Comparison of Flow-Distribution Models for Design of Water Distribution Networks with Redundancy

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Abstract

Five flow-distribution models are compared for optimal design of looped water distribution networks (WDNs) with redundancy. A two-stage iterative methodology, in which a WDN designed using LP in the first stage for the selected flow-distribution corresponding to all-pipes working condition, is checked in the second stage for its performance under failure conditions. The LP problem of first stage is modified to additionally include the flow-distribution for the most critical pipe-failure condition. The methodology is continued until network becomes satisfactory under multiple single pipe failures. It is observed that the Chiong’s flow-distribution model provided minimum cost design with redundancy for illustrative network.

Keywords: Design; Optimization; Redundancy; Reliability; Water Distribution Networks;

1. Introduction

The total cost of a water distribution network (WDN) is a function of the length, discharge and head loss in different links. Treating the pipe diameter as a continuous variable, the total cost of a gravity network can be written as [1-2]

\[
C_T = \sum_{i=1}^{X} K_{2} K_{1}^{m/r} L_{i}^{1+m/r} Q_{i}^{m/r} (h_i)^{-m/r} = \sum_{i=1}^{X} K_{2} K_{1}^{m/r} L_{i}^{1+m/r} Q_{i}^{m/r} (H_{i} - H_{j})^{-m/r}
\]

(1)
in which \( C_T = \) total cost of network; \( K_2 = \) link cost constant; \( K_1 = \) coefficient in head loss formula which depends upon the pipe material and diameter, type of flow, and the units of other terms; \( L = \) length of link \( x; \) \( m = \) exponent of diameter in cost-diameter relationship; \( p \) and \( r \) = exponents of discharge and diameter in pipe head loss formula; \( X = \) total number of links in the network; and \( H_i \) and \( H_j = \) hydraulic gradient level (HGL) values at upstream node \( i \) and downstream node \( j \) of link \( x \).

To obtain optimal flows resulting in a least cost network, it is necessary that all the parameters in Eq. (1) are considered simultaneously. However, this problem is NP-hard and no method exists that can consider all of them simultaneously and provide directly the global optimum solution. Recognizing this fact different researchers suggested different approaches to select good (not necessarily optimal) flow distributions. Since the index of \( L \), i.e., \((1+m/r)\), is much more than the indices of \( Q \) and \( h (pm/r \) and \(-m/r, \) respectively) it is natural to assume that the total cost of a network would be less if \( L \) is less. Considering this, Ridgik and Lauria [3] suggested “minimum spanning tree concept”; while Bhave [1] suggested “path concept” to identify a good branching configuration. The links retained in the identified branching configuration are designed to carry major flows and termed as primary links; while those necessary for only loop forming are designed to carry some minimum flow and termed as secondary links. Suribabu and Neelkanthan [4] suggested fixing flow-distribution such that \( SLQ \) in the network is minimized.

One of the basic purposes of a looped WDN is to meet consumer demands through alternate paths in case of failure of any of the pipe. When loop forming links are provided such that they are of either minimum sizes or designed to carry some minimum pipe, the basic purpose is defeated. Chiong [5], Tanyimboh and Templeman [6], Bhave and Gupta [7] suggested different flow-distribution models to have better flexibility in the network design. Chiong [5] proposed an algorithm to calculate pipe flows under maximization of their uniformity. To maximize flow uniformity, the set of pipe flows are assumed as a statistical series and are calculated in such a way so as to minimize the variance of the series subject to satisfying the constraints of node flow continuity at nodes Martinez [8]. Tanyimboh and Templeman [6] suggested path entropy model for deciding flow distribution for flexible network design; while Bhave and Gupta [7] suggested distribution of flows to incoming links of any node (starting from sink node) by assigning weights in inverse proportions to the path length for the design of networks that can accommodate fuzzy demands.

Basic Linear Programming (LP) methodology can be used to design a looped WDN for fixed flow distribution [9-10]. The basic LP method is extended to design level-1 redundant network [11] with the assumptions that nodal demands are concentrated at nodes and valves are provided at either ends of pipe. Thus, in the event of pipe failure a single link can be removed without cutting off any consumer from the system. In this paper, a comparison of designs for the two looped WDN [12] and Hanoi WDN [13] are carried out using five different flow-distribution models to achieve level-1 redundant design using the optimal design methodology of Gupta et al. [11].

2. Flow-distribution algorithms

2.1 Model Based on Path Concept

In a single-source branched WDN, there is one path from source to any demand node. Therefore, there is a unique flow-distribution. Let us assume that this branched network is converted to looped network by adding loop forming links. Now, in looped network alternate paths from source to demand nodes are available. For such a looped network infinite flow distributions are possible depending upon the direction of flow and the flow assigned to loop forming links.

Bhave [1] suggested an optimization methodology in which a looped network is converted to a branched network using path concept in which all demand nodes are connected to source node through shortest path. This branching configuration is designed to carry maximum flows by selecting loop forming links either of some minimum size or carrying some minimum flows. Flow direction in a loop-forming link is decided from node having higher hydraulic gradient level (HGL) requirement to that having lower HGL requirement.

2.2 Entropy Based Model

Tanyimboh and Templeman [6] suggested an optimization model for flexible design of WDNs based on the concept of entropy used with a goal of maximizing flow uniformity. The proposed method is path based and assumes that the
directions of flows in pipes are already known. Let us say, by fixing the same flow direction as obtained by analyzing the network with all links of minimum size. The methodology of flow-distribution is explained with an example network (Example Network 1) of Alperovits and Shamir [12] as shown in Fig. 1.

![Fig. 1. Example network 1 [12]](image)

All pipes in the network are of 1000 m length. Source head and nodal demands along with minimum required HGL values are shown in Fig. 1. The network is analyzed with all links of minimum size and obtained flow directions are shown in Fig. 1. To decide the flow distribution in the network, number of paths to each demand nodes is obtained. The number of paths to each demand nodes is equal to the summation of number of paths of its upstream nodes. Therefore, a simple procedure to calculate the number of paths is as follows: (1) First assign the number of path for source node as 1; (2) Select any node whose all upstream nodes have been processed. Add the numbers assigned to all nodes immediately upstream of the chosen node. Assign the total to the present node; and (3) Repeat step (2) until all nodes have been processed. In Fig. 1, number of paths to each demand node is shown in square box. The procedure of flow assignment starts from terminal node(s). Water demand at the terminal node is divided to upstream links in proportion to number of paths on upstream nodes. For example, water demand of node 7 is assigned to links 6 and 8 in a proportion of 1:2. These link flows are added to water demand of upstream nodes and process of flow-assignment continues until source node is reached. Yassin-Kassab et al. [14] extended the entropy based model for multi-source networks.

### 2.3 Bhave and Gupta Model

Bhave and Gupta [7] suggested an optimization model for design of WDNs considering uncertainty in design demands as fuzzy parameters. They suggested path concept for identifying the branched networks of primary pipes. Flow direction in secondary pipes is decided arbitrarily or based on HGL requirements as discussed above. Flow distribution in incoming pipes at a node, starting from the terminal (sink) node, is fixed by assigning weights inversely proportional to the path lengths. The process is continued until flows in all pipes are fixed. Path concept provided four alternative trees for the example network of Fig. 1. Herein, tree selected by Bhave and Gupta [7] is considered and the flow directions in secondary links are fixed as fixed by them.
2.4 Suribabu and Neelakantan Model

Suribabu and Neelakantan [4] suggested a two-stage optimization process in which flow distribution decided in the first stage is optimized using LP in second stage. Flow distribution for initially selected flow direction is obtained by optimizing the summation of product of length of the pipe and discharge through it subject to the constraints of node flow continuity and minimum and maximum flow in a pipe. They suggested that direction of flow can be fixed based on the natural gradient of nodal points and shortest possible path from source to maximum demand point. For the example network of Fig. 1, node 5 has maximum demand and minimum GL.

Flow direction in links 4, 7 and 8 is therefore decided towards node 5. Removal of these pipes leads to branched network in which flow direction can be fixed from source to demand point. Since pipe 1 has to carry the total flow in the example network, the maximum flow in a pipe is restricted to 1120 m³/h, i.e. total demand of the network and the minimum flow in a link is considered as 10 m³/h.

2.5 Chiong’s Model

Martinez [8] used Chiong’s model in optimal design of looped WDNs. Chiong [5] proposed an algorithm to calculate pipe flows under maximization of their uniformity. To maximize flow uniformity, the set of pipe flows are assumed as a statistical series and are calculated in such a way so as to minimize the variance of the series subject to satisfying the constraints of node flow continuity at nodes. The solution to the above optimization is set of equations describing algebraic summation of pipe flows in every loop to be zero. The solution methodology is simple and can be applied to multi-source network. There is no need to assign flow direction in advance and provides a unique solution.

<table>
<thead>
<tr>
<th>Pipe No.</th>
<th>Flows in pipes (m³/hr) using Path Concept</th>
<th>Entropy-based Model</th>
<th>Bhave and Gupta Model</th>
<th>Suribabu and Neelakantan Model</th>
<th>Chiong’s Model</th>
</tr>
</thead>
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<td>1120.00</td>
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<td>10.00</td>
<td>133.33</td>
<td>10.00</td>
<td>10.00</td>
<td>195.33</td>
</tr>
</tbody>
</table>

3. Network design with redundancy

3.1 Topologic Redundancy

To develop a level-1 redundant WDN, two separate types of redundancies must be established: topologic and hydraulic. While topologic redundancy assures the availability of a continuous physical path from a source to each demand nodes in the event of pipe failure, the hydraulic redundancy will ensure supply of required quantity of water at desired pressures. In a level-1 topological redundant single-source network each node must be connected with at least two pipes. Kessler et al. [15] suggested an algorithm to achieve level-1 topological redundancy and can be used. Herein, it can be observed from the two-loop network that the source node 1 is connected by only one pipe.

To achieve a level-1 topological redundancy, a parallel pipe of same size for pipe 1 is considered as no information about possible connectivity of node 1 with other nodes is available.
3.2 Hydraulic Redundancy

The methodology consists of two stages. In the first stage, an initial design of the system is obtained using LP for link flows obtained by considering one of the flow-distribution models (this initial design is also referred later as design under 0-pipe failure (0-PF) condition). In the second stage, the performance of network is evaluated using a Node Flow Analysis (NFA) under multiple single-pipe failure (1-PF) conditions.

The iterative procedure is terminated if the performance of the network is found satisfactory under both 0-PF and 1-PF conditions. Otherwise, the most critical pipe failure condition is identified and a new additional flow-distribution corresponding to critical pipe failure is passed on to first stage to improve the design. The iterative method is continued till network becomes satisfactory to sustain any single-pipe failure.

3.3 LP model for design of looped WDNs

The optimization model for a single source gravity fed looped WDN can be formulated as

Minimize $C_y = \sum_{x=1}^{X} \sum_{y=1}^{Y} c_{xy} L_{xy}$  \hspace{1cm} (2)

in which $L_{xy}$ = length of pipe size $y$ in link $x$. Note that the second summation gives the sum of the costs of $Y$ pipes in a link, while the first summation gives the sum of the costs of $X$ links.

Subjected to

$X$ link-length constraints

$$\sum_{y=1}^{Y} L_{xy} = L_x \text{, for } x = 1, ..., X$$  \hspace{1cm} (3)

$N$ path-headloss constraints.

$$\sum_{x \in P_j} \sum_{y=1}^{Y} S_{xy} L_{xy} \leq H_0 - H_j^\text{min} \text{, for all paths } P_j, j = 1, ..., N$$  \hspace{1cm} (4)

in which $S_{xy}$ = hydraulic slope for pipe size $y$ in link $x$; $H_0$ = HGL at the source node; and $H_j^\text{min}$ = minimum required HGL at the end node of path $P_j$.

$M$ loop-headloss constraints.

$$\sum_{x \in \mathcal{P}_m} \sum_{y=1}^{Y} S_{xy} L_{xy} \leq 0 \text{, for all loops } M, m = 1, ..., M$$  \hspace{1cm} (5)

and $XY$ non-negativity constraints

$L_{xy} \geq 0 \text{, for } x = 1, ..., X; y = 1, ..., Y$  \hspace{1cm} (6)
3.4 Node Flow Analysis for assessing network performance under pipe-failure conditions

The LP model for an assumed flow-distribution will provide a satisfactory network design for a normal working condition. This network may not be able to perform satisfactorily under failure conditions. The NFA provides available nodal flows by considering both available heads and nodal demands simultaneously, and is used to assess the performance of deficient network.

3.5 Identification of a most critical pipe failure condition

There could be several single pipe-failure conditions under which network may become deficient. However, total available flows would be different for different pipe-failure conditions. The pipe failure condition that shows the minimum total available flows (maximum deficiency) is the considered herein as most critical pipe failure condition.

3.6 Improvement in design and repetitions

To improve the design further the most critical pipe failure condition is considered. The most critical pipe is removed and a new flow-distribution is obtained by the selected flow-distribution model. The LP problem is reformulated by considering $M+N$ path head loss constraints for this new flow-distribution along with those previously incorporated. Thus, the total path head loss constraints at the $F$th iteration becomes $F \times (M+N)$, where $F$ is number of flow-distributions considered. Thus, an improved design will be satisfactory under normal working conditions and also under the considered critical pipe failure conditions.

The improved design is checked for performance under other pipe-failure conditions, and the above procedure is continued until no pipe is found to be critical (i.e. when demands at all the nodal are satisfied under failure of any single pipe).

4. Network designs for different flow-distributions for example network

The two-stage methodology is used to design example network by considering different flow-distribution model. Initially, flow-distribution with all pipes in working condition is obtained and network is designed using LP. It is observed that flow-distribution based on path concept provided minimum cost solution (416820 units). Since the flow distribution using Suribabu and Neelakantan’s model is also same as that obtained by model based on path concept, both the designs are similar. The design solution obtained by Chiong’s model which achieves more uniformity in flow distribution is costliest (431473 units).

The iterative methodology is continued to obtain level-1 redundant design solutions with various flow-distribution models. The iteration details are provided in Table 2. It can be observed from Table 2 that models based on path concept and Chiong’s model required 3 iterations to reach to final design values, while other models required only 2 iterations. For iterative design based on path concept, node flow analysis indicated deficient conditions during failure of 1, 2, 3, 5, 6, and 7. The most critical failure condition is observed to be of failure of pipe 1. Under this condition a parallel pipe of same diameter is added. On imposing the constraints for failure of pipe 1 in LP and redesigning, failure of pipes 2, 3, 5, 6 and 7 are observed to be critical. Failure of the pipe 3 is most critical. Again, on imposing the additional constraints for failure of pipe 3 in LP, and redesigning the network is observed to be deficient during failure of pipe 5. So, finally the constraints for failure of pipe 5 are added and with this the network is observed to be completely satisfactory. It can be observed from the layout of the network that pipes 1, 2 and 3 are near to the source and their failure is more critical. The LP formulation requires simultaneous consideration of only few failure conditions and it is more appropriate to include them iteratively [11].

The final design solutions for example networks 1 for various flow distribution models are given in Table 3. The minimum cost level-1 redundant design is obtained with the flow-distribution using Chiong’s model (845202 units), while flow-distribution based on path concept provided maximum cost solution (896164 units). Thus, level-1 redundant design solution by Chiong’s model is about 5.7% cheaper. Even though initial solutions for flow distribution based on path concept and model based on Suribabu and Neelakantan are same, the final solution is quite different.
Table 2. Critical pipe failure conditions for Example Network 1

<table>
<thead>
<tr>
<th>Iteration Number</th>
<th>Critical pipe failure conditions for flow-distributions based on Path Concept</th>
<th>Entropy Model</th>
<th>Suribabu and Neelakantan</th>
<th>Bhave and Gupta Model</th>
<th>Chiong’s Model</th>
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<td>-</td>
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<td>2</td>
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</table>

Bold face indicates the most critical failure pipe

Table 3. Final Level-1 Redundant Designs of Example Network 1

<table>
<thead>
<tr>
<th>Pipe No.</th>
<th>Network design for flow-distributions using Path Concept</th>
<th>Entropy Model</th>
<th>Suribabu and Neelakantan</th>
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5. Summary and Conclusions

Maintaining a looped nature of WDN through the constraints of minimum pipe size or minimum pipe flow provides least cost design, however, the performance of such network under pipe failure conditions would be poor. To provide flexibility in design by providing redundancy, several flow-distribution models have been suggested based on different heuristics. Such flexible designs result in higher costs. Gupta et al. (2014) proposed iterative use of LP to convert these flexible designs to level-1 redundant designs. Flow distribution models have been compared herein for providing level-1 redundant designs. It is observed that flow distribution model of Chiong that consider uniformity in flows in deciding flow distribution is best in providing level-1 redundant design.

^1 Indicates links with parallel pipes of same sizes.
References