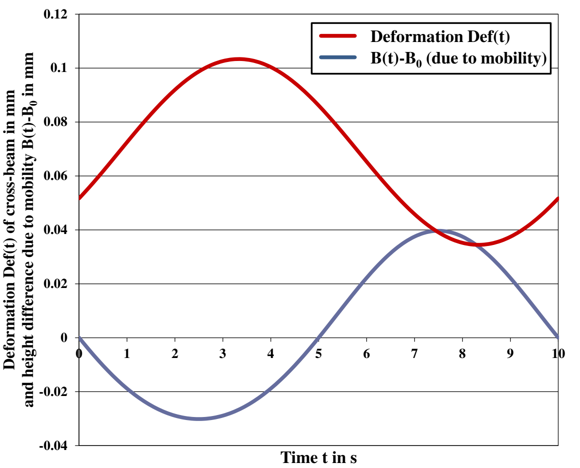


Fig. 6. Deformation Def(t) and height difference $B(t)$-$B_0$ (due to mobility of crank mechanism) of ideal demonstrator

*5.1.3. Wear(t)*

The frictional contact between the lower moving part and the horizontal branch of the support causes wear of the moving part during use. The following relation is assumed, which leads to a total wear of 11.2 µm during one motion sequence of the crank mechanism (Figure 7).

$$Wear(t) = 0.005mm \cdot \sqrt{\frac{t}{0.2 \cdot T_{total}}} \quad (17)$$

*5.1.4. Thermal Expansion $\Delta E_T(t)$*

According to Figure 3, the upper red part of the stack-up assembly underlies a time-variant thermal load Temp(t) during $T_{total}$:

$$Temp(t) = 80°C - 20°C \cdot sin\left(\frac{3}{4} \cdot \omega \cdot t\right) \quad (18)$$

The time-variant temperature difference between Temp(t) and the ambient temperature of $\vartheta_0 = 20$ °C causes a thermal expansion $\Delta E_{Temp}(t)$ [42] of the red part

$$\Delta E_{Temp}(t) = \alpha_{Alu} \cdot E_0 \cdot (Temp(t) - \vartheta_0) \quad (19)$$

whereas $\alpha_{Alu} = 0.000238\ K^{-1}$ is the coefficient of expansion of aluminum [40] and $E_0 = 10$ mm is the height of the considered part at the ambient temperature of 20 °C. Figure 7 shows wear and thermal expansion during one motion sequence of the crank mechanism.

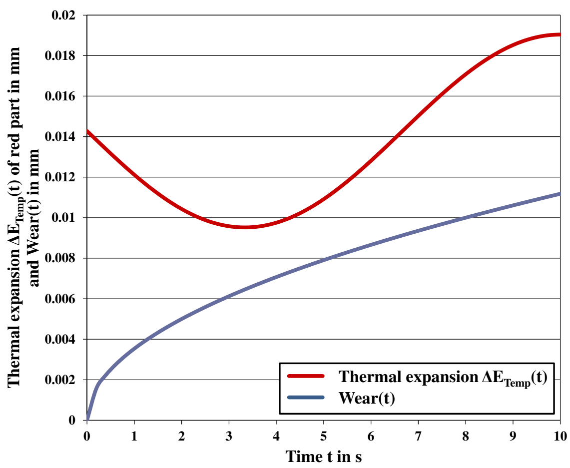


Fig. 7. Wear(t) and thermal expansion $\Delta E_{Temp}(t)$ of ideal demonstrator

*5.2. Monte-Carlo-Simulation and Result Representation*

The second step of a tolerance analysis involves the application of a tolerance analysis method [19]. In this case, Monte-Carlo-Sampling is used to generate n virtual systems – the so-called samples. These samples represent systems which only differ in the dimensions of their parts which underlie random deviations, according to their corresponding probability distributions (as detailed in Table 1). Based on equations (8), (16), (17) and (19) the systematic deviations and finally the time-variant FKC(t) (Equation (6)) for each virtual sample are determined.

*5.3. Representation and Discussion of the Results*

The representation and interpretation of the results is the final step of the "statistical tolerance analysis of systems in motion" [5, 19]. These are usually based on histogram plots of the resulting probability distribution of the FKC as well as the results of a contributor analysis [38]. However, due to the time dependence of the system and thus the FKC, the FKC's frequency distribution may change during the motion sequence of the system. Hence, an additional dimension (time in s) is required to visualize the time-dependent FKC (Figure 8). Furthermore, the given specification limits (LSL and USL) of the FKC are detailed.

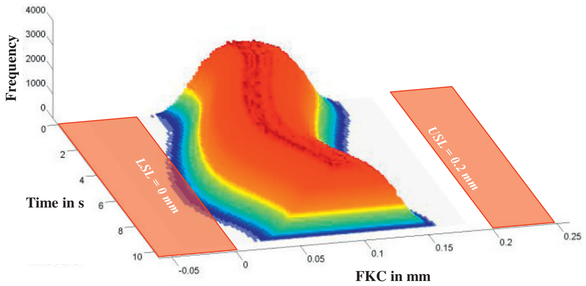


Fig. 8. FKC during the time of $T_{total} = 10$ s: The distribution violates the specification limit LSL = 0 mm

This visualization reveals that the FKC underlies both a variation (caused by the random deviations; Table 1) and a time-dependent mean shift (caused by the time-variant systematic deviations). As a consequence, the FKC exceeds its lower specification limit of LSL = 0 mm. Furthermore, there still remains a certain "unused safe distance" between the probability distribution and the USL = 0.2 mm. In conclusion, the FKC of the initial tolerance design neither stays within the given limits (and thus causes scrap) nor takes advantage of the FKC's entire acceptable tolerance [LSL; USL]. Consequently, a tolerance redesign is highly recommended to optimize the tolerance specification and thus to reduce the scrap rate as well as the resulting manufacturing costs.

## 6. Statistical Tolerance-Cost Optimization

Usually, the tolerance design is modified and an additional tolerance analysis is undertaken to evaluate the achieved improvement of FKC(t). However, instead of this iterative process, the presented tolerance-cost-optimization is used to determine the "optimal tolerance design" and to illustrate the methodology's practical use.

*6.1. Tolerance-Cost-Relations*

According to Figure 2, first, the technical system and all appearing deviations must be well defined (Section 4). Furthermore, corresponding tolerance-cost-relations are required, which describe the dependencies between the manufacturing costs and the tolerances of the six random deviations (Table 1). Therefore, reciprocal (k = 1) [8] and reciprocal squared (k = 2) [43] tolerance-cost-models, which both can be derived from SUTHERLAND's reciprocal tolerance-cost-model [44], are applied:

$$K(T_i) = K_{fix;i} + \frac{K_{ind;i}}{T_i^{\,k}} \qquad (20)$$

In this model, $K_{fix}$ represents the fixed manufacturing costs and $K_{ind}$ represents the individual costs of the tolerance $T_i$ during manufacture [10]. The cost terms $K_{fix}$ and $K_{ind}$ for single-stage (Tolerances of A, C and R) and multi-stage (D, h and L) manufacturing processes are detailed in Table 2. However, due to the limited availability of realistic "tolerance vs. cost"-data, the components $K_{fix}$ and $K_{ind}$ are assumed. The tolerance-cost-relations are further illustrated in Figure 9.

Table 2. $K_{fix}$, $K_{ind}$ and k for tolerance-cost-models of random tolerances

| Tolerance $T_i$ | $K_{fix}$ | $K_{ind}$ | k |
|---|---|---|---|
| A (single-stage manufacture) | 8.0 € | 0.05 € | 1 |
| C (single-stage manufacture) | 5.0 € | 0.02 € | 2 |
| D (manufacturing process 1) | 7.0 € | 0.002 € | 2 |
| D (manufacturing process 2) | 0.2 € | 0.01 € | 2 |
| D (manufacturing process 3) | 1.0 € | 0.03 € | 2 |
| R (single-stage manufacture) | 2.0 € | 0.01 € | 2 |
| h (manufacturing process 1) | 4.0 € | 0.01 € | 1 |
| h (manufacturing process 2) | 1.0 € | 0.06 € | 1 |
| L (single-stage manufacture) | 0.5 € | 0.03 € | 1 |

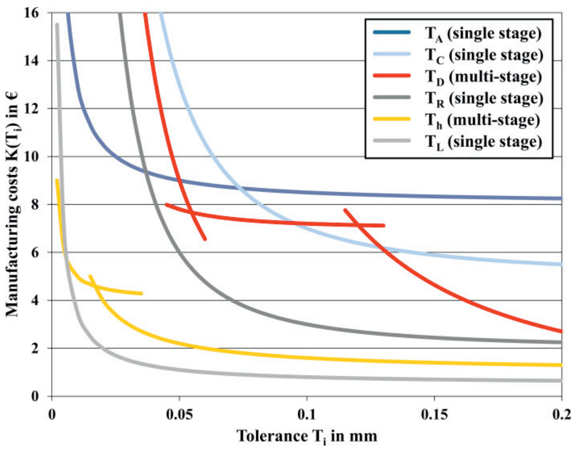


Fig. 9. Visualization: Tolerance-cost-relations of random tolerances

*6.2. Formalization of the Optimization Problem*

The objective of the considered tolerance-cost-optimization problem is to minimize the resulting total manufacturing costs of the tolerance design:

$$min(K_{total}) \qquad (21)$$

However, a certain maximum scrap rate of 0.62 % should not be violated. This means, that 99.38 % of the systems should not exceed the given specification limits of the FKC of LSL = 0 mm and USL = 0.2 mm at any point in time t.

$$\int_{LSL}^{USL} \rho[FKC_t(x)]dx \geq c \;\; for \; t \in [0; T_{total}] \quad (22)$$

The term $\rho(FKC_t)$ is the probability density of the FKC at any point in time, while the constant c corresponds to a certain percentage of fully functional (non-scrap) systems. In this case study, c = 0.9938, which corresponds to a ±4σ requirement of a normally distributed FKC and a maximum of 0.62 % defects, respectively. Further constraint is the limitation of the tolerances $T_i$ due to:

$$T_i > 0 \quad (23)$$

### *6.3. Application of Optimization Algorithm*

Optimization algorithms are used to identify the "optimal solution" between diverging requirements. In this case study, Particle Swarm Optimization (PSO) is applied to determine the optimal tolerance design. In contrast to local optimization, global optimization approaches allow the identification of global instead of local minima within the search space. Furthermore, the choice of an appropriate optimization algorithm is based mainly on the considered mathematical problem. Since Equation (22) represents a highly non-linear constraint, PSO is chosen. PSO is able to deal with non-linear constraints and has already been successfully applied to several tolerance-optimization problems (Section 2.2). Moreover, in comparison with genetic algorithms, PSO is a more universal approach which is easier to implement as well as to apply [45].

#### *6.3.1. Particle Swarm Optimization (PSO)*

PSO is a stochastic optimization technique that was invented by KENNEDY in 1995 [23].

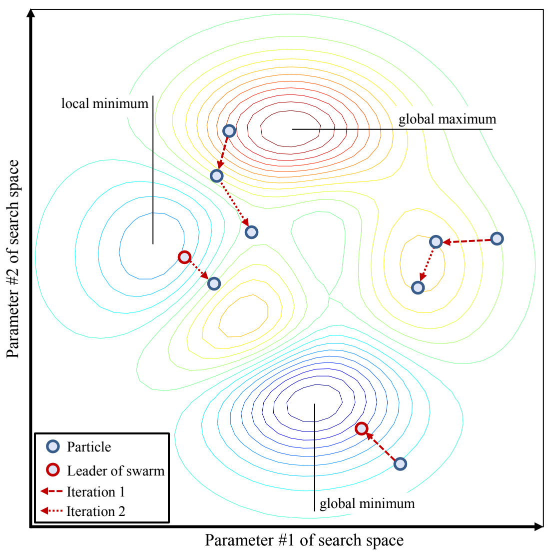


Fig. 10. Behavior of a particle swarm during two iterations of PSO (two-dimensional search space, population of swarm = 4 particles)

The concept of the algorithm goes back to the social behavior of biological decentralized collectives, such as bird flocks or fish swarms. The algorithm distributes the particles of the swarm (population) among a certain search space. These particles move in the search space and gravitate towards a leading particle. The role of the leading particle is subject to switching among the particles during the application and thus can cause quick changes of the swarm's overall movement. The successive iterations of each particle's upcoming position and velocity are defined by its own observations and by information from the remaining particles of the population. Further information on the mathematical background of PSO in tolerance-related applications is detailed in [24]. Figure 10 illustrates two iterations of a particle swarm (population = 4) within a two-dimensional search space.

#### *6.3.2. Application to Tolerance-Optimization Problem*

In order to apply PSO to the detailed tolerance-cost-optimization problem, the search space must be defined. This 12-dimensional space is limited by lower and upper bounds of the tolerances $T_i$ and mean shifts $MS(T_i)$ of the six non-ideal dimensions (Table 3).

Table 3. Search space for PSO ($T_i$ = tolerance; $MS(T_i)$ = mean shift)

| Parameter | Lower bound in mm | Upper bound in mm |
|---|---|---|
| $T_A$ (height of support) | 1e-10 | 0.4 |
| $MS(T_A)$ (height of support) | -0.05 | 0.05 |
| $T_C$ (height of white part) | 1e-10 | 0.4 |
| $MS(T_C)$ (height of white part) | -0.05 | 0.05 |
| $T_D$ (height of blue part) | 1e-10 | 0.4 |
| $MS(T_D)$ (height of blue part) | -0.05 | 0.05 |
| $T_R$ (crank radius) | 1e-10 | 0.4 |
| $MS(T_R)$ (crank radius) | -0.05 | 0.05 |
| $T_h$ (height; cross-beam) | 1e-10 | 0.4 |
| $MS(T_h)$ (height; cross-beam) | -0.05 | 0.05 |
| $T_L$ (force distance) | 1e-10 | 2 |
| $MS(T_L)$ (force distance) | -0.05 | 0.05 |

Moreover, several parameter settings are required to start the optimization. The tolerance-cost-optimization is performed in Matlab 2011 using the PSO toolbox of [46]. The initial parameter settings are listed in Table 4. The optimization was performed with 10,000 samples to analyze the violation of the constraint (Equation (22)).

Table 4. Parameter settings in Matlab PSO Toolbox [46]

| Parameter | Value |
|---|---|
| Population of swarm | 20 |
| Tolerance on the objective function violation in € | 1e-3 |
| Termination tolerance on the constraint violation in - | 1e-6 |
| Maximum number of iterations before algorithm stops | 1500 |
| Maximum number of "identical" generations (tolerances of function and constraint) before algorithm stops | 100 |

### 6.4. Representation and Discussion of the Results

The PSO algorithm successfully concluded after 92 iterations since the variation of the constraint was less than 1e-6 for 100 particle generations (= 5 iterations) (as defined in Table 4). Table 5 details the initial and the optimal resulting tolerance design. Furthermore, the resulting time-dependent probability distribution of the FKC during one motion sequence is shown in Figure 11.

Table 5. Comparison of initial and optimal tolerance design

| Parameter | initial tolerance design | | optimal tolerance design | |
|---|---|---|---|---|
| | Mean in mm | Tolerance in mm | Mean in mm | Tolerance in mm |
| A | 30.000 | ±0.040 | 30.0118 | ±0.0289 |
| C | 7.000 | ±0.030 | 7.0192 | ±0.0600 |
| D | 7.850 | ±0.030 | 7.8190 | ±0.0509 |
| R | 4.000 | ±0.050 | 3.9678 | ±0.1955 |
| h | 12.000 | ±0.010 | 11.9792 | ±0.1410 |
| L | 50.000 | ±0.500 | 50.0368 | ±0.1423 |

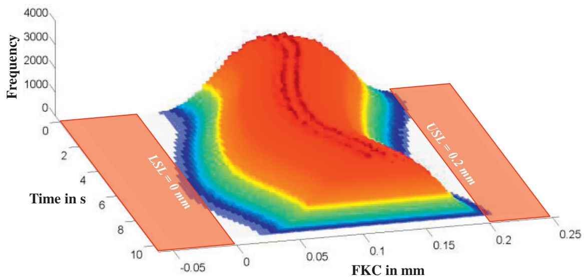


Fig. 11. FKC during the duration of $T_{total} = 10$ s: The ±4σ-quantiles stay within the specification limits LSL = 0 mm and USL = 0.2 mm.

While the initial tolerance design causes manufacturing costs of 29.72 €, the optimal tolerance design leads to a significant cost reduction to 20.39 €. Moreover, the optimal tolerance design causes fewer defects and thus fulfills the given ±4σ-requirement.

The main modifications of the tolerance design are changes in the dimensions' mean values by certain mean shifts. These mean shifts aim to compensate the time-variant mean shifts of the FKC caused by the systematic deviations (deformation, thermal expansion, mobility of parts and wear). Furthermore, the mean shifts aim to center the nominal FKC within the specification limits LSL and USL. Hence, the remaining range of the FKC towards LSL and USL can be allocated among the tolerated random variations of the six dimensions.

## 7. Closing Words

This paper has presented a methodology for the statistical tolerance-cost-optimization of systems with time-variant deviations, such as deformation, thermal expansion, mobility of parts and wear. By means of the optimization the product developer can identify the optimal tolerance design against the backdrop of the diverging requirements: wide tolerances to reduce costs vs. narrow tolerances to ensure functionality.

Despite the successful application of the methodology in an easy to follow walkthrough case study, several limitations and thus potentials for future research arise. Two of these are discussed briefly:

- Restrictions concerning the considered time $T_{total}$: The consideration of time-variant mechanisms aims to evaluate the effects of tolerances on the FKC during the product's entire life. However, the numerical expense may drastically increase. For instance, the consideration of the demonstrator system just covered one motion sequence of 10 s. Hence, appropriate methods must be developed to extend tolerance analysis of time-variant systems towards a "lifelong" consideration. Inspirations and suggestions on this topic may be gained in the field of reliability analysis.
- Unavailability of up-to-date "tolerance vs. cost"-data: The resulting optimal tolerance design is essentially affected by the considered tolerance-cost-relations of the random deviations. However, hardly any reasonably current publication that deals with the relation between costs and tolerances provides sufficient empirical data. In particular, many publications reference quite old books [47, 48] and thus use out-of-date data. Furthermore, we assume that industrial professionals have (at least some) present and validated data, but may keep those undisclosed due to their explosiveness against the backdrop of global competition in quality and costs.

In conclusion, the proficient use of a least cost tolerance allocation reduces or even avoids time- and money-consuming iterations during product development [49, 50, 51]. Nevertheless, the identified optimal tolerance design, however, highly depends on the definition of objectives, constraints and restrictions. Consequently, an unbiased and critical evaluation of the gained results of the product developer is, a fortiori, indispensable.

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