Analysis and study of the aerodynamic turbulent flow around a blade of wind turbine

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Abstract:
The flow around a wind turbine is a set of forces applied by the wind on the blades defined by the most significant parameters from a dimensional analysis detailing the power of wind turbine. This later is criss-crossed by changes of turbulent intensity unlike the Reynolds number (essential parameter and not unique for the transition to turbulent flow). Thus the performance of the wind turbine depends on the specific speed that allows to get the most power coefficient by controlling the speed. So it’s studying the aerodynamics of the wind turbine side in a controlled environment without any uncertainties due atmospheric phenomena.

The equations of fluid mechanics (Navier-Stokes) allow the aerodynamic analysis of any flow that satisfy the minimum conditions around a wind turbine as well as factors related to the lift and drag airfoil describing geometry of the blade to the same profile as a function of the Reynolds number and the angle of attack formed between the relative flow and the chord of the blade section.

Keywords: wind turbine; aerodynamic; strain rates; turbulent intensity.

1. Performance of wind turbine

The possibility of regulating power of modern high-speed wind turbine (by adjusting blade angle and/or speed of rotation of the blades), unlike the older generation (passive stall control of the blade), will lead to a study of the stress of the blade by the incident wind (aerodynamic forces) and therefore of the flow around a wind turbine and the identification of the most revealing parameters for dimensional analysis of the power [12].

\[ P = \frac{1}{2} U_H^3 \cdot S_R \cdot \rho \cdot \left[ \frac{\alpha D}{U_H} \cdot \frac{\sigma_H}{U_H} \cdot \frac{\sigma_H}{U_H} \cdot \frac{\sigma_H}{U_H} \cdot \frac{\beta}{\psi} \cdot M \cdot \omega \cdot u' \right] \]  \hspace{1cm} (1.1)

\( U_H \): Speed of incident wind measured at the height (H) of the hub.
\( \Omega \): rotational speed of the blades.
\( D \): diameter of the wind turbine.
\( \sigma \): standard deviation measured on the horizontal component of wind during a period of average.
\( \rho, \mu \): density and dynamic viscosity of air.
Area swept by the blades and is \(\frac{\pi D^2}{4}\) for a rotor having a taper angle of zero.

**Cp**: Power coefficient is a dimensionless function of the following six parameters:

- **\(\Omega = \frac{aD}{U_H}\)**: Specific speed. In general, it is defined with the radius of the rotor rather than the diameter.
- **\(R_e = \rho U_H D/\mu\)**: Reynolds number based on the diameter of the wind turbine speed and the measured incident flow to the hub varies according to temperature: for the same wind speed, for example, it varies between 30% and -25 °C +25 °C.
- **\(\mathcal{E} = \frac{\sigma_H}{U_H}\)**: turbulence intensity at hub height;
- **\(G = \frac{U_Z}{U_H} = \frac{\sigma_Z}{\sigma_H}\)**: Normalized distributions and averaged in the time of the wind speed and its variance according to the trihedral (x, y, z), x being oriented in the direction of flow, z being the axis pointing vertically, while y is perpendicular to x and z.
- **G and J**: depend on the local topography as well as exchange sénérégétiques radiative, conductive and convective associated with the atmosphere or the ground surface.
- **\(\beta\)**: pitch angle of the blade.
- **\(\psi\)**: yaw angle (angle made by the incident flow relative to the axis of rotation of the turbine).

2. Aerodynamic analysis of the flow around the wind turbine:

The equations of fluid mechanics [continuity equation, quantity of movement (Navier-Stokes) and Bernoulli] are appropriate for the analysis of the aerodynamic flow satisfying minimal conditions (continuous medium, moderate Mach number, Newtonian fluid) around a wind turbine [6]. The analysis of the Navier-Stokes equations is based on the geometry of solid surfaces exposed to the flow of air particles, which are functions of time t and the volume dv. In wind turbine, turbulence plays a major role throughout the flow as well as length and temporal scales are in the millimeter and tenth of a second [1].

The equation of motion of an incompressible viscous fluid is

\[
\frac{D}{Dt} \int_V \rho \ u \ d\tau = \int_V \ f \ d\tau + \int_S \ \sigma \ n \ dS
\]

Where S is the surface bounding the volume V, ds is a surface element of normal n, f is the force exercised per volume unit and \(\sigma\) is the stress tensor.

The temporal derivation of the first term is a Lagrangian derivation, i.e. the fluid particles motion.

The mass of the fluid element remains constant. Thus it is possible to write the first term in the following form:

\[
(f_V \rho \frac{Du}{Dt} \ d\tau).
\]

The sum of surface forces could be written as (Theorem of Ostrogradsky):

\[
\int_V \ \nabla \ \sigma \ d\tau
\]

Where \(\nabla \sigma\) is the divergence of the stress tensor, a vector whose component (i) is: \(\frac{\partial \sigma_{ij}}{\partial x_j}\).

Letting the volume (V) to zero, the equation of motion becomes:

\[
\frac{\rho Du}{Dt} = f + \nabla \ \sigma.
\]
The decomposition of the Lagrangian derivative of the velocity components (sum of the Eulerian derivative and convective acceleration). In a flow, acceleration of a particle of fluid comprises, in general, two inputs, the first is, due to the variation over time of the speed at each point of the flow (non-stationary flow), the second is due to the exploration of a non-uniform velocity field even when the flow is stationary, if the flow is not uniform, a particle of fluid will recognize during its displacement more areas high or low speed, resulting in an acceleration term "convective"[4]. The first contribution to the acceleration is simply the time derivative of the Eulerian velocity: \( \frac{\partial \mathbf{u}}{\partial t} \)

If a fluid particle is at \( (r_0, t) \), it traveled in \( (\delta t) \) a distance

\[ |\delta r = u(r_0, t)\delta t + O(\delta t^2)|. \]

The fluid speed at point \( (r_1 = r_0 + \delta r) \) will then be:

\[ u(r_1, t) = u(r_0, t) + \nabla u \cdot \delta r \quad (2.3) \]

The fluid particle acceleration is: \( \nabla u \cdot \delta r / \delta t \) with \( \lim_{\delta t \to 0}(\frac{\delta r}{\delta t}) = \mathbf{u} \).

and the convective acceleration is written: \( (u, \nabla u) \) such as the fluid particle total acceleration which is the particle derivate of speed in a Galilean reference is written:

\[ \frac{D\mathbf{u}}{Dt} = \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \quad (2.4) \]

\[ \frac{du}{dt} = \frac{\partial u}{\partial t} + u_j \frac{\partial u}{\partial x_j} \]

Hence the distribution of an index implies addition over that index.

\[ u_j \frac{\partial u}{\partial x_j} = \frac{\partial u_i}{\partial x_j} = u_1 \frac{\partial u}{\partial x_1} + u_2 \frac{\partial u}{\partial x_2} + u_3 \frac{\partial u}{\partial x_3} \]

\( (\frac{du}{dt}) \): is the rate of change per unit time of the speed related to a point driven by the movement and the velocity gradient \( \nabla \mathbf{u} \) is a tensorial quantity with components \( (\frac{\partial u_i}{\partial x_j}) \) and the component is:

\[ \frac{D\mathbf{u}_i}{Dt} = \frac{\partial \mathbf{u}_i}{\partial t} + \sum_j u_j \frac{\partial \mathbf{u}_i}{\partial x_j} \]

convective acceleration projection of the velocity gradient on the local direction of flow.

The contribution of surface forces at the i component of the equation of motion (function of the torque constraints for a Newtonian fluid in motion) is written:

\[ \sigma_{ij} = -p \delta_{ij} + 2\eta e_{ij} \quad (2.5) \]

The movement equation is:

\[ \rho \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \eta \Delta \mathbf{u} + \mathbf{f} \quad (2.6) \]

Where

\[ \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho} \nabla p + \theta \Delta \mathbf{u} + \frac{1}{\rho} \mathbf{f} \quad (2.7) \]

The Newtonians fluid behavior (Navier-Stokes). Where \( \theta \) is the fluid kinematic viscosity.
3. **Torsor of strain rates (stress tensor) in a moving fluid:**

Let \((x_i)\) the coordinates of the points:

\[ x_{iA}' = x_{iA} + u_{iA} \cdot dt \cdot dx_{iA} \cdot dx_{iA}' \]

Coordinates of \(\overline{AB}\) and \(\overline{AB}'\), hence:

\[ dx_{iA}' = dx_{iA} + \frac{\partial u_{i}}{\partial x_{j}} dx_{jA} \cdot dt \]

The velocity gradient tensor \((\frac{\partial u_{i}}{\partial x_{j}})\) allows the calculation of the evolution of a length element during its movement, which has a symmetrical component \(\delta_{ij}\) (torsor strain rates or another deformation and antisymetric \(d_{ij}\) which is related to tourbillion vector \((\omega = \frac{1}{2} \boldsymbol{\omega} \cdot \boldsymbol{\omega})\).

\[
\omega_i = \frac{1}{2} \varepsilon_{ijkl} \frac{\partial u_{l}}{\partial x_{k}}
\]

\(\varepsilon_{ijkl}: \text{Alternator tensor} \)

\[
\varepsilon_{ijkl} = \begin{cases} 
1 & \text{(ijkl : all different in the cyclic order)} \\
-1 & \text{(ijkl : all different in other order)} \\
0 & \text{(at least two indices are equal)} 
\end{cases}
\]

\(d_{ij} = -\varepsilon_{ijkl} \cdot \omega_k \) and \(\omega_i = \frac{1}{2} \varepsilon_{ijkl} \cdot d_{kl}\) hence the velocity gradient is:

\[ \frac{\partial u_{i}}{\partial x_{j}} = \delta_{ij} - \varepsilon_{ijkl} \cdot \omega_k \]

\(\delta_{ij}\): Fluid element deformation.

\(-\varepsilon_{ijkl} \cdot \omega_k\): Simple deformation.

The rotation of the blade is produced by the total moment resulting from the sum of elementary moments tangential forces exerted at different distances \(r\) along the blade. For a number of blades \(B\), the elementary time, \(dQ\) of a section located at a distance \(r\), with wide \((dr)\) and the curl \((c)\), is given by:

\[ dQ = \frac{1}{2} Br u r^2 (C_L \sin \phi - C_D \cos \phi) c \cdot r \cdot dr \]

The concept of Newtonian fluids shows that the viscosity constraints are consequences of deformation. Global rotation does not produce viscous stresses. Constraints for a Newtonian compressible fluid:

\[ \tau_{ij} = \lambda_{\mu} \frac{\partial u_{i}}{\partial x_{j}} \cdot \delta_{ij} + 2\mu \cdot \delta_{ij} \]

where \(3\lambda_{\mu} + 2\mu = 0\) (hypothesis of Stokes). (3.1)

In incompressible, only the coefficient of dynamic viscosity occurs in computation since \(\frac{\partial u_{i}}{\partial x_{j}} = 0\). And the viscous forces are the force of pressure forces to sense the surface having the outer normal volume bounded by the surface of their application.
The movement equation is in (2.6) and (2.7):

\[ \frac{\partial \sigma_{ij}}{\partial x_j} = -\frac{\partial \rho}{\partial x_i} + \eta \frac{\partial^2 u_i}{\partial x_j \partial x_j} \] (3.2)

which describes the Newtonian fluid behavior (Navier-Stokes) where \( \theta \) is the fluid kinematic viscosity.

In the absence of flow, if the only force in this volume is the gravity, we find from the Navier-Stokes law of fluid statics:

\[ \nabla \! p = \rho \, g \] (3.3)

In a fluid at rest, the stress tensor is isotropic (no tangential stress). On each axis the normal stress is the opposite of the pressure \( (\sigma_{ii} = -p) \)

The contribution of the pressure is negative, because the fluid is compressed (normal stress is positive traction). The pressure thus defined is identical to the pressure, in a thermodynamic view it can be defined from the equation of state of fluid. In a fluid in motion, the stress tensor is a sum of two contributions, one isotropic (pressure \( p \)) and another anisotropic (fluid viscosity), for the component \( \sigma_{ij} \) of the tensor [5]. Considering that the quantity speed is defined per unit mass:

\[ \frac{d}{dt} \int_V \rho \, u \, d\tau = \int_V \frac{\partial \rho u}{\partial t} \, d\tau + \int_S \rho \, u_j \cdot \vec{u}_{nj} \cdot ds = \int_V \left( \frac{\partial \rho u}{\partial t} + \frac{\partial \rho u_j u}{\partial x_j} \right) \] (3.4)

Which expresses the rate of change of the quantity \( u \) contained in the volume \( V \) driven by the movement with \( (\int_V \frac{\partial \rho u}{\partial t} \, d\tau ) \): rate of change of the quantity \( u \) within the field \( (\int_S \rho u_j \cdot \vec{u}_{nj} \cdot ds) \) Outflow of the quantity \( (u) \) across the surface \( S \).

Where \( (\vec{u}_{nj}) \) is the unit vector normal to the surface \( S \) and directed towards the outside this. S: area identifying the volume \( V \) of air particles formed. The balance of amount of fluid into and out of a reference volume \( (V) \), fixed relative to the coordinate system which is expressed in the Eulerian speed. The variation per unit of time of the mass contained in the volume \( (V) \) is equal to that traversing, per unit of time, the surface \( S \) that delimits the volume \( (V) \) driven by the movement. is:

\[ \int_V \frac{\partial \rho u}{\partial t} \, d\tau = - \int_S \rho \, u_j \cdot \vec{u}_{nj} \cdot ds \] (3.5)

Using the theorem of Ostrogradsky to transform the second member in integral volume and inverting the temporal differentiation and integration in the first part, we obtain

\[ \int_V \frac{\partial \rho}{\partial t} + \text{div}(\rho u) \] (3.6)

This equality gives us the local expression of the conservation of mass: \[ \frac{\partial \rho}{\partial t} + \text{div}(\rho u) = 0 \]

\[ \frac{\partial u}{\partial t} + u \cdot \nabla \rho + \rho \cdot \text{div}u = \frac{\partial \rho}{\partial t} + \rho \cdot \text{div}u = 0 \] (3.7)

The total of first two terms on the left is the derivative "particulate" (following the movement of fluid) density. If the fluid is incompressible, the density does not change over time and the equation of mass conservation reduces to:

\[ \text{div}u \equiv \nabla \! u = 0 \]

This equation expresses the conservation of volume of a fluid element during its deformation by the flow.

The equation of mass conservation for \( u = 1 \)ds, we obtain \( \frac{d}{dt} \int_V \rho d\tau = 0 \). So the continuity equation is:

\[ \frac{\partial \rho}{\partial t} + \frac{\partial \rho u_j}{\partial x_j} = 0 \] (3.8)
The rate of change of the quantity contained in the volume \( V \) in motion is equal to the integral of the amount \( \rho \frac{du}{dt} \).
In incompressible turbulent flow, the speed is decomposed according to the method of ensemble mean: average:
\[
[u_i] = \langle u_i \rangle + u'_i,
\]
from which the continuity equation is written:
\[
\frac{\partial}{\partial x_j} \langle u_j \rangle + \frac{\partial}{\partial x_j} u'_j = 0 \quad \text{and} \quad \frac{\partial}{\partial x_j} \langle u_j \rangle = 0,
\]
demonstrating that the fluctuating flow is divergence zero.

On the other hand, the mean particle derived:
\[
\frac{du}{dt} = \frac{\partial u}{\partial t} + \langle u_j \rangle \frac{\partial}{\partial x_j} u,
\]
which expresses the rate of change of the quantity related to a point which moves with the movement means. The derived mean particle has the same properties as those of the particular derivative.

4. Equations for solving the incompressible flow:

The resolution is based on the treatment of local and global equations, therefore it is made incompressible by the equations of continuity and momentum principles for a mean field.

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\] (4.1)

\[
\rho \frac{\partial u}{\partial t} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x} \left( \mu \frac{\partial u}{\partial x} - \rho < u' v' > \right)
\] (4.2)

Pressure is a function of \( x \), connected to the external velocity by:
\[
\frac{dp}{dx} = -\rho_e U_e \frac{du_e}{dx}
\] (4.3)

The unknown factors of the equation (4.2) are \([U(x, y), V(x, y)\) and \(< u' v' > (x, y)\) so we have two equations for three unknowns which gives us an open system (closure performed by Reynolds): \(-\rho < u' v' > \) the average speed field.

\[
-< u' v' > = \theta_L \frac{\partial u}{\partial y}
\] (4.4)

\( \theta_L \): eddy viscosity which is not a physical property of the fluid so the equation (4.1) does not solve the problem, it is the product of a velocity \( u \) and a length \( l \) \((\theta_L = u. l)\) local turbulence characteristics

5. Equation of vorticity (vortex):

In turbulent flow (where the vorticity is significant) the velocity field is given by the vorticity: \( \omega = \nabla \times u \) Taking the curl of the Navier-Stokes equations, we obtain the evolution equation of vorticity. Where it is assumed that the force derives from a potential:

\[
\frac{\partial \omega}{\partial t} + u. \nabla \omega = \omega. \nabla u + \delta \omega + \frac{1}{\rho^2} \nabla p \nabla p
\]

Where the derivative of the Lagrangian vorticity is a sum of two terms of production and another dissipation due to viscosity.

A first-term production which is the dot product of vorticity and velocity gradient, reflecting the reorientation of the vector dot product \((\omega. \nabla u)\) and extension (or compression) of vorticity; this extension process is vital for procreation of the turbulence.

Second involving the density gradient. This term \( \text{génaration barocline} \) of the vorticity is zero if the pressure gradient and density gradient are parallel (Refers to a vertical structure of the lower atmosphere in which the isobars and isotherms intersect). This term occurs in a vertically stratified fluid by gravity and subjected to a horizontal pressure gradient, where the dense fluid starts to move more slowly than the lighter fluid, resulting ungradient speed and a vertical component of vorticity perpendicular to the plane defined by \( \nabla p \) and \( \nabla p \).

In a boundary layer:

\[
( -< u' v' > \sim u^2).
\]

Who can translate this viscosity by:
\[ -\langle u'_i, u'_j \rangle + \frac{2}{3} k \delta_{ij} = \partial_t \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \]  

(5.1)

\[ k = \frac{\langle u'_i, u'_i \rangle}{2} \text{: Turbulence kinetic energy.} \]

So (2.3) became:

\[ \left| \frac{u}{L} \right| \sim \frac{\partial u}{\partial y} \]  

(5.2)

Average velocity field requires the time of the turbulence).

An air particle in transverse movement from A to B, L apart, at the initial time (position A), with a speed \( U_A \), time t (in B), having a speed \( (U_B) \).

Speed gap \( (U_A - U_B) \) is a turbulent fluctuation (\( u' \)):

\[ u' = U_A - U_B = -L \frac{\partial u}{\partial y} \]  

(5.3)

An average quadratic

\[ \langle u'^2 \rangle = \langle L^2 \rangle \left( \frac{\partial u'}{\partial y} \right)^2 \]

if we assume that:

\[ -\langle u'v' \rangle \sim \alpha < u'^2 > \]

So,

\[ \frac{\alpha u'v'}{\partial y} \sim \frac{l}{c^2} \left( \frac{\partial u}{\partial y} \right)^2 \]

where \( l = c^{1/2} < L^2 >^{1/2} \)

is a length and a mixture of (characteristic length of turbulence to a boundary layer), expressions of mixing length:

\[ \frac{l}{\delta} = 0.085 \theta (\frac{x}{0.085 \cdot y}) \]  

(5.4)

For this report with all its variations is a link between the scale of turbulence and boundary layer thickness and almost constant equal to 0.085 respecting the term of the linear variation with y, given as: \( [l = \chi, y] \); where the apparent viscosity

\[ \partial_l = l^2 \left( \frac{\partial u}{\partial y} \right) \]  

(5.5)

which linear function of y and

\[ \partial_l = \alpha U_e \delta_1 y_i \]  

(5.6)

\( \delta_1 \); is a thickness of displacement and the constant \( \alpha = 0.0168 \)

Are only valid in the turbulent area concerned, so at the border of the flat plate, it is essential to consider the Viscosity:

\[ \gamma_i = \frac{1}{1 + \frac{5.5(\delta_1)^6}{\delta}} \]

\( \gamma_i \); Intermittency factor, usually taken equal to (1) since there is little difference on the profile of average speeds. [7]

The damping of a fluctuation in speed under the effect of viscosity near the plate is governed by Stokes in the case of a harmonic oscillation for a flow parallel to a plate infinite [10].

The expression of the amplitude of the oscillation [7]:

\[ u' = u_0 (1 - e^{-\gamma y}) \]

\[ y_s = \frac{\gamma u_s}{\nu} \text{: Variable Stokes} \]

\( U_s \) : speed related to the angular frequency \( \omega \) and the viscosity, the comments and turbulent velocity which is proportional to the friction velocity \( U_f \).

\[ U_s = (\omega \theta)^{1/2} \]  

(5.7)

where
around airfoils

The presence of a solid body moving in traffic gives rise to a lift force which will be a calculation of potential flow and moving (seat of discontinuities in the flow velocity) then determine the relative speed, it is necessary to remove

\[ -\langle u' v' \rangle = -\langle u_0' v_0' \rangle (1 - e^{-ys})^2 \]

\[ -\langle u' v' \rangle = \chi^2 y^2 \frac{\partial n}{\partial y} (1 - e^{-ys})^2 \]

(5.8)

These models mixing length or apparent viscosity can process the boundary layer equations, and consequently the velocity field which reflects the average thickness of displacement or the coefficient of friction [7].

6. The bearing surfaces:

The presence of a solid body moving in traffic gives rise to a lift force which will be a calculation of potential flow around airfoils so this is the calculation of the angle of incidence of the mean flow which is identical in the presence or absence of this movement the absence of movement, the stagnation point is located on the rear upper part of the wing and not on the trailing edge. The power line adjacent to the low side of the wing must overcome the sharp trailing edge which explains the divergence of the velocity and velocity gradient at the edge, really viscous flow. The current lines on the top, near the leading edge become choked. This fluid acceleration induced, according to Bernoulli's law, a significant depression at the top of the wing.

The surface is a porous actuatrice area on which establish discontinuities velocity and pressure. The translation occurs or absence of this movement the absence of movement, the stagnation point is located on the rear upper part of the wing and not on the trailing edge. The power line adjacent to the low side of the wing must overcome the sharp trailing edge which explains the divergence of the velocity and velocity gradient at the edge, really viscous flow.

The model is based on a actuatrice area the validation of the airfoil is mathematic model treating the wording on the kinematics and dynamics of the flow to solve the problem of blade rotation.

The surface is a porous actuatrice area on which establish discontinuities velocity and pressure. The translation occurs along the X axis, while the surface is flat and perpendicular to the direction of the axis Z. By convention, the discontinuities are applied in the direction given by the Z axis, that is to say that in appointing the Zp actuatrice along the X axis, while the surface is flat and perpendicular to the direction of the axis Z. By convention, the discontinuity of a variable (May be replaced à u, v ou w : the components of the speed of flow or the pressure p) the surface through is equal à : (\( \Delta = Zp_+ - Zp_- \)) is a function of position X, Y of the actuatrice area and quantities are limit values of both sides of the actuatrice area.

Since the surface is a actuatrice area vortex, it must meet certain criteria, the net flux of vorticity, which must remain zero through any closed surface of the flow. For the actuatrice area, this translates into the following constraint on the discontinuities (\( \Delta u \) and \( \Delta v \)) speed components along the axes X et Y:

\[
\frac{\partial \Delta v}{\partial x} - \frac{\partial \Delta u}{\partial y} = 0.
\]

Is a constraint that has its equivalent in flow using panel methods and analytical methods using the principle of lifting line, when one considers the wake, this constraint ensures the continuity of the vertical component of vorticity through actuatrice the surface, namely: [\( \omega_{Zp+} = \omega_{Zp-} \)]

At the time contact of a particle on a surface porous actuatrice supposed, is the momentum of a particle that will be a sudden change (solicited by a pulse accompanying the change of momentum).

\[
f_x = \rho \omega_{moy} \Delta u, \quad f_y = \rho \omega_{moy} \Delta v \quad \text{and} \quad f_z = -\rho (u_{moy} \Delta u + v_{moy} \Delta v)
\]

(6.1)

\( f_x \) et \( f_y \): Components of the system of forces associated with the actuatrice area.
\( \rho \omega_{moy} \): Mass flow through the actuatrice area (per unit area).
\( \Delta u, \Delta v \): The discontinuity quantity of movement along X and Y.

\( u_{moy}, v_{moy} \) et \( \omega_{moy} \): Components of the speed (Training) of the flow over the actuatrice area and since this surface and moving (seat of discontinuities in the flow velocity) then determine the relative speed, it is necessary to remove the actuatrice area speed of flow which will pose a problem whose only solution is to consider the actuatrice area as static.

\[
u_{moy} = \frac{u_{Zp+} + u_{Zp-}}{2} = \omega_{Zp+} - \frac{\Delta u}{2} = \omega_{Zp-} + \frac{\Delta u}{2}
\]

(6.2)

\( W_{moy} \): Component orthogonal to the surface actuatrice continues (along \( \bar{Z} \)) and is intense at the tips of the wings, important in their studies and that of the induced drag the actuatrice area wif the streamlines of the flow (no not the fluid particle crosses) becomes equivalent to a vortex sheet justifying the orthogonal component to the water (Bernoulli) with the existence of a static pressure gap(\( \Delta P \)) and a total pressure constant.

\[
P_{Zp-} + \frac{1}{2} \rho (u_{Zp-}^2 + v_{2p-}^2 + w_{2p-}^2) = P_{Zp+} + \Delta P + \frac{1}{2} \rho (u_{Zp+}^2 + v_{2p+}^2 + w_{2p+}^2)
\]

(6.3)

The power exerted by the surface forces on the particles actuatrice fluid the disposal is:
\[ W = f_X \cdot u_{moy} + f_Y \cdot v_{moy} + f_Z \cdot w_{moy} = 0 \]

for a total pressure constant.

7. Concept of actuator disc

The rotor of a helix can be represented by a vortex sheet distributed on the porous surface of a disc, result of the process when making soft limit the number of blades to infinity \[6\], while maintaining constant the rotor solidity \[Y\]. The solidity (\(\sigma\)) rotor is the dimensionless number which kept constantly. As we increase the number of blades the problem persists, the form factor of each blade increases and the drag takes the form of canceling. Disc actuator is no longer able to model the physics of a wind turbine rotor on the influence of the wake on the aerodynamics of the blade. \((\sigma = \frac{Bc}{\pi R})\),  

B : the number of blades  
c: length of the chord of a section of the blade  
R: radius of the rotor.  

The actuator disk made propellers \[3\] or helicopter rotors \[2\] can be schematically represented by a surface on which actuatrice introduce discontinuities in velocity equal to the average value of the circulation around the profile with the rotor blades (Fig. 3). These discontinuities in speed have an azimuthal component, with an additional radial component in the case where the actuator disk has a non-zero thickness \[11\], which vanishes if the thickness becomes zero.  

The flow of a fluid through a section of the blade is analyzed by treatment of the aerodynamic forces (lift and drag) produced by a flow on a section of the blade are calculated using the third Newton law (principle of action and reaction) giving the expression of forces acting at the actuator disk.
Conclusion

Boundary layer in the static pressure is constant along the orthogonal to the wall. The velocity profile measured in a flat plate boundary layer is the general effect of turbulence, the latter improves the mixing of the flow. (Fig.1) and (fig.2).

The study of the flow of a viscous incompressible fluid around the wind turbine blade is similar to using and applying the equations of fluid mechanics (continuity equation, momentum-and Navier-Stokes-Bernoulli) for a flat plate.

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