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Mathematical Model of the Straight-line Rolling Tire – Rigid Terrain Irregularities Interaction

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Abstract

The article discloses a semi-empirical model of the zero-camber straight-line rolling tire – rigid terrain irregularities' interaction. Tire enveloping properties are modeled by a system of radial springs. During validation of the model, the authors compared the experimental and computed tire I-247 static load – deflection curves for the step and triangular prisms. The comparison showed that the model gives radial reactions which are lower than the experimental ones. The authors developed the method for the model correction based on the analysis of the tire – terrain overlap in the contact area at each calculation step. After the correction, the model showed a good accuracy for the tire I-247 deflected on the step and on triangular prisms. The authors plan to use the developed model for analysis of the cross-country capacity and dynamic loads of a wheeled vehicle driving across off-road obstacles.

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Keywords: wheeled vechicle; simulation; tire enveloping properties; radial spring tire model.

1. Introduction

During the period from the end of the 1960s to the beginning of the 1970s, a number of semi-empirical tire – rigid terrain interaction models based on the radial spring tire enveloping model and calculation of "equivalent volume" [1-3] were developed. Such models perform the three following operations at each calculation step:

- calculate magnitude and direction of the radial reaction force on the base of the tire elastic characteristics obtained during deflection on a flat surface;

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- replace uneven terrain with the equivalent plane providing the same magnitude and direction of the radial elastic reaction force;
- calculate friction forces in the equivalent plane by one of the friction models for the flat surface.

The main disadvantage of this approach is the underestimation of the reaction of the terrain irregularities when their curvature radius is considerably lower than the tire radius (specific tire radius – irregularity curvature radius ratio highly depends on the tire inflation pressure) [4, 5]. During comparative analysis of the cross-country capacity, vibration isolation and dynamic loads for different designs of the wheeled vehicles, when the qualitative behavior of the reactions is more important than the accurate values of the reactions, models of this type can be tolerated since they should simulate at least qualitative behavior of the reactions well. This feature together with the advantage of the small number and availability of the required initial parameters and high computational efficiency explains the renewed interest of the MBS software developers in these models in the mid1980s [6] and in the early 2000s [7, 8]. Models of this type are included into the libraries of the standard tire – road interaction models of several commercial MBS software packages. However, even the qualitative accuracy of these models turns out to be rather low, especially for irregularities with sharp edges. One of the possible ways of improving their accuracy is to correct radial reactions according to the shape of the terrain irregularities.

The present article discloses the variant of the radial spring tire model developed by the authors and shows the method for correction of the radial reactions by using the tire – terrain overlap factor obtained from the analysis of the tire – terrain contact area on each calculation step.

2. Model description

The model uses the following coordinate systems (see Fig. 1):

- road fixed coordinate system (FCS) $O_r X_r Y_r Z_r$ – rectangular coordinate system fixed to the terrain;
- wheel stability coordinate system (WSCS) $OXYZ$ – movable orthogonal coordinate system whose origin coincides with the wheel center, Z axis is directed along the Z_r axis of the FCS, X axis is perpendicular to the wheel rotation axis.Fig. 1 shows the diagram of the straight-line motion of the wheel in the plane $X_r Z_r$ of the FCS.

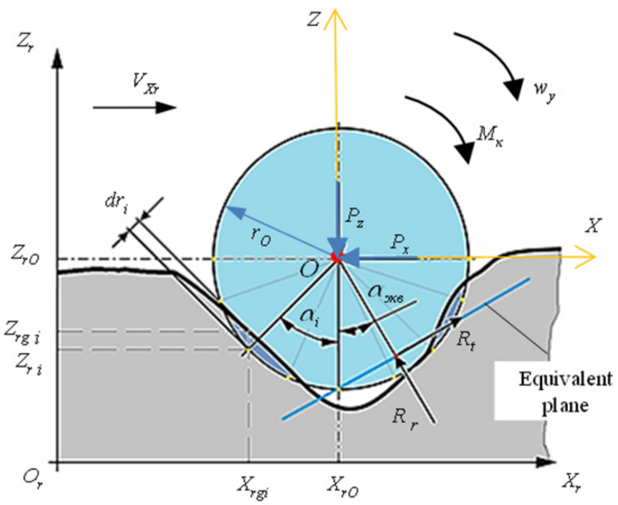

Fig. 1. Diagram of the straight-line motion of the wheel over irregular terrain.

Let us define n points on the lower half-circle of the non-deflected tire in-plane profile. Position of each point is defined by the angle α_i between the vertical line drawn from the center of the zero-camber wheel onto the X_r axis and by the ray connecting the point on the profile with the wheel center (see Fig. 2). The actual number of the selected points is the trade-off between the model accuracy and computational efficiency. Let us calculate the coordinates X_{ri} and Z_{ri} of the selected points of the tire profile in FCS:

$$X_{ri} = X_{r0} + r_0 \cdot \sin(\alpha_i);$$
$$Z_{ri} = Z_{r0} - r_0 \cdot \cos(\alpha_i); \qquad (1)$$
$$-\frac{\pi}{2} \le \alpha_i \le \frac{\pi}{2};$$

where X_{r0}, Z_{r0} – coordinates of the wheel center in the FSC.

Vertical coordinate Z_{ri} of the *i*th point of the non-deflected tire profile in the FCS can be computed by the following formula:

$$Z_{ri} = Z_{r0} - r_0 \cdot \cos(\alpha_i), \qquad (2)$$

where Z_{r0} – vertical coordinate of the wheel center in the FSC.

Tire radial deflection dr_i at the *i*th point of the profile can be calculated from the following equations:

$$dr_i = \begin{cases} 0, \; Z_{rgi} \le Zr_i \\ Z_{rgi} - Z_{ri} \sin|\alpha_i|, \; Z_{rgi} > Z_{ri} \end{cases} \qquad (3)$$

where Z_{rgi} – vertical coordinate of the terrain profile under the *i*th point of the tire.

First, let us compute the angle α_{eqv} between the vector of the resultant radial reaction R_r and the vertical line drawn from the wheel center on the X_r axis of the road FCS (see Fig. 3). The angle α_{eqv} can be found as the sum of the angles between the same vertical line and the radii of the tire profile points in contact with the terrain, taken with their weight factors:

$$\alpha_{eqv} = \sum_{i=1}^{n_C} (\alpha_i \cdot k_i), \qquad (4)$$

where n_c – number of the tire profile points in the contact with the terrain; k_i – weight factor:

$$k_i = \frac{dr_i}{\sum_{i=1}^{n_c} dr_i}. \qquad (5)$$

Radial reaction R_r is the sum of the elastic and damping components R_{re} and R_{rd}:

$$R_r = R_{re} + R_{rd}. \qquad (6)$$

R_{re} is the resultant force of the elastic components at every contact point and depends on the sum of the radial deflections of the tire points. Usually, the only data available is the “vertical reaction – tire vertical deflection” characteristic curve obtained on a flat surface. Typical curve for a flat surface is shown in Fig. 2. For a given unloaded tire radius and given number of points on the tire profile lower half-circle we can easily obtain the "vertical reaction – total radial deflection" curve (see Fig. 2). This new curve will be used for the computation of the elastic radial reaction R_{re} on an arbitrary irregular terrain on each calculation step:

$$R_{re} = f(\sum_{i=1}^{n_c} dr_i) \tag{7}$$

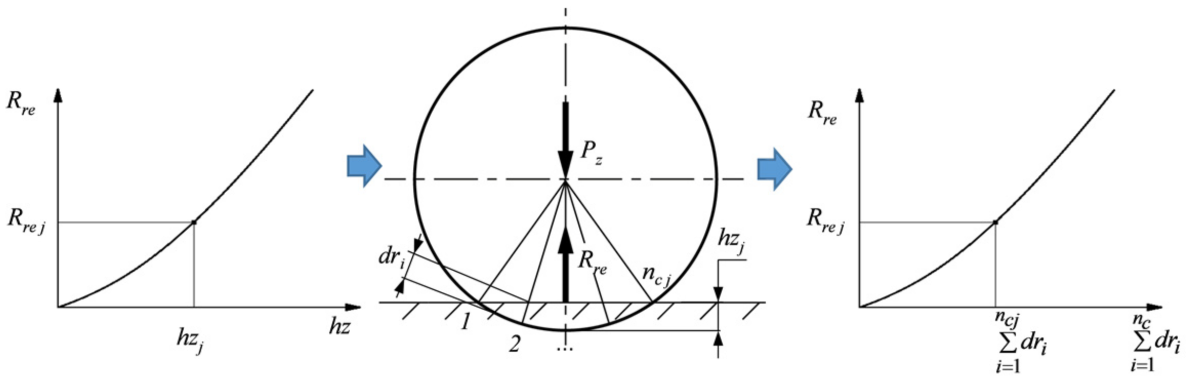

Fig. 2. Using the "vertical reaction – vertical deflection" curve to obtain the "vertical reaction – total radial deflection" curve.

Component R_{rd} depends on the tire radial deflection rate. Let us define the velocities of the tire in-plane profile points along the X_r and Z_r axes:

$$\begin{aligned} V_{Z_r i} &= \omega_y (r_0 - dr_i) \sin\alpha_i + V_{Z_r o}, \\ V_{X_r i} &= \omega_y (r_0 - dr_i) \cos\alpha_i + V_{X_r o}; \end{aligned} \tag{8}$$

where ω_y – wheel rotation velocity; $V_{X_r o}$ and $V_{Z_r o}$ – velocities of the wheel center *O* along the corresponding axes of the FCS.

Vector of the radial velocity of the *i*th point of the unloaded tire profile:

$$V_{ri} = V_{X_r i} \sin\alpha_i + V_{Z_r i} \cos\alpha_i. \tag{9}$$

Radial deflection rate of the *i*th point:

$$\frac{d}{dt}(dr_i) = \dot{Z}_{rgi} \cos\alpha_i - V_{ri}. \tag{10}$$

Equivalent deflection rate:

$$\frac{dr_{eqv}}{dt} = \frac{\sum_{i=1}^{n_c} \left(\frac{d}{dt}(dr_i) \right)}{n_c}. \tag{11}$$

Then, from the linear damping model we can find the damping component R_d :

$$R_d = -b_t \frac{dr_{eqv}}{dt}, \tag{12}$$

where b_t – tire damping factor.

Tangent reaction in the equivalent plane can be found from any friction model for a flat surface, for instance, from the model [1]:

$$R_\tau = \mu_S R_r . \tag{13}$$

Terrain reactions projected onto the axes of the WSCS:

$$\begin{aligned} &R_X = R_\tau \cos\alpha_{eqv} - R_r \sin\alpha_{eqv};\\ &R_Z = R_\tau \sin\alpha_{eqv} + R_r \cos\alpha_{eqv};\\ &M_Y = R_\tau \cdot r_{eeqv} + R_r \cdot r_{eeqv} \cdot f \cdot sign\left(\omega_y\right); \end{aligned} \tag{14}$$

where r_{eeqv} – distance from the wheel center to the equivalent plane.

3. Model validation and correction

Fig. 3a and 4 show load – deflection curves for the tire I-247 with internal pressure 3.5 atm. obtained from experiment [10] and calculated with the model on a step and on triangular prisms with different angles.

As can be seen from the figures, the load – deflection curves calculated by the model are passing too low when the contact surface is different from the flat surface, the less being the tire – terrain overlap area the lower passing the calculated curve in comparison to the experimental one. The most obvious way to correct the model is introduce into the equation (6) a tire – terrain overlap factor:

$$R_{re} = k_R \cdot f(\sum_{i=1}^{n_c} dr_i), \tag{15}$$

where k_R – tire – terrain overlap factor.

Factor k_R reflects the difference between the tire – terrain overlap on the given terrain and the overlap on the flat surface at the same deflection, i.e. implicitly reflects the difference of the terrain shape from the flat shape. Function for this factor calculation should meet the following criteria:

- its value should increase with the decrease of the overlap difference;
- its value should be equal to 1 for the flat surface.

One of the possible variants is to use the following simple function:

$$k_R = \frac{n_c}{n_{c0}}, \tag{16}$$

where n_c – number of the tire profile points in contact with the terrain; n_{c0} – number of the tire profile points in contact with the flat surface at the same total radial deflection;

Fig. 3b, 4b show load – deflection curves for the same tire calculated by the corrected model. Introduction of the adjustment factor increased the accuracy of the model. The maximum increase in accuracy was for the case of the low overlap between the tire and the irregularity. Maximum average relative error of the elastic reaction calculation is 24% (pos. 3) which is just about a halve of the error obtained with the initial model.

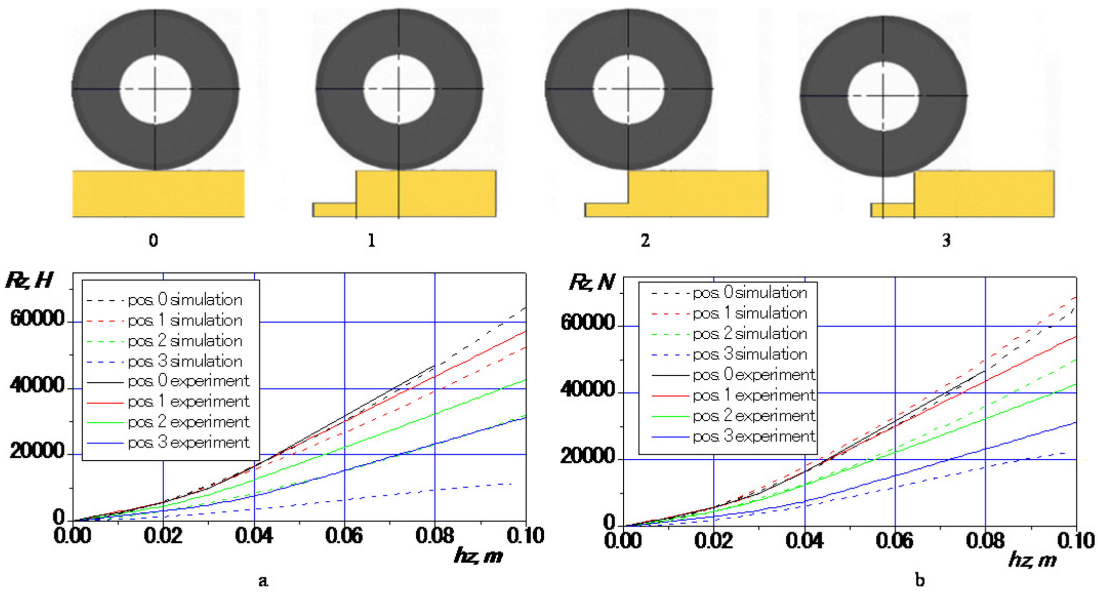

Fig. 3. I-247 tire load – deflection curves on the step for positions 0 – 3 (solid lines – experiment, dashed lines – simulation): (a) original model; (b) corrected model.

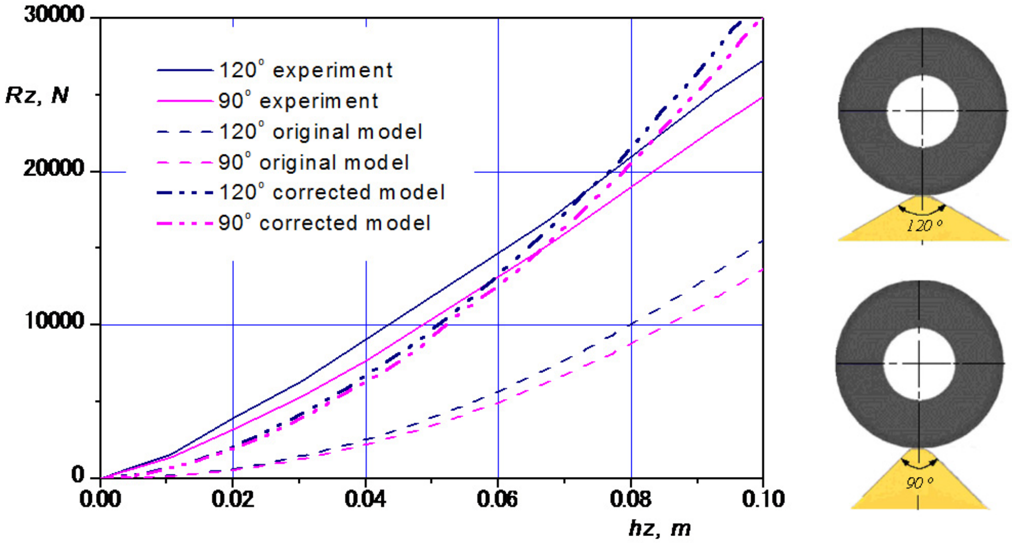

Fig. 4. I-247 tire load – deflection curves on the triangular prisms: solid lines – experiment; dashed lines – original model; dashed-dot lines – corrected model.

Insignificant difference between the calculated and experimental curves for the flat surface can be explained by the fact that the characteristics (7) was approximated by the following function:

$$R_{re} = P_{zs} \cdot \left(\frac{h_z}{h_{zs}} \right)^{1.5} = P_{zs} \cdot \left(\frac{\sum_{i=1}^{n_c} dr_i}{\sum_{i=1}^{n_{cs}} dr_i} \right)^{1.5}, \tag{17}$$

where P_{zs} – maximum static load on the tire; h_{zs} – deflection at the maximum static load; n_{cs} – number of the tire points in the contact with the terrain when the tire deflection is h_{zs}. This method of the tire load – deflection curve description is convenient since it requires low number of input tire parameters.

The accuracy of the radial reactions calculation can be further increased by using a more sophisticated exponential function for the tire – terrain overlap coefficient. However, in this case, each tire would need calculation of several additional parameters which is not always possible due to lack of the experimental data.

4. Conclusion

The method of the known radial-spring tire model correction provided considerable improvement of the model accuracy during calculation of the tire I-247 radial static reactions on the rigid uneven terrain without using additional model input data. The accuracy is improved due to analysis of the tire-terrain overlap at every calculation step.

The authors plan to use the corrected model for analysis of the cross-country capacity and dynamic loads of a wheeled vehicle driving across uneven rigid terrain.

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