A logic for deontic dilemmas

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Abstract

The possibility of deontic dilemmas poses a significant problem for deontic logic. Here I review some proposals to resolve this problem, and then offer a new account. This is a simple modification of standard deontic logic that enables the system to accommodate deontic dilemmas without inconsistency and without deontic explosion, while at the same time accounting for the range of genuinely valid inferences.

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In what follows I describe the problem that deontic dilemmas pose for deontic logic, and look briefly at some attempts to resolve that problem. These are not entirely successful. I then offer a new way to accommodate such dilemmas that works better. This proposal will be simple, modest, even conservative; the logic proposed will be an elementary modal logic of a quite ordinary sort. It is my aim to show that such a logic can accommodate deontic dilemmas in a reasonable way despite some objections that have been raised, especially by Horty in a number of works, [18–20].

With that in mind, throughout this discussion, I shall suppose a propositional language with formulas \( A \) in the usual vocabulary, and a single monadic modal operator \( O \) to represent ‘it ought to be that …’ with \( OA \) well-formed whenever \( A \) is. I thus use the idiom of ought-to-be, which some would distinguish from ought-to-do. I do not discriminate between the two here since analogous issues arise for both locutions. I also abstract from

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considerations of agency and action, and from such issues as the time of an obligation, the
authority that institutes it or the person to whom it might be directed, if any. These are
all significant factors of normative discourse and so deserve to be treated in deontic logic.
But it is reasonable to suppose that they do not have particular bearing on the question of
deontic dilemmas before us. The same issues should arise with any further sophistication
of the logic, and should probably be treated in much the same way. Thus, what I present
here might be taken as a blueprint for a more detailed treatment in richer contexts. In a
similar vein, I do not distinguish between so-called prima facie oughts and actual, or all-
things-considered, oughts, since here too similar problems arise with respect to both. Let
O be an operator for whatever sort of ‘ought’ one likes to consider.

1. The problem of deontic dilemmas

By a ‘deontic dilemma’ I mean a situation in which, in a univocal sense of ‘ought’,
some state of affairs, A, both ought to be and ought not to be, in which, that is, both OA
and O¬A are true. More broadly, a deontic dilemma would be a situation in which there
are inconsistent states of affairs, A and B, both of which ought to be, that is, a case where
\( \vdash A \rightarrow \neg B \), or \( \vdash \neg (A \land B) \), and yet both OA and OB are true. More broadly still, a
deontic dilemma would be a situation in which it is impossible for both A and B to be
realized even though both ought to be, where the sense of impossibility could be anything
appropriate to the context of discourse, from some metaphysical impossibility to a more
mundane practical incompatibility.1

The first sense of deontic dilemma is easily seen to be a case of the second, and the
second a case of the third. In addition, given seemingly innocent assumptions, like the
inheritance, or monotonicity, rule

\[
\text{RM) } \quad \vdash A \rightarrow B \text{ then } \vdash OA \rightarrow OB
\]

or natural variants of it, then the second broader sense of dilemma reduces to the first, in
the sense that any case of the second will imply a case of the first. Hence, any deontic logic
that eschews the first sort of dilemma, must also eschew the second. Similarly, if the sense
of impossibility in the third description is such that the logic contains anything like

\[
\text{NM) } \quad \vdash \neg \diamond (A \land \neg B) \rightarrow (OA \rightarrow OB)
\]

1 In a language for conditional obligation, we would also allow for conditional deontic dilemmas. These would
be situations in which both it ought to be that A on a condition B, and also it ought to be that not-A on the same
condition, i.e., where both O(A/B) and O(¬A/B) are true, and similarly for the other senses of incompatible
requirements. Since the issues raised by such conditional dilemmas are much the same as for the monadic cases,
I shall not discuss them further here. (In [13] I did introduce operators for conditional norms in order to show
how a maneuver similar to that proposed here for dealing with dilemmas could also apply to another, independent
problem that Horty has raised for standard logics of conditional obligation, the problem of how to eat asparagus;
cf. [18,19]. That, however, raises further questions that are too tangled, and too tangential, to address here; hence,
I will now reserve that discussion for another place.)
(with $\Diamond$ for the appropriate sense of possibility), then the yet broader sense of dilemma also reduces to the first, and any logic that eschews the one must eschew the other. As a result, we can focus our attention primarily on the first sort of dilemma, since it is easiest to discuss, though natural examples might well take the form of the second or third.

It is convenient to note here that, given the rule of replacement for logical equivalents

$$\text{RE) }\quad \text{If } \vdash A \leftrightarrow B \text{ then } \vdash OA \leftrightarrow OB$$

which seems a *sine qua non* for any reasonable deontic logic, the inheritance rule (RM) is equivalent to each of these principles

$$\text{M) }\quad \vdash O(A \wedge B) \rightarrow (OA \wedge OB)$$

$$\text{OR) }\quad \vdash OA \rightarrow O(A \vee B)$$

That is to say, given (RM) then both (M) and (OR) are derivable (and also (RE), of course), and given (RE) and either (M) or (OR), then (RM) is derivable. Hence any discussion of (RM) applies as well to (M) and (OR).

It is plausible that there are deontic dilemmas, indeed that they are very common. I shall not argue that here, however. Instead, I will simply take it for granted that there are, or could be, such cases in order to investigate how deontic logic should accommodate them.

Any deontic logic that accommodates deontic dilemmas must, of course, not contain the principle

$$\text{D} \quad \vdash OA \rightarrow \neg O \neg A, \text{ or}$$

$$\vdash \neg (OA \wedge O \neg A)$$

lest it license contradictions. (D) is central to standard deontic logic, SDL, in its many variants, and so SDL can be thought of as denying the possibility of dilemmas. More generously, one might think of commitment to (D) not so much as *denying* the possibility of dilemmas, but rather as *delimiting* the range of application of the logic. One might think that, while deontic dilemmas might be possible, standard deontic logic only applies to the logic of normative structures that are in fact deontically consistent or dilemma-free. Such

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2 This is especially so when one considers the norms that apply to multiple agents, for one agent might be required to do one thing while another agent is required to do something else that is incompatible with the first. E.g., two people might each have promised something that precludes the other’s fulfilling his promise. Of course, since its inception, standard deontic logic has denied the possibility of conflicts of obligation, and there is a long standing philosophical tradition that argues against the possibility at least of *moral* dilemmas. Hory [20] examines several such arguments; see also Forrester [3] for more sources. Many in that tradition nowadays maintain that what look like cases of dilemmas, where it seems a person ought to do something $A$ and ought to do something else $B$ but can’t do both, are not really dilemmas; rather they are situations where it is not the case that the person ought to do $A$ and not the case that the person ought to do $B$, but only the case that the person ought to do ($A$-or-$B$). I am unconvinced, and all the more so when we consider that the agents of the obligations might be distinct. See Routley and Plumwood [28] for more discussion of how normative conflicts pervade our lives.

3 This is analogous to the way one might preserve the inference from (A), All $S$ are $P$, to (I), Some $S$ are $P$, by saying that the logic that contains it applies only to that part of the language in which all terms $S$ have existential import. C.f. Lambert [23, p. 261f] for discussion of this, and the reasons why this sort of maneuver should be rejected. I develop this theme further in Section 3 below.
an approach has significant drawbacks, however, and, I think, it should not be maintained for long. Nevertheless, it does suggest a measure of adequacy that we might apply to a logic that does accommodate dilemmas, namely that it should be equivalent to SDL in case there aren’t any. That is, for purposes of this inquiry, we should make minimal changes to SDL in order to tolerate deontic dilemmas. One way to put this is to say

\[
(\ast) \quad \text{A logic for deontic dilemmas should be such that if (D) is added as an axiom scheme the result is equivalent to SDL.}
\]

I think this is a worthwhile criterion, but I do not insist on it. Most proposals to accommodate dilemmas do not meet this condition; one I offer below does.

Any deontic logic that, like SDL, contains the rule (RM) mentioned above and also the aggregation principle

\[
\text{AND) } \vdash (O A \land O B) \rightarrow O(A \land B)
\]

and the principle of *ex falso quodlibet* from classical logic

\[
\text{EFQ)} \quad \vdash (A \land \neg A) \rightarrow B
\]

will *ipso facto* contain a principle of “deontic explosion”, such as

\[
\text{DEX)} \quad \vdash (O A \land O \neg A) \rightarrow OB
\]

This says that if there is any instance of a deontic dilemma then *everything* is obligatory. (And similarly for the other broader senses of dilemma.) That seems clearly unacceptable. It is plausible that there are deontic dilemmas; it is not plausible that everything ought to be the case. Thus, any deontic logic that accommodates dilemmas must not contain deontic explosion. We might make this too a condition of adequacy.

\[
(\ast\ast) \quad \text{A logic for deontic dilemmas should not contain (DEX), or anything like it.}
\]

(In what follows I will be more concerned with meeting (\ast\ast) than (\ast).)

The *problem* that deontic dilemmas present for deontic logic can now be seen as simply the question of how to avoid deontic explosion, and (D), while at the same time accounting for the full range of inferences that do seem valid for normative concepts. Since (EFQ), (RM) and (AND) suffice for (DEX), to be adequate for deontic dilemmas, a logic must reject or restrict at least one of these. The question is, What is the best way to do that?

This question is distinct from asking how such dilemmas should be resolved, or how one should decide to act in the face of a dilemma. It is also distinct from the kind of case often considered in the literature of defeasible reasoning, whereby one might have information that seems to lead to inconsistent conclusions, but where some of that information defeats the application of other information, so that no conflict is actually generated. (Birds fly; emus are birds; emus don’t fly; Ezra is an emu. One concludes that Ezra doesn’t fly, because he’s an emu; one does not conclude that he flies, even though he’s a bird.) Similarly, one might have reasons to think that one ought to do A, but other reasons to think that one ought to do B, when A and B are incompatible, where the latter reasons override, or defeat, the former. For example, one set of regulations might enjoin A and another forbid A, but there might be some mechanism of priority that make only one injunction operative in a
particular case, so that perhaps the prohibition of $A$ defeats the injunction for $A$ under those conditions. A deontic dilemma is a case where neither claim of obligation, $OA$ and $OB$ for incompatible $A$ and $B$, is defeated; both genuinely apply. How a deontic logic should accommodate that sort of situation is what concerns me here.

2. Some proposed solutions

The problem posed by deontic dilemmas has two sides. On the one hand, one wants a logic that is not too strong; it must avoid (D), which is easy, and it must avoid deontic explosion (DEX), which is also easy, though less so. On the other hand, it must not be too weak; it must capture all the inferences one wants for the operator $O$. It is this that makes the problem of deontic dilemmas a challenge.

As noted above, to avoid (DEX), at least one of the principles (EFQ), (RM) and (AND) must be rejected or restricted. This suggests three ways one might try to weaken standard deontic logic. Let us consider them briefly in turn.\footnote{This will be a very brief review of some proposals that have been made; it is not meant to be exhaustive. Nor do I present the full motivation behind these accounts, which might go beyond the question of deontic dilemmas.}

2.1. Reject ex falso quodlibet

Perhaps the most direct way to avoid the derivation of (DEX) is just to deny the principle of \textit{ex falso quodlibet}, that a contradiction implies everything. This means basing one’s deontic logic on a paraconsistent logic, rather than the classical propositional calculus, PC, that is usually assumed. A natural alternative is to apply a relevant logic instead. Routley and Plumwood recommend this approach in [28], and in my own [4] and [5] I proposed systems based on the logic $R$ for this purpose. I find relevant logic attractive, and think that (EFQ) is indeed the real culprit behind deontic explosion. Nevertheless, this is a radical departure from standard deontic logics, and requires defending a non-standard approach to logic in general that goes well beyond deontic considerations. Rather than enter that debate here, for present purposes I will simply set this approach aside.\footnote{Other types of paraconsistent deontic logic appear in Priest [27, Chapter 13], da Costa and Carnielli [2], and Kouznetsov [22]. McGinniss’s [24] ‘semi-paraconsistent’ deontic logic treats non-modal contexts classically and the contents of deontic contexts paraconsistently; this preserves (EFQ) and restricts (RM). These and the systems based on relevant logics are all vulnerable to the challenge given in Section 2.3 below.}

2.2. Reject modal inheritance

Keeping all of classical PC, including (EFQ), but denying the rule of monotonicity or inheritance for $O$, the rule (RM), and its partners (M) and (OR), will also clearly block the derivation of (DEX). Various authors have, for various reasons, called this rule into question, e.g., Jackson [21], Hansson [16], [17, pp. 141ff.], and myself in [6–8,10], amongst others. Generally speaking, however, the reasons for questioning (RM) have little to do with the question of deontic dilemmas, and more to do with other paradoxes of deontic
logic. Indeed, my own proposals along these lines contained (D) and so are incompatible with accepting the possibility of deontic dilemmas.

While I remain suspicious of (RM), I will not now suggest abandoning it altogether. As van Fraassen, [33, p. 419], remarked, “[RM] is not easily given up”. In a discussion, not of deontic dilemmas, but the other deontic paradoxes, Nute and Yu, [26, p. 5], comment on my earlier rejection of this rule,

But the principle of inheritance of obligations is one of the most fundamental principles of SDL and has strong intuitive appeal. It requires the agent to take moral responsibility for the logical consequences of what he/she has committed to do. The rejection of the principle, therefore, seems to be contrary to one of our basic moral reasoning patterns.

I grant that (RM) does have strong intuitive appeal, and perhaps it is hard to give up. Nonetheless, to anticipate later discussion, I shall propose modifying it. This will not be a complete rejection of the principle, as in the works cited above, but rather a limitation on it that should take the intuitive appeal of the rule into account. That is the subject of Section 3 below.

2.3. Reject aggregation

Given the appeal of (RM) and attachment to PC with (EFQ), perhaps the most natural suggestion for avoiding deontic explosion is to reject the aggregation principle (AND), especially since, regardless of explosion, with this principle a deontic dilemma, \( OA \) and \( O\neg A \), would yield \( O(A \land \neg A) \), in violation of ‘ought implies can’.

In [9], [11] and [12], I recommend such a logic, called \( P \), precisely for this purpose. It is axiomatized by PC, with closure under modus ponens, the inheritance rule (RM) and two minimal axioms, (N) \( \vdash O\top \) and (P) \( \vdash \neg O\bot \), where \( \top \) is any tautology and \( \bot \) is \( \neg\top \). Since \( P \) lacks (AND), neither (D) nor (DEX) is derivable. \( (P + \text{AND}) \) yields full SDL.\(^6\)

\( P \) is very well-behaved. It has natural interpretations in terms of neighborhood semantics, after Segerberg [30] or Chellas [1], and in terms of preference-based models, [9,11,12], as well as in an extension of Kripke-models, [9,29]. Nevertheless, it is a very weak deontic logic, and perhaps it is too weak. This is the concern to which I alluded above as the second side of the problem posed by deontic dilemmas. This is what motivates the present discussion the most.

Van Fraassen [34] and after him Horthy [18–20] have argued that systems like \( P \) fail to account for inferences that seem unobjectionable and that seem to require aggregation. Horthy often gives the example of someone who recognizes these obligations for a person, Smith,

\(^6\) Others have also proposed this, or a very similar system, to accommodate deontic dilemmas. For example, Schotch and Jennings [29] introduced the same system, and \( P \) is very like the first system van Fraassen proposed in [34, p. 16], though he backed away from it for reasons we will discuss next. \( P \) contains (N) while van Fraassen’s does not, but this is a minor difference. Chellas [1, p. 202] proposes the same system as van Fraassen’s for a minimal deontic logic, again in order to permit dilemmas.
i) Smith ought to fight in the army or perform alternative service to his country.—$O(F \lor S)$

ii) Smith ought not to fight in the army.—$O\neg F$

and who then reasons to the conclusion

iii) Smith ought to perform alternative service to his country.—$OS$

Whether this is a case of Smith deliberating what he himself should do, or someone else describing Smith’s situation, the inference from (i) and (ii) to (iii) seems valid. Given the principle of aggregation, that is easy to explain. By (AND), (i) and (ii) entail $O((F \lor S) \land \neg F)$. Since $\vdash ((F \lor S) \land \neg F) \rightarrow S$, $\vdash O((F \lor S) \land \neg F) \rightarrow OS$, by (RM). So, given $O((F \lor S) \land \neg F)$, (iii) $OS$ follows. Nothing in $P$ licenses this inference, however, and this seems a significant shortcoming of the system and others like it that lack rules like (AND).\(^7\)

Considerations like this make the problem of deontic dilemmas a difficult problem. Van der Torre and Tan [32] call it ‘van Fraassen’s paradox’. To resolve it, we might hope to steer a middle course between a normal logic like SDL, which is clearly too strong, and a minimal logic like $P$, which appears to be too weak. Since (AND) seems to be what distinguishes $P$ from SDL, perhaps there is a way to restrict this rule without rejecting it altogether.

2.4. Restrict aggregation

Here we look briefly at some ways to limit the aggregation principle; the first and last do not work at all, and though the others do better, they might accomplish less than one thinks at first.

2.4.1. Consistent aggregation

A first natural suggestion for a way to accommodate both situations in which there are deontic dilemmas and the cases of innocent inferences using aggregation is to adopt a principle that allows $OA \land OB$ to entail $O(A \land B)$ except when that would get one into trouble, as when $A$ and $B$ are incompatible, which, as we have seen, would lead to deontic explosion, not to mention a violation of principle (P). Hence, it seems plausible simply to restrict aggregation to those cases where $A$ and $B$ are consistent (or jointly possible). Call this the principle of Consistent Aggregation or

$\text{CAND}$) If $\not\vdash A \rightarrow \neg B$ then $\vdash (OA \land OB) \rightarrow O(A \land B)$\(^8\)

\(^7\) The same shortcoming befalls the systems mentioned earlier without (EFQ) or (RM), though in a different way since they might contain (AND).

\(^8\) As with the principle (NM), mentioned in Section 1, if the language has alethic modalities, this rule might be replaced with a stronger postulate $\vdash \Diamond (A \land B) \rightarrow ((OA \land OB) \rightarrow O(A \land B))$, with $\Diamond$ for any appropriate sense of possibility. All the remarks to follow would apply mutatis mutandis to this as well.
While this move might seem natural, (CAND) is still too strong and will not serve as it is supposed to. Counterexamples are easy to find. Here is one adapted from Horty [20, p. 581]. Suppose that someone, Jones, ought to visit his daughter Abby at a certain time and in preparation for that, notify her he is coming, $O(V_a \land N_a)$. But it could also be that Jones ought also to visit his daughter Beth at that same time and notify her he is coming, $O(V_b \land N_b)$. Because of circumstances, however, such as that Abby and Beth live on opposite sides of the country, it is impossible for Jones to visit both at that time. Thus he faces a deontic dilemma. Both $O(V_a \land N_a)$ and $O(V_b \land N_b)$ hold, although presumably $O((V_a \land N_a) \land (V_b \land N_b))$ does not. But from $O(V_a \land N_a)$, $O N_a$ follows by (RM), and similarly $O N_b$ follows from $O(V_b \land N_b)$. $N_a$ and $N_b$ are jointly possible, and so they are candidates for (CAND). Hence we infer $O(N_a \land N_b)$, that Jones ought to notify both his daughters he is coming to visit, and indeed that he ought to notify both that he is coming even if he only goes to see one of them. That seems implausible.

This sort of example should make one suspicious of (CAND). Moreover, we can make a stronger, more general case against this rule. Consider a case of a deontic dilemma where $O A$ and $O \neg A$ hold, and let $B$ be any consistent proposition, so that $\models \neg B$. We show that $O B$. $B$ must be consistent with either $A$ or $\neg A$; suppose it is $\neg A$, so that $\not\models B \rightarrow A$, and argue:

\begin{itemize}
  \item[i)] $O A$ \quad \text{hyp}
  \item[ii)] $O \neg A$ \quad \text{hyp}
  \item[iii)] $\not\models B$ \quad \text{hyp}
  \item[iv)] $\not\models B \rightarrow A$ \quad \text{hyp}
  \item[v)] $O (A \lor B)$ \quad \text{i, PC, RM}
  \item[vi)] $\not\models (A \lor B) \rightarrow \neg \neg A$ \quad \text{iv, PC}
  \item[vii)] $O ((A \lor B) \land \neg A)$ \quad \text{ii, v, vi, CAND}
  \item[viii)] $\models ((A \lor B) \land \neg A) \rightarrow B$ \quad \text{PC}
  \item[ix)] $\models O ((A \lor B) \land \neg A) \rightarrow OB$ \quad \text{viii, RM}
  \item[x)] $OB$ \quad \text{vii, ix, PC}
\end{itemize}

In case $B$ is consistent with $A$ the argument is similar, and so we may discharge the hypothesis at (iv). Thus we conclude that if there is any deontic dilemma, then anything consistent is obligatory. Call this rule:

**DEX-1**  If $\not\models \neg B$ then $\models (OA \land O \neg A) \rightarrow OB$

(DEX-1) does not go quite as far as full deontic explosion (DEX) that follows from full aggregation, where $B$ could be anything at all, but it is still absurd. It still means the collapse of normative distinctions in plausible circumstances. Clearly, a logic adequate for deontic dilemmas must reject (DEX-1) no less than (DEX). Hence, consistent aggregation, (CAND), is far too strong, not much better than complete aggregation, (AND), itself.

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9 I know of no published source that actually adopts this rule (along with unrestricted (RM)), and for good reason. Nevertheless, it is the sort of proposal that comes to mind first when considering how to handle deontic dilemmas. At least, it often comes up in conversations. (Van der Torre and Tan, [32, p. 411], attribute this principle, with this name, to van Fraassen [34], but I do not find it there.)

10 Van der Torre and Tan, [32, p. 412], observe much the same.
2.4.2. Constrained consistent aggregation

Van Fraassen [34], and after him Hory [18–20], took the need to account for inferences like that concerning Smith’s service to his country while also allowing for deontic dilemmas to require a rather different foundation for deontic logic.11 This supports a weaker version of consistent aggregation that avoids the argument leading to (DEX-1). Hory [20, p. 580], calls it ‘consistent consequent aggregation’, or we might say ‘constrained consistent aggregation’, (CCA). It is difficult to describe or motivate this proposal without more machinery than we have available, but very roughly, according to [19], it goes like this.12 Let \( \Gamma \) be a set of ought statements \( OA \), and let \( \overline{\Gamma} \) be the set of their enjoined contents, \( \overline{\Gamma} = \{ B : OB \in \Gamma \} \). Then Hory specifies that \( OA \) is an F-consequence of \( \Gamma \) — \( \Gamma \vdash F OA \) — just in case there is a consistent subset of \( \overline{\Gamma}, \overline{\Gamma}^* \), such that \( \overline{\Gamma}^* \vdash A \). The rule (CCA) can now be (roughly) stated: Suppose a number of oughts \( OB_1, \ldots, OB_n \) are F-consequent on a set \( \Gamma \), i.e., \( \Gamma \vdash F OB_1, \ldots, \Gamma \vdash F OB_n \), then the aggregate \( OB_1 \land \cdots \land OB_n \) is F-consequent on \( \Gamma \), i.e., \( \Gamma \vdash F OB_1 \land \cdots \land OB_n \) if the set \( \{ B_1, \ldots, B_n \} \) is both (i) consistent and (ii) a subset of \( \Gamma \). (Cf. [20, p. 580].)

Condition (i) of (CCA) is like the rule of consistent aggregation (CAND); condition (ii) lets (CCA) escape the problems that confronted that rule. In the example of Jones visiting his daughters, we can suppose that the setup is such that the relevant set \( \Gamma = \{ O(V_a \land N_a), O(V_b \land N_b) \} \). Then \( \Gamma \vdash F O(V_a \land N_a) \) and \( \Gamma \vdash F O(V_b \land N_b) \), though \( \Gamma \vdash F O((V_a \land N_a) \land (V_b \land N_b)) \) since \( V_a \) and \( V_b \) are presumed to be incompatible. Also \( \Gamma \vdash F O(N_a) \) and \( \Gamma \vdash F O(N_b) \) but, importantly, \( \Gamma \vdash F O(N_a \land N_b) \) just as we would want. For, though \( N_a \) and \( N_b \) are consistent, their set \( \{ N_a, N_b \} \) is not a subset of \( \overline{\Gamma} \). The more general problem that led to (DEX-1) would be treated similarly. The crucial step there is step (vii) by (CAND), but this step does not follow from (ii) and (v) by (CCA) even given (vi) since \( \{ A \lor B, \neg A \} \) is not a subset of the propositions enjoined by the operative set of background oughts, which we can take now to be just \( \Gamma = \{ OA, O\neg A \} \), so that \( \overline{\Gamma} = \{ A, \neg A \} \). By contrast, when aggregation is wanted, it is available. Thus in the example of Smith and his service to his country, let us suppose the operative set of oughts is \( \Gamma = \{ O(F \lor S), O\neg F \} \). Then, obviously, \( \Gamma \vdash F O(F \lor S) \) and \( \Gamma \vdash F O\neg F \), and so too \( \Gamma \vdash F O((F \lor S) \land \neg F) \) since \( \{ (F \lor S), \neg F \} \) is a consistent subset of \( \overline{\Gamma} \). From this \( \Gamma \vdash F OS \) follows since (RM) holds for \( O \) in Horty’s system. Thus the rule (CCA) seems to do what is asked of it without getting into trouble.

We can state a more general form of (CCA), namely: Given \( \Gamma \), if \( \overline{\Gamma} \) is consistent, then if \( \Gamma \vdash F OB_1, \ldots, \Gamma \vdash F OB_n \), then \( \Gamma \vdash F OB_1 \land \cdots \land OB_n \). (This follows immediately from the definition of \( \vdash F \).) The requirement that \( \overline{\Gamma} \) be consistent is important, though, for it is not in general true that if \( \Gamma \vdash F OB_1, \ldots, \Gamma \vdash F OB_n, \) then \( \Gamma \vdash F (OB_1 \land \cdots \land OB_n) \), even when all the \( B_i \)’s are mutually consistent. For example, and a propos the argument about Smith’s service, let \( \Gamma'' = \{ O(p \land (F \lor S)), O\neg p \land \neg F) \} \), where \( p \) might be quite unrelated to \( F \) or \( S \). Then \( \Gamma'' \vdash F O(F \lor S) \) and \( \Gamma'' \vdash F O\neg F \), but \( \Gamma'' \vdash F (O(F \lor S) \land \neg F) \), despite the consistency of \( F \lor S \) and \( \neg F \). Indeed, \( \Gamma'' \vdash F OS \). This reveals that whether or

11 See Hansen [15] (also [14]) for another development of van Fraassen’s ideas; Paul McNamara points toward something similar at the end of [25].
12 The accounts of [18] and [20] are a little different; they will be briefly described in the next subsection. Hansen’s [15] and McNamara’s [25] proposals mentioned in Footnote 11 are more like those accounts.
not an ought statement is established is highly sensitive to the formulation of the members of \( \Gamma \), and with that the validity of arguments containing such statements. Hence, whether or not aggregation is operative depends on the particular specification of the members of \( \Gamma \), rather than simply the properties of the candidates for combination.

The concern here is more philosophical than formal. Horty describes \( \Gamma \) as a background context of commands or oughts. He suggests that Smith’s obligation to fight or perform alternative service and his obligation not to fight might issue from distinct authorities, such as the law and religion or conscience, respectively (e.g., [19, p. 21], [20, p. 578]). Presumably, \( \Gamma \) represents these, amongst others. In the context of normative reasoning, however, one might have little awareness of the exact forms of these background injunctions, as seems required, and since the authorities are distinct, one has little assurance that they will be mutually consistent. If they are not, one could have a situation like that with \( \Gamma \) above, rather than \( \Gamma \) as originally given, in which case the conclusion \( O(F \lor S) \) will not be forthcoming even though both \( O(F \lor S) \) and \( O \rightarrow F \) obtain, and thus the argument \( O(F \lor S), O \rightarrow F \vdash OS \) will not be valid, as it was said to be.

\( \vdash_F \) is not \( \vdash \)-F-consequence is not logical consequence in an ordinary sense. Although this account provides for \( O(F \lor S), O \rightarrow F \vdash_F OS \), it does not give \( O(F \lor S), O \rightarrow F \vdash OS \). It does not give an account of the ordinary validity of the original argument regarding Smith’s service. Instead it offers a weaker substitute, perhaps to explain the appearance of (ordinary) validity. Perhaps that is all one can expect; perhaps that is enough.

2.4.3. Two-faced deontic logic

In the accounts of [18] and [20], unlike [19], Horty distinguishes two sorts of ought statements, \( !A \) and \( OA \). In [18] \( !A \) represents an imperative, ‘Do \( A \)!’ and \( OA \) represents a statement ‘it ought to be that \( A \)’ that might be derived from such imperatives. In [20] \( !A \) represents a statement of a \textit{prima facie} ought that itself might be derived from some background imperatives, while \( OA \) represents a statement of ‘all-things-considered’ ought that would be determined by such \textit{prima facie} oughts that are binding in the situation. Horty’s question is then how such (all-things-considered) ought statements \( OA \) are derived from sets of commands or \textit{prima facie} obligations, \( !B \), especially in the face of normative conflicts, and it is answered as before, with \( \Gamma \vdash_F OA \) defined the same but with \( \Gamma \) now a set of formulas \( !B \). This means that, although (CCA) continues to hold, we no longer have \( O(F \lor S), O \rightarrow F \vdash_F OS \), for that is not defined; instead we have \( !F \lor S, !F \vdash_F !S \), which is not defined either.

There is a similar doubling of ‘oughts’ in Hansen’s development of van Fraassen’s ideas [15, Section 5], where he distinguishes an operator \( O^2 \) that functions somewhat like Horty’s \( ! \) and an operator \( O^F \) that is like Horty’s \( O \). In contrast to Horty, however, Hansen retains classical consequence, \( \vdash \). The rule (RE) applies to \( O^2 \), but neither (CAND) nor (RM) does. (RM) applies to \( O^F \), but (CAND) does not. This blocks deontic explosion as

\[
\vdash (O^F A \land O^F \neg A) \rightarrow O^F B,
\]

with consistent \( B \), and also \( \vdash (O^2 A \land O^2 \neg A) \rightarrow O^2 B \) and

\[
\vdash (O^2 A \land O^2 \neg A) \rightarrow O^F B,
\]

etc. At the same time, this system contains this mixed form of (CAND): If \( \neg (B_1 \land \cdots \land B_n) \) then \( \vdash (O^2 B_1 \land \cdots \land O^2 B_n) \rightarrow O^F (B_1 \land \cdots \land B_n) \) [15, p. 51]. This supports the (ordinary) validity of the argument regarding Smith’s service, at least in the sense that \( O^2 (F \lor S), O^2 \neg F \vdash O^F S \) (given the consistency of \( F \lor S \) and
Nevertheless, this account too still does not yield validity for $O^F(F \lor S)$, $O^F \neg F \therefore O^F S$ (or for $O^2(F \lor S)$, $O^2 \neg F \therefore O^2 S$).

Although it works differently, the ‘two-phase deontic logic’ of van der Torre and Tan [32] (cf. also van der Torre [31]) employs a similar device. This presents two ought-operators, $\circledast_1$ and $\circledast_2$, and specifies that (CAND) applies to the first without further qualification, but not at all to the second, whereas (RM) applies to the second, but not to the first. The two are related as $\circledast_1 A$ entails $\circledast_2 A$. Hence we also have this mixed form of (CAND), $\vdash (\circledast_1 B_1 \land \cdots \land \circledast_1 B_n) \rightarrow \circledast_2 (B_1 \land \cdots \land B_n)$ when $B_1, \ldots, B_n$ are mutually consistent. Dividing the application of the rules in this way bars the derivation of deontic explosion (DEX-1) quite elegantly, even while it generates the validity of the argument of Smith’s service. That is, it yields $\circledast_1 (F \lor S)$, $\circledast_1 \neg F \vdash \circledast_2 S$. But, like the others, this approach too does not provide validity for $\circledast_2 (F \lor S)$, $\circledast_2 \neg F \therefore \circledast_2 S$, or for $\circledast_1 (F \lor S)$, $\circledast_1 \neg F \therefore \circledast_1 S$.

Some such division of ‘ought’s, and the rules that apply to them, seems necessary if one is to block (DEX-1) while still supporting the argument regarding Smith’s service, at least so long as one takes that to rest on aggregating its premises, given their consistency, and applying (RM) to the result. It is not obvious, however, that there is any such ambiguity of ‘ought’ as it occurs in the discourse that gives us the argument about Smith’s service and inclines us to accept its being valid. Nor is it obvious that, as that argument is given, the premises should be taken in the first of the two senses rather than the second, even while the conclusion is taken in the second and not the first.

2.4.4. Permitted aggregation

Here is another way to restrict aggregation, one that does not require introducing double ought-operators. The rules (CAND) and (CCA) screen candidates for combination by conjunction through the logical property of consistency (or more broadly possibility). In place of that one might consider screening for normative consistency (or normative possibility). This is simply the notion of (joint) permissibility, which is already available in the language. Then one could say that aggregation is permitted when the aggregate is itself permitted. This would be the rule

$$
PAND \vdash P(A \land B) \rightarrow ((OA \land OB) \rightarrow O(A \land B))$$

(where, as usual, $PA =df \neg O \neg A$). Add this principle to the logic $P$ to form the system $PA$.15

Consider how $PA$ handles the previous examples. For the case of Smith and his service to his country, one wants to infer $O((F \lor S) \land \neg F)$ from $O(F \lor S)$ and $O \neg F$ in order to

13 Another way to look at the two phases requires only a single operator but constraining the application of the rules so that (CAND) cannot be applied after (RM) in a derivation. This makes the validity of an argument dependent on a particular proof procedure, and it leaves a non-standard consequence relation. Cf. [32, Section 2].

14 Likewise, for Horry and Hansen: $A \vdash O A$ and $\vdash O^2 A \rightarrow O^F A$, respectively, provided that $A$ is consistent.

15 This system is new. I once thought that, since (PAND) is weaker than (CAND), it would provide an adequate way to accommodate deontic dilemmas, but, as we shall see, it does not. Nevertheless, I present it here both as a cautionary tale, and as a step to the new proposal I develop in the next section.
conclude OS. Implicit in the example is that it is all right, i.e., permitted or not forbidden, that Smith perform alternative service to his country and not fight in the army; without that, the example has no intuitive appeal. Thus we can take \( P(\neg F \land S) \) as an implicit premise in the setup. So we have the assumptions (i) \( O(F \lor S) \), (ii) \( O\neg F \), and (iii) \( P(\neg F \land S) \), and reason as follows: \( \neg F \land S \) is logically equivalent to \( (F \lor S) \land \neg F \), hence (iii) yields \( P((F \lor S) \land \neg F) \), which with (i) and (ii) gives \( O((F \lor S) \land \neg F) \) by (PAND). Since \( \vdash ((F \lor S) \land \neg F) \rightarrow S \), \( \vdash O((F \lor S) \land \neg F) \rightarrow OS \) by (RM), which allows us to conclude OS by modus ponens, as desired. This seems as natural as the original argument for OS from (i) and (ii) by way of (AND).

In the case of Jones notifying his daughters he is coming to visit, for the example to count against (AND) or (CAND) it must be assumed that there is something wrong with his notifying them both (when he will visit at most one). That is, \( O(\neg(N_a \land N_b)) \) is implicit in the setup. Since we are thus given \( \neg P(N_a \land N_b) \), we cannot have the condition necessary to apply (PAND) to infer \( O(N_a \land N_b) \) from \( ON_a \) and \( ON_b \) under pain of having a contradictory premise set. Hence, the undesirable conclusion is not forthcoming. A similar point applies to the argument that gave rise to (DEX-1) from (CAND). As we look at that derivation (i)–(x), and suppose (i) \( OA \) and (ii) \( O\neg A \), and now consider some proposition \( B \) for which we assume that it is not only logically consistent (or compossible) with \( \neg A \) (or \( A \) as the case might be) but normatively consistent with it, i.e., we assume (iii) \( P(B \land \neg A) \), then we find that, given (RM), (i) entails (iv) \( \neg P(B \land \neg A) \), which contradicts (iii). Hence, the attempt to derive the explosive conclusion \( OB \) along these lines fails because it requires inconsistent premises. So long as the premise set of the argument is logically consistent, no problem should arise. (Or so it might appear.) In this way PA might seem an appropriate logic for deontic dilemmas.

Unfortunately, this proposal really fares no better than the rule (CAND) above, and for very similar reasons. Like (CAND), (PAND) also yields a form of deontic explosion, namely that if there is any case of a deontic dilemma, then anything that is permitted will be obligatory, which seems absurd. The argument for this consequence is much like the one against (CAND) given above. Suppose a deontic dilemma, \( OA \) and \( O\neg A \), and any proposition \( B \) such that \( PB \), and reason thus:

\[
\begin{align*}
  \text{i)} & \quad OA & \text{hyp} \\
  \text{ii)} & \quad O\neg A & \text{hyp} \\
  \text{iii)} & \quad PB & \text{hyp} \\
  \text{iv)} & \quad O(A \lor B) & \text{i, PC, RM} \\
  \text{v)} & \quad O(\neg A \lor B) & \text{ii, PC, RM} \\
  \text{vi)} & \quad \vdash B \rightarrow ((A \lor B) \land (\neg A \lor B)) & \text{PC} \\
  \text{vii)} & \quad P((A \lor B) \land (\neg A \lor B)) & \text{iii, vi, RE} \\
  \text{viii)} & \quad O((A \lor B) \land (\neg A \lor B)) & \text{iv, v, vii, PAND} \\
  \text{ix)} & \quad OB & \text{vi, viii, RE} \\
\end{align*}
\]

This yields the principle (by conditional proof)

\[ \vdash (OA \land O\neg A) \rightarrow (PB \rightarrow OB) \]

This is equivalent to

\[ \vdash (OA \land O\neg A) \rightarrow (OB \lor O\neg B) \]
that if there is a deontic dilemma, then everything is either required or prohibited. Though weaker than both the original (DEX) and (DEX-1), these are still unacceptable. They are enough to bar (PAND) in the presence of (RM). Hence we must reject the system PA.

Moreover, the pattern of this argument would seem to generalize. As applied to consistent aggregation, it would show that if there is a deontic dilemma then any consistent (or possible) proposition is obligatory. As applied here, it shows that if there is a deontic dilemma then any permitted proposition is obligatory. Consider then any proposed restriction on aggregation, any principle that if OA and OB and Cond(A ∧ B), then O(A ∧ B), where Cond(A ∧ B) is some condition that could be met by the conjunction of A and B to reflect the limitation of aggregation, such as their mutual consistency, compossibility, or co-permissibility, etc. If the condition Cond is preserved under replacement for logical equivalents, then the preceding sort of argument would seem to apply, regardless of the particular condition. If Cond is such that it is plausible that a proposition, unrelated to the A of the dilemma, could meet it without itself being obligatory, then the restricted principle of aggregation based on it will be in trouble, for we will then have this general form of deontic explosion

\[ \text{DEX-gen) } \vdash \text{Cond}(B) \rightarrow ((OA \land O\neg A) \rightarrow OB), \text{ or } \]
\[ \text{If Cond}(B) \text{ then } \vdash (OA \land O\neg A) \rightarrow OB \]

(depending on whether the condition is expressed in the object-language or the meta-language). This should cast doubt on any attempt to accommodate deontic dilemmas simply by limiting, but not excluding, aggregation (AND) in this kind of way, at least so long as (RM) is preserved in full.16

3. Another proposal: permitted inheritance

In this section I present my new proposal, drawing on the discussion of permitted aggregation, but redirecting its device. That is, instead of limiting the aggregation rule (AND), I propose now to limit the inheritance principle (RM) in much the same way. The idea behind (CAND) and (PAND) was that aggregation should be allowed except when it gets one into trouble, by producing deontic explosion, (DEX) or its variants. The same idea can be applied to the inheritance rule. Thus, I propose to replace (RM) with a Rule of Permitted Inheritance:

\[ \text{RPM) if } \vdash A \rightarrow B \text{ then } \vdash PA \rightarrow (OA \rightarrow OB) \]
where, as before, \( P A =_{df} \neg O\neg A \). Thus, if A entails B then if one ought to do A then one ought to do B, provided that A is permitted or not barred by the normative system. In other words, if A is an unconflicted obligation and it entails B, then B too is obligatory.17

16 This remark does not apply to Horty’s rule (CCA) of Section 2.4.2 or to the mixed versions of (CAND) of Hansen or of van der Torre and Tan of Section 2.4.3.

17 Like (NM) and (CAND) as adapted in Footnote 8, (RPM) could be strengthened to \( \vdash \square(A \rightarrow B) \rightarrow (PA \rightarrow (OA \rightarrow OB)) \) if the language contains alethic modalities with \( \square \) for an appropriate necessity.
In keeping with the motivation behind this restriction on inheritance, we might equally well adopt the rule

\[
\text{RUM)} \quad \text{if } \vdash A \rightarrow B \text{ then } \vdash UA \rightarrow (OA \rightarrow OB)
\]

where \( UA \) says that \( A \) is ‘unconflicted’, i.e., \( UA \equiv (OA \land O\neg A) \). These two rules are equivalent; given either one, the other is derivable. Hence they can be used interchangeably.\(^{18}\)

Since (RPM) is weaker than the original (RM), it cannot just be added to the weak logic \( P \), as was done with rules like (CAND) or (PAND). Instead, let us build a system from scratch. There are three plausible ways to do this. I present all three.

Let \( \text{DPM.1} \), the first Deontic logic with Permitted Inheritance, be given by adding to classical PC, with closure under modus ponens,

\[\begin{align*}
\text{RE)} & \text{ if } \vdash A \leftrightarrow B \text{ then } \vdash OA \leftrightarrow OB \\
\text{RPM)} & \text{ if } \vdash A \rightarrow B \text{ then } \vdash PA \rightarrow (OA \rightarrow OB) \\
\text{N)} & \vdash O\top \\
\text{AND)} & \vdash (OA \land OB) \rightarrow O(A \land B)
\end{align*}\]

(\( \text{RE} \)) is simply a replacement rule for logical equivalents; I consider it a prerequisite for any plausible deontic logic, regardless of the question of deontic dilemmas. In systems with unrestricted (RM), (\( \text{RE} \)) is derivable; here it must be postulated separately. (N) is included primarily so that the logic will approximate SDL; given (\( \text{N} \)) and (\( \text{RE} \)), the rule form of necessitation (\( \text{RN} \)), if \( \vdash A \) then \( \vdash OA \), is derivable. One might dispense with either form, but given (RPM), and PC, alone, \( \vdash (OA \land PA) \rightarrow O\top \) would be derivable. Thus, if there were anything that was both obligatory and permitted, i.e., any unconflicted obligation, as no doubt there is, then \( \top \) would be obligatory. So one gains little by not including (N). \( \text{DPM.1} \) has an unrestricted principle of aggregation (AND), yet it will still avoid (D) and especially deontic explosion (DEX) (as well as (DEX-1) and (DEX-2)). Because it has (AND) without restriction, \( \text{DPM.1} \) must not posit (P), \( \vdash \neg O\bot \), since otherwise (D), \( \vdash \neg (OA \land O\neg A) \), would be derivable, contrary to our desire to allow for deontic dilemmas.

If one wanted (P), in order to maintain that although there could be conflicts of obligation, there could be no obligatory contradictions, ‘ought implies can’ and all that, then one should restrict (AND) along the lines of (PAND) in Section 2.4.4 above. This yields the second variation \( \text{DPM.2} \), with

\[\begin{align*}
\text{RE)} & \text{ if } \vdash A \leftrightarrow B \text{ then } \vdash OA \leftrightarrow OB \\
\text{RPM)} & \text{ if } \vdash A \rightarrow B \text{ then } \vdash PA \rightarrow (OA \rightarrow OB) \\
\text{N)} & \vdash O\top
\end{align*}\]

\(^{18}\) The corresponding, derived, rule for permission is: (RUPM) If \( \vdash A \rightarrow B \text{ then } \vdash UB \rightarrow (PA \rightarrow PB) \). In the interest of parity between obligation and permission with respect to inheritance, one might propose a new rule, (RUMP) If \( \vdash A \rightarrow B \text{ then } \vdash UA \rightarrow (PA \rightarrow PB) \), or the corresponding (RUMO) If \( \vdash A \rightarrow B \text{ then } \vdash UB \rightarrow (OA \rightarrow OB) \). Given either (AND) or (PAND), however, these further rules would return deontic explosion as (DEX-2), and so they must be rejected.

We might think of modal inheritance, especially as expressed by the postulate (M) of Section 1, and aggregation, (AND), as converses. DPM.1 qualifies inheritance with a permission clause; DPM.2 applies that intuition to both. In DPM.2 again neither (D) nor (DEX), or its variants, is derivable. Hence this too is a viable candidate for a logic to accommodate deontic dilemmas.

The third variant is the intersection of the other two. One might not want to exclude entirely the possibility of a contradiction or impossibility being obligatory, and so one might want to reject the axiom (P), as with DPM.1, while at the same time one might want to restrict the aggregation principle as well as the inheritance rule, as with DPM.2. So one could have DPM.3 axiomatized by (RE), (RPM), (N), and (PAND). Henceforth, I will use DPM to refer indiscriminately to all three versions.

In classically-based logics with unrestricted (RM) the distribution principle (K), $O(A \rightarrow B) \rightarrow (OA \rightarrow OB)$ is indistinguishable from the aggregation principle (AND); that is, given one, the other will be derivable. That is not so for DPM. None of the versions contains (K), and DPM.1 has (AND). If DPM contained (K), then the unrestricted inheritance rule (RM) would be derivable, and then (DEX) (or (DEX-2)) would reoccur, and DPM.2 would contain (D) and be equivalent to SDL. That (RM) is derivable given (K), is very quick from (N) and (RE), but even without (N), unrestricted (K) would yield the rule: if $\vdash A \rightarrow B$ then $\vdash (OC \land PC) \rightarrow (OA \rightarrow OB)$, and from this another form of deontic explosion would follow, namely

$$\text{DEX-3)} \quad \vdash (OC \land PC) \rightarrow ((OA \land O\neg A) \rightarrow (PB \rightarrow OB))$$

This says that if there were anything that was both obligatory and permitted, any unconflicted obligation, as there surely is, then if there were any deontic dilemma, then whatever is permitted is obligatory. (Derivation is left to the reader for fun.) While less than full (DEX), or even (DEX-1) or (DEX-2), (DEX-3) is also unacceptable. Hence (K) too is unacceptable in this context.

Although (K) is unacceptable, and not derivable in DPM, this restricted form, ‘permitted (K)’, is derivable:

$$\text{PK) } \vdash P(A \land B) \rightarrow (O(A \rightarrow B) \rightarrow (OA \rightarrow OB))$$

---

19 One could also combine (RPM) with (CAND), instead of (PAND), in order to allow for (P). The remarks below apply as well mutatis mutandis to this variant, which shows that what is most operative in dealing with deontic dilemmas is the rule (RPM). One might also consider systems with a stronger rule, (RCM), that restricts inheritance to consistent or possible, rather than permitted, antecedents, i.e., if $\vdash A \rightarrow B$ then if $\not\vdash \neg A$ then $\vdash OA \rightarrow OB$, or if $\vdash A \rightarrow B$ then $\vdash OA \rightarrow OA \rightarrow OB$. This, however, returns (DEX) or (DEX-1) or (DEX-2), depending on the form of aggregation used, by an argument like that of Section 2.4.4, for contingent A of the dilemma. Hence, this latter suggestion will not do.
This follows with (RPM) and either (AND) or (PAND). Conversely, given (PK), along with (N) and (RE), the rule (RPM), hence also (RUM), can be derived. So either might be taken as primitive. Likewise, given (RE), either (RPM) or (PK) could be replaced by either of

\[
\text{PM) } \vdash P(A \land B) \rightarrow (O(A \land B) \rightarrow (O A \land O B))
\]

\[
\text{POR) } \vdash PA \rightarrow (OA \rightarrow O(A \lor B))
\]

or similarly for (UM) or (UOR) with U in place of P. (Derivations are again left to the reader.)

I will present DPM in full formal dress in Appendix A, where I will also demonstrate the claim that indeed neither (D) nor (DEX), including the several variations described above, is derivable. Now, however, let us consider informally how DPM responds to the concerns raised for the other systems in Section 2.

With the failure of (D) and (DEX) in its several forms, one primary issue is resolved. DPM is able to tolerate deontic dilemmas without blowing up. Moreover, it escapes the problematic cases from Section 2.4.1 that arise for systems with consistent aggregation (CAND), such as the scenario of Jones visiting his daughters.

For logics with the rule (RM) if there is a case of a deontic dilemma where OA and OB are both true but A is incompatible with B, it follows that OA and O¬A are both true, and so there is a dilemma in the narrow sense. This result does not quite hold for DPM, but something similar does, namely that if OA and OB are both true and A and B are incompatible, then either OA and O¬A are both true or else OB and O¬B are both true. Hence, if there is a deontic dilemma with regard to A and B, then at least one of the conflicting obligations is self-conflicted; either OA and ¬PA holds or else OB and ¬PB holds. This will suffice to block the application of (RPM). Thus, with Jones and his daughters, we are given that O(V_a \land N_a) and O(V_b \land N_b) when it is impossible to have both V_a \land N_a and V_b \land N_b (because it is impossible to have both V_a and V_b). This posed a problem for (CAND) because, with (RM) one could infer both ON_a and ON_b, and then the undesirable O(N_a \land N_b) follows since N_a and N_b are jointly possible. With (RPM) in place of (RM), however, one cannot infer both ON_a and ON_b since, by the above, one will not have both of the initial conditions P(V_a \land N_a) and P(V_b \land N_b) that are required for the two applications of the rule. Hence, even with the unrestricted aggregation rule (AND) of DPM, the unwanted O(N_a \land N_b) is not derivable.

In this way DPM avoids not only deontic explosion, but also other untoward cases that troubled earlier proposals. So it seems these logics are not too strong. The real question, though, is whether they are too weak, as, for example, the system P of Section 2.3 seemed to be. Since these systems restrict (RM), and since (RM) has strong intuitive appeal (cf. the comment of Nute and Yu in Section 2.2), won’t DPM automatically fail to capture all the inferences one expects? Further, is DPM adequate to represent the sort of inference that was brought against the system P, which originally raised this concern?

Regarding the intuitive force of (RM), like all appeals to primitive intuition, it is difficult to respond to this argument. In this case, however, we are faced with a conflict of intuitions. One might equally well claim intuitive force for the principle of aggregation (AND), and yet we cannot have both, under pain of deontic explosion in the face of dilemmas. Thus at least one of these intuitions must be revised. As we have seen, however, (AND) cannot be
easily restricted in a way that avoids deontic explosion whereas (RM) can. Hence, we have reason to question, and revise, our initial intuitions about this rule.

Furthermore, I suspect that those initial intuitions are drawn primarily from considering cases in which the antecedent $A$ of the entailment will be considered normatively consistent, i.e., unconflicted, and thus the intuitive support for a rule of inheritance applies more to (RPM) or (RUM) than the unrestricted (RM). I doubt whether we have clear intuitions about inheritance in conflict cases. Hence, the limitation on the rule does not automatically mean the system is too weak; more argument would be required.

Here is an analogy to illustrate what the restriction on the inheritance principle accomplishes. Consider free logic.\(^{20}\) In classical first-order logic, the rule of universal instantiation, (UI), from $\forall x A x$, infer $A t$, is valid for all individual constants $t$. And it certainly has strong intuitive appeal. Nevertheless, the free logician maintains that (UI) is not valid; it fails in cases where the individual constant $t$ does not refer to anything that exists. For example, the argument

\[
\begin{align*}
&\text{For all objects, } x, \text{ there is an object, } y, \text{ identical to } x. \quad \forall x \exists y (y = x) \\
&\text{There is an object } y \text{ identical to the planet Vulcan.} \quad \exists y (y = \text{Vulcan})
\end{align*}
\]

has a true premise and a false conclusion. (The quantifiers here are construed classically, as ranging over existent entities.) The plausibility, the intuitive appeal, of (UI) derives from the presupposition that the singular term $t$ refers to an existent. By adopting this rule, classical logic, in effect, limits its range of application to languages containing no terms that fail to refer in this way. This is a severe and artificial limitation. In its place, the free logician recommends letting the language contain singular terms that lack existential import, and building the presupposition required for the application of (UI) into the rule itself. Thus, although free logic rejects the classical (UI), it accepts the restricted rule (RUI), from $\forall x A x$ and $t$ exists, infer $A t$.

With the logics DPM I recommend something analogous. We accept inferences based on deontic inheritance (RM), we find that they have strong intuitive appeal, under the presupposition that the situations described are conflict free. Standard deontic logic, with its unrestricted rule (RM), in effect limits itself to reasoning with normative structures that exclude deontic dilemmas. This is a severe and artificial limitation. In its place I propose that we recognize the possibility of such conflicts, and then build the presupposition required for the application of the rule into the rule itself. That is what (RPM) or (RUM) does. Given that $A$ entails $B$, we accept that $OA$ entails $OB$, provided that $A$ is not itself conflicted. The standard rule from (M), From $O(A \land B)$ infer $OA$, is analogous to (UI); the restricted rule from (UM), From $O(A \land B)$ and $UA$ infer $OA$, is analogous to (RUI). DPM rejects the first, and adopts the second.

That is just how DPM treats arguments like that of Smith’s obligation to serve his country. This is similar to the argument in Section 2.4.4 for PA to illustrate the restricted rule of permitted aggregation (PAND). We are given that Smith ought to fight in the army or perform alternative service, $O(F \lor S)$, and that he ought not to fight in the army, $O\sim F$, and we take it as an implicit premise that it really is all right, i.e., permitted or not forbidden,

\(^{20}\) See, e.g., [23] for a useful introduction to this kind of logical system and its motivations.
that Smith not fight but perform alternative service, $P(\neg F \land S)$. We then argue that Smith ought to perform alternative service ($OS$). In DPM.1:

i) $O(F \lor S)$  
ii) $O\neg F$  
iii) $P(\neg F \land S)$  
iv) $\vdash (\neg F \land S) \leftrightarrow ((F \lor S) \land \neg F)$  
v) $P((F \lor S) \land \neg F)$  
vi) $O((F \lor S) \land \neg F)$  
vii) $\vdash (O((F \lor S) \land \neg F) \rightarrow S)  
ix) OS$

In DPM.2 or DPM.3 the argument inserts (v)$' P((F \lor S) \land \neg F) \rightarrow O((F \lor S) \land \neg F)$ by (PAND) from (i) and (ii) to conclude (vi) by PC from (v).

Thus these systems seem well equipped to handle arguments like this, provided one is prepared to accept the implicit premise (iii). 21

It is worth noting that in case there were no deontic dilemmas, no violations of (D), then DPM.1 would agree wholly with SDL. That is, if (D) were added as an axiom to DPM.1, the result is equivalent to SDL. Hence, DPM.1 satisfies the criterion of adequacy (*) mentioned in Section 1. (This is not so for DPM.2 or DPM.3. Although (RM) can be derived from (RPM) and (D), the full proposition (AND) does not follow just given the weaker (PAND), along with (D) and (RM), etc. Thus, in a sense, DPM.2 and DPM.3 do not correspond to SDL in a dilemma-free universe. DPM.2 and DPM.3 plus both (D) and (AND) are, however, equivalent to SDL.)

4. Conclusion

The project of this paper was to develop a simple monadic deontic logic that would allow for deontic dilemmas without triggering deontic explosion in their presence, while it would also accept and account for the apparent validity of arguments like that of Smith’s service to his country:

I) a) $O(F \lor S)$  
b) $O\neg F$  
\therefore c) OS

As we saw in Section 2, this is not an easy task. Simple solutions, like denying ex falso quodlibet (EFQ) or the inheritance rule (RM) or the aggregation principle (AND), fail the latter aspect, if not worse, while more refined solutions, like restricting aggregation to consistent (CAND) or co-permissible (PAND) combinations, succumb to versions of deontic explosion.

21 For DPM.1 it suffices to have just $P\neg F$, or even $U\neg F$, and for DPM.2 $P\neg F \land PS$ will serve as well as the stronger $P(\neg F \land S)$, though the arguments are different.
The proposal that van Fraassen [34] put forward, as developed by Horty [18–20], briefly described in Sections 2.4.2 and 2.4.3, fares better, but at the cost of a significant shift in the foundations of deontic logic. Amongst other things, this framework requires that the status of a deontic sentence $O\alpha$ must always be relativized to an explicitly given set $\Gamma$ of basic norms. Then, while it avoids deontic explosion in the face of deontic dilemmas, it accounts for the argument (I) only in the sense that $O(F \lor S), O\neg F \vdash F OS$ in [19], or $!\neg F \vdash F OS$, but not $O(F \lor S), O\neg F \vdash OS$, in [18,20]. This does not even extend to have that $\Gamma \vdash F O(F \lor S)$ and $\Gamma \vdash O\neg F$ imply $\Gamma \vdash OS$, nor especially that $O(F \lor S), O\neg F \vdash OS$. Similarly, in Hansen’s reconstruction of van Fraassen’s ideas [15], and in the two-phase deontic logic of van der Torre and Tan [32], which also distinguish two discrete senses of ‘ought’ at work in the argument, we have only $O^2(F \lor S), O^2\neg F \vdash O^2 S$ or $\oplus(F \lor S), \oplus\neg F \vdash \oplus S$, respectively, but not their univocal counterparts. Consequently these approaches must hold that, taken at face value, the argument (I) is not, strictly speaking, valid, in an ordinary sense of validity. Rather, the appearance of validity is explained either by invoking a weaker notion of consequence, F-consequence, as in Horty [19], or by distinguishing the sense of ‘ought’ as it occurs in the premises from the sense as it occurs in the conclusion, or both.

The proposal I offer in Section 3 is simpler than these; it does not require a distinction between basic norms and derived norms and the explicit representation of the former. It does not require two senses of ‘ought’. It allows for deontic dilemmas and avoids the various versions of deontic explosion. It does this by restricting the inheritance rule to (RPM) or (RUM). As with the accounts derived from van Fraassen, this proposal too will say that the argument (I) is not, strictly speaking, valid as it stands. But it explains the appearance of validity quite differently. Although (I) is not valid, the argument

\[
\text{I'} \quad \begin{align*}
a) & \quad O(F \lor S) \\
b) & \quad O\neg F \\
d) & \quad P(\neg F \land S) \\
\therefore c) & \quad OS
\end{align*}
\]

is, by (RPM) and either (AND) or (PAND). If (I) appears to be valid, as it seems to, and as it seems to without referring to a specific set of background norms, $\Gamma'$, and without equivocating on the sense of $O$, that is because the additional premise (d) of (I)' is easily considered tacit in the context in which we view the argument. Thus we explain the apparent validity of (I) by taking it to be enthymematic for (I)', which is valid, in a quite ordinary sense of validity, under the rules of DPM. If, however, one tries to derive explosive consequences from a deontic dilemma in a similar way, by appealing to some implicit premise, that additional premise would contradict one of the propositions in the dilemma, and so could not be considered a tacit presupposition in that context. By limiting the rule of inheritance in this way, we can remain within the realm of elementary modal logic and keep to our basic intuitions, even as we accept the possibility of deontic dilemmas, but avoid deontic explosion, and still provide for the inferences that do seem valid.
Appendix A

In this appendix I dress the logics DPM in more formal clothing, their axiomatics and their semantics, and demonstrate that the forms of deontic explosion described in the main text are not derivable.

The language, $\mathcal{L}$, for DPM is a propositional language adequate for classical propositional logic plus the monadic deontic operator $O$ such that $OA$ is well-formed whenever $A$ is. ‘$A$’, ‘$B$’, ‘$C$’, etc. are variables for arbitrary formulas of $\mathcal{L}$. $A \rightarrow B$ is understood to be equivalent to $\neg A \lor B$ and to $\neg(A \land \neg B)$. $A \leftrightarrow B$ is $(A \rightarrow B) \land (B \rightarrow A)$. As usual, $PA \equiv \neg O \neg A$. $\top$ is any classical tautology in $\mathcal{L}$ and $\bot$ is $\neg \top$.

DPM.1 is the least set of formulas containing all classical tautologies of formulas of $\mathcal{L}$, plus all instances of

\[
\begin{align*}
\text{N)} & \quad O \top \\
\text{AND)} & \quad (OA \land OB) \rightarrow O(A \land B)
\end{align*}
\]

and closed under the rules

\[
\begin{align*}
\text{MP)} & \quad \text{if } \vdash A \rightarrow B \text{ and } \vdash A \text{ then } \vdash B \\
\text{RE)} & \quad \text{if } \vdash A \leftrightarrow B \text{ then } \vdash OA \leftrightarrow OB \\
\text{RPM)} & \quad \text{if } \vdash A \rightarrow B \text{ then } \vdash PA \rightarrow (OA \rightarrow OB)
\end{align*}
\]

DPM.2 is the least set of formulas containing all classical tautologies of formulas of $\mathcal{L}$, plus (N) as above and also all instances of

\[
\begin{align*}
\text{P)} & \quad \neg O \bot \\
\text{PAND)} & \quad P(A \land B) \rightarrow ((OA \land OB) \rightarrow O(A \land B))
\end{align*}
\]

and closed under the rules (MP), (RE) and (RPM), while DPM.3 is just like DPM.2 but without axiom (P). In all cases $\vdash$ represents membership in the appropriate system.

Since these are non-normal, but still classical modal logics, they are most easily interpreted in the neighborhood semantics familiar from Segerberg [30] or Chellas [1, Chapters 7–9]. The key idea is to take obligatoriness, or normative requirement, to be a property or attribute of propositions. A formula $OA$ is then true just in case the proposition expressed by $A$ has this property. More precisely, consider a proposition to be a set of possible worlds, and accordingly the proposition expressed by $A$ to be the set of worlds at which $A$ is true; designate that set $|A|$. Consider a property of such propositions extensionally, as a set of propositions, and thus a set of sets of possible worlds. Each possible world $a$ has associated with it a set, $O_a$, of propositions; these are the propositions that are obligatory (from the point of view of $a$). Hence, if $|A|$ is the proposition expressed by $A$ (on a model), i.e., the set of possible worlds where $A$ is true (on the model), then $OA$ is true (at $a$ on the model) just in case $|A|$ is a member of $O_a$.

More formally, define a neighborhood frame, $F$, to be a pair $(W, O)$ in which $W$ is a non-empty set of points, e.g., possible worlds, and $O$ is a function assigning every $a \in W$ a set, $O_a$, of subsets of $W$; i.e., $O_a \subseteq \wp W$. A model, $M$, is a pair $(F, v)$ where $F$ is a
neighborhood frame \( \langle W, O \rangle \), and \( v \) is a function assigning every atomic formula \( p \) of \( \mathcal{L} \) a subset of \( W \), i.e., \( v(p) \subseteq W \). A satisfaction relation \( \models \) is defined as usual, so that for any model \( M = \langle F, v \rangle \) on a frame \( F = \langle W, O \rangle \), for any \( a \in W \),

\[
T(p) \quad M, a \models p \text{ iff } a \in v(p) \\
T(\neg) \quad M, a \models \neg A \text{ iff } M, a \not\models A \\
T(\wedge) \quad M, a \models A \wedge B \text{ iff } M, a \models A \text{ and } M, a \models B \\
T(\vee) \quad M, a \models A \vee B \text{ iff } M, a \models A \text{ or } M, a \models B
\]

and in particular

\[
TO) \quad M, a \models OA \text{ iff } |A|_M \in O_a
\]

where \( |A|_M = \{ a \in W : M, a \models A \} \). \( |A|_M \) is the proposition expressed by \( A \) on the model \( M \).

As usual, when \( F = \langle W, O \rangle \) and \( M = \langle F, v \rangle \), \( M \) satisfies \( A \)--- \( M \models A \)--- iff \( M, a \models A \) for every \( a \in W \). \( A \) is valid on a frame \( F \)--- \( F \models A \)--- iff \( M \models A \) for every model \( M = \langle F, v \rangle \) on \( F \), and \( A \) is valid in a class of neighborhood frames \( \mathcal{F} \)--- \( \mathcal{F} \models A \)--- iff \( F \models A \) for every \( F \in \mathcal{F} \). A set of formulas \( S \) is sound with respect to a class of frames \( \mathcal{F} \) iff \( \mathcal{F} \models A \) for every \( A \in S \). \( S \) is complete with respect to \( \mathcal{F} \) iff for every \( A \) such that \( \mathcal{F} \models A \), \( A \in S \).

In [13] I proved the logics \( \text{DPM} \) sound and complete with respect to the classes of neighborhood frames \( F = \langle W, O \rangle \) that meet these conditions. For \( \text{DPM.1} \), for all \( X, Y \subseteq W \) and all \( a \in W \),

\[
a) \ W \in O_a \\
b) \text{If } X \in O_a \text{ and } Y \in O_a \text{ then } X \cap Y \in O_a \\
c) \text{If } X \subseteq Y \text{ and } X \in O_a \text{ and } \neg X \not\in O_a \text{ then } Y \in O_a
\]

Condition (a) validates (N), condition (b) validates (AND) and condition (c) validates (RPM). (RE) comes for free. For \( \text{DPM.2} \), take conditions (a) and (c) above, but modify condition (b) to

\[
b') \text{ If } X \in O_a \text{ and } Y \in O_a \text{ and } \neg (X \cap Y) \not\in O_a \text{ then } X \cap Y \in O_a
\]

for (PAND), and add the condition

\[
d) \emptyset \not\in O_a
\]

for (P). For \( \text{DPM.3} \), take conditions (a), (b)', and (c).

The proof of soundness for these systems is routine. The proof of completeness is complicated; it is not necessary, or possible, to repeat it here, but see [13] for details. There it was also established that these systems have the finite model property and hence are decidable.
Given the soundness of the systems, it is easy to demonstrate that neither (D) nor the various forms of deontic explosion described in the text are derivable in DPM. The following models, the first for DPM.1 and DPM.3, the second for DPM.2 and DPM.3. Let F1 = ⟨W, O⟩ with W = {a, b, c}, and Oa = {W, ∅, {a, b}}, and Ob = Oc = {W, ∅}. It is not difficult to show that F1 satisfies the conditions (a), (b) and (b)', (c), and (d) and is thus a frame for DPM.1 and DPM.3. Let F2 = ⟨W, O⟩ with W = {a, b, c}, as before, and Oa = {W, {a}, {b, c}} (and Ob = Oc = {W}). Here it is easy to show that F2 meets the requisite conditions (a), (b)', (c), and (d) for a frame for DPM.2 and DPM.3. Let M1 = ⟨F1, v1⟩, with v1(p) = W, v1(q) = {a, b} and v1(r) = {a} (and v1(s) = W for any other atomic formula s); let M2 = ⟨F2, v2⟩, with v2(p) = {a}, v2(q) = W and v2(r) = {b} (and v2(s) = W for any other s).

If M is M1 or M2, then M, a ⊨ Op, M, a ⊨ Op, p, M, a ⊨ Pr and M, a ⊭ Or. (Verification may be left to the reader.) Hence each model falsifies this instance of (DEX-3), (Op ∨ Oq) → ((Op ∧ O¬p) → (Pr → Or)), and corresponding instances of the other variants of deontic explosion (DEX), (DEX-1) and (DEX-2). These models also clearly falsify this instance of (D), Op → O¬p. Hence, by the soundness of the systems, neither (D) nor any of the variants of deontic explosion is derivable in DPM.22

References


22 This is a revised version of a paper [13] I presented to the annual meeting of the Society for Exact Philosophy, May 2004, and to DEON 2004. My thanks to all who participated in those discussions, and especially to John Horty at the SEP and Jörg Hansen at DEON for their comments.
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