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Quantum holonomies for an electric dipole moment

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ABSTRACT

In this Letter, we obtain the quantum holonomies for a neutral particle with a permanent electric dipole moment based on the analogue effects of the He–McKellar–Wilkens effect and the Scalar Aharonov–Bohm effect and show a new proposal for implementing one-qubit quantum gates.

1. Introduction

At present days, geometric quantum phases [1] and quantum computation [2] are the center of several studies around the world. The possibility of implementing the quantum computation via non-abelian geometric quantum phases [3] was proposed by Zanardi and Rasetti [4], and it has became important due to its stability [5]. In recent years, a great deal of works has shown different applications of quantum holonomies in quantum computation [6–15]. The holonomic quantum computation is based on adiabatic evolutions, where unitary operators $U(\lambda)$, called holonomies, act on the subspace spanned the eigenvectors of a family of Hamiltonians $\mathcal{F}=\{H(\lambda)=U(\lambda)H_0U^{\dagger}(\lambda);\lambda\in\mathcal{M}\}$ (λ corresponds to the control parameter [4]). When the unitary operator $U(\lambda)$ acts on an initial state $|\psi_0\rangle$ belonging to the control manifold $\mathcal{M}$, it brings the initial state to a final state $|\psi\rangle=U(\lambda)|\psi_0\rangle$, where this action gives rise to a quantum gate [16]. The general expression of the action of this unitary operator is given by: $|\psi\rangle=U(\lambda)|\psi_0\rangle=e^{-i\int_0^t E(t')\,dt'}\Gamma(\lambda)|\psi_0\rangle$. The first factor $e^{-i\int_0^t E(t')\,dt'}$ corresponds to the dynamical phase, while the second factor $\Gamma(\lambda):=\mathcal{P}\exp\int_C\mathcal{A}$ is the holonomy. The object $\mathcal{A}=A(\lambda)\,d\lambda$ corresponds to a connection 1-form called the Mead–Berry connection 1-form, and the quantity $A(\lambda)$ corresponds to the Mead–Berry vector potential [1]. The components of $A(\lambda)$ are defined as: $A^{\alpha\beta}=\langle\psi^{\alpha}(\lambda)|\partial/\partial\lambda|\psi^{\beta}(\lambda)\rangle$. It has been shown in Ref. [17] that the dynamical phase can be omitted if we redefine the energy levels, and study the appearance of geometric phases in any cyclic evolution of the quantum system. Hence, it has became interesting to study the holonomic quantum computation either in cyclic evolutions or in noncyclic evolutions [18,19]. Examples of non-adiabatic cyclic evolutions are the Aharonov–Bohm effect [20], the Scalar Aharonov–Bohm effect [21–23], the dual of the Aharonov–Bohm effect [24,25], the Aharonov–Casher effect [26], and the He–McKellar–Wilkens effect [27]. Analogue effects of the Scalar Aharonov–Bohm effect, the Aharonov–Casher effect and the He–McKellar–Wilkens effect have been studied in noncommutative quantum mechanics [28], Lorentz-symmetry violation background [29] and in the presence of topological defect [30–33]. Recently, the holonomic quantum computation has been studied based on the Aharonov–Casher effect [34–36].

In this Letter, we consider a neutral particle with a permanent electric dipole moment, thus, and based on the analogue effects of the Scalar Aharonov–Bohm effect for neutral particle with permanent magnetic dipole moment [30] and the He–McKellar–Wilkens effect [31–33], we calculate the quantum holonomies associated with these analogue effects and show a new proposal for implementing one-qubit quantum gates. We start by doing a brief review of the mathematical formulation of the spinor theory in the presence of curvature and torsion. Thus, we present the background of this work based on a torsion field, and review the analogue effect of the He–McKellar–Wilkens effect in the presence of torsion. In the following, we obtain the analogue effect of the Scalar Aharonov–Bohm effect for a neutral particle with a

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permanent electric dipole moment in the presence of a dislocation. At the end, we discuss the quantum holonomies based on the analogue effects of the Scalar Aharonov–Bohm effect and the He–McKellar–Wilkens effect, and how to implement the holonomic quantum computation.

Spinors in curved spacetime must be defined locally, where spinors transform under infinitesimal Lorentz transformations [37]. Thus, we introduce a local reference frame for observers through the noncoordinate basis given by a 1-form basis $\hat{\theta}^a = e^a{}_\mu(x)\,dx^\mu$, which satisfies the condition: $g_{\mu\nu}(x) = e^a{}_\mu(x)e^b{}_\nu(x)\eta_{ab}$, where $\eta_{ab} = \mathrm{diag}(+++)$. The components $e^a{}_\mu(x)$ are called triads, and have an inverse defined as $dx^\mu = e^\mu{}_a(x)\hat{\theta}^a$. Triads are related to the inverse via $e^\mu{}_a(x)e^a{}_\nu(x) = \delta^\mu{}_\nu$ and $e^a{}_\mu(x)e^\mu{}_b(x) = \delta^a{}_b$. The Latin indices indicate the local reference frame (flat space), while the Greek indices indicate the curved space. When we consider the presence of torsion, the same mathematical formulation to define the spinors is used [38]. However, in the presence of torsion, the expression of the covariant derivative of a spinor changes in relation to that one given only in the presence of curvature. The components of the covariant derivative of a spinor in the presence of curvature and torsion are given by [38]

$$\nabla_\mu = \partial_\mu + \frac{i}{4}\omega_{\mu ab}(x)\Sigma^{ab} + \frac{i}{4}K_{\mu ab}(x)\Sigma^{ab}, \tag{1}$$

where we have a connection 1-form $\omega^a{}_b = \omega_\mu{}^a{}_b(x)\,dx^\mu$ related to the curvature and a connection 1-form $K_{\mu ab}(x)$ related to the torsion. By following Ref. [38], the connection 1-form $K_{\mu ab}(x)$ can be defined in terms of the contortion tensor via $K_{\mu ab}(x) = K_{\beta\nu\mu}[e^\nu{}_a(x)e^\beta{}_b(x) - e^\nu{}_b(x)e^\beta{}_a(x)]$. Moreover, the contortion tensor is related to the torsion tensor via $K^\beta{}_{\nu\mu} = \frac{1}{2}(T^\beta{}_{\nu\mu} - T_\nu{}^\beta{}_\mu - T_\mu{}^\beta{}_\nu)$, where the torsion tensor is antisymmetric in the last two indices while the contortion tensor is antisymmetric in the first two indices. An interesting effect is the coupling between the torsion and spinors [38]. By writing the torsion tensor into three irreducible components, the trace 4-vector $T_\mu = T^\beta{}_{\mu\beta}$, the axial 4-vector $S^\alpha = \epsilon^{\alpha\beta\nu\mu}T_{\beta\nu\mu}$, and the tensor $q_{\beta\nu\mu}$, which satisfies the conditions $q^\beta{}_{\mu\beta} = 0$ and $\epsilon^{\alpha\beta\nu\mu}q_{\beta\nu\mu} = 0$, the torsion tensor becomes: $T_{\beta\nu\mu} = \frac{1}{3}(T_\nu g_{\beta\mu} - T_\mu g_{\beta\nu}) - \frac{1}{6}\epsilon_{\beta\nu\mu\gamma}S^\gamma + q_{\beta\nu\mu}$. It was shown in Ref. [38] that the trace 4-vector T^μ and the tensor $q_{\beta\nu\mu}$ decouple to fermions, while the axial 4-vector S^μ couples to spinors. We shall show this in the following.

Let us discuss the quantum dynamics of a neutral particle with a permanent electric dipole moment interacting with external fields. The quantum dynamics of this neutral particle is described by introducing a nonminimal coupling $i\gamma^\mu\partial_\mu \to i\gamma^\mu\partial_\mu - i\frac{d}{2}\Sigma^{\mu\nu}\gamma^5 F_{\mu\nu}(x)$ into the Dirac equation [39,40], where $F_{\mu\nu}(x)$ is the electromagnetic tensor and $\Sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$. In the presence of curvature and torsion, the partial derivative must be changed by the covariant derivative of a spinor defined in (1). Thus, the γ^μ matrices correspond to the Dirac matrices defined in curved space, and are related to that in flat space via $\gamma^\mu = e^\mu{}_a(x)\gamma^a$, where γ^a corresponds to the usual Dirac matrices in flat spacetime [41]. It has been shown in Ref. [32] that the nonrelativistic limit of the Dirac equation in the presence of curvature and torsion is given by (with $\hbar = c = 1$)

$$i\frac{\partial\psi}{\partial t} = \frac{1}{2m}(\vec{p} + \vec{\Xi})^2\psi - \frac{d^2B^2}{2m}\psi + d\vec{\sigma}\cdot\vec{E}\psi + \frac{1}{8}\vec{\sigma}\cdot\vec{S}\psi \tag{2}$$

where $\vec{\sigma}$ are the Pauli matrices, and we have introduced the vector $\vec{\Xi}$ whose components are [31,32]

$$\Xi_i = -d(\vec{\sigma}\times\vec{B})_i - i\xi_i - \frac{1}{8}S^0\sigma_i. \tag{3}$$

We have defined the components of the vector $\vec{\xi}$ in (3) as $\xi_i = -\frac{1}{4}e^\varphi{}_i(x)\omega_{\varphi jk}(x)\Sigma^{ij}$. Note the spin–torsion coupling given by the last term of Eq. (2).

From now on, we will work with a background given by a topological defect in a crystalline solid. In crystalline solids, linear topological defects are built by applying the Volterra process [42], which consists in of a process of “cut and glue”. The analogy between linear topological defects in solids and the three-dimensional gravity was proposed by Katanaev and Volovich [43], where a defect corresponds to a torsion, a singular curvature, and both of them along the line of the defect. By using the approach of Katanaev and Volovich [43], a continuous distribution of defects can be described through the differential geometry, where all the information about the stress and strain induced by the defect in an elastic media are contained in the geometric quantities like the metric, the curvature tensor, etc. Studies of linear topological defects can be found in the literature in the context of classical mechanics [44–47], and as a background in quantum systems [48–53].

Let us present the background of this work. In this work, we work with a background made of a linear topological defect called dislocation. By using the Katanaev–Volovich approach [43], a dislocation is described by the following line element [45,46]

$$ds^2 = d\rho^2 + \rho^2\,d\varphi^2 + (dz + \chi\,d\varphi)^2, \tag{4}$$

where the parameter χ is a constant, and it is related to the Burgers vector [42]. Note that, there is no presence of curvature in this defect, but there exists the presence of torsion [46]. By using the formulation of the spinor theory in curved space [37], we define the triads in the form:

$$\begin{aligned}
\hat{\theta}^1 &= \cos\varphi\,d\rho - \rho\sin\varphi\,d\varphi;\\
\hat{\theta}^2 &= \sin\varphi\,d\rho + \rho\cos\varphi\,d\varphi;\\
\hat{\theta}^3 &= dz + \chi\,d\varphi.
\end{aligned} \tag{5}$$

Now, we need to get the information about the torsion of the defect. This can be obtained by solving the Cartan structure equations [54] $T^a = d\hat{\theta}^a + \omega^a{}_b \wedge \hat{\theta}^b$, where $\omega^a{}_b = \omega_\mu{}^a{}_b(x)\,dx^\mu$ corresponds to the connection 1-form given in (1), and T^a is the torsion 2-forms which is related to the connection 1-form $K_{\mu ab}(x)$ in (1) and, consequently, it is related to the 4-axial vector given in (2). Moreover, the operator d is the exterior derivative and the symbol $\wedge$ means the wedge product. Thus, by using the triads (5) and by solving the Cartan structure equations, we do not obtain any non-null component of the connection 1-form $\omega_\mu{}^a{}_b(x)$, but we obtain one non-null component of the torsion 2-form [32]

$$T^3 = 2\pi\chi\,\delta(\rho)\delta(\varphi)\,d\rho\wedge d\varphi. \tag{6}$$

It has also been shown in Ref. [32], if we consider a linear distribution of magnetic charges λ_m on the symmetry axis of the dislocation and a neutral particle with a permanent electric dipole moment, that this linear distribution of magnetic charges produces a radial magnetic field $\vec{B} = \frac{\lambda_m}{\rho}\hat{\rho}$ which interacts with the electric dipole moment, and a phase shift on the wave function arises from this interaction. The phase shift associated with this interaction in the presence of a dislocation is

$$\begin{aligned}
\phi_{\mathrm{HMW}} &= -d\oint(\vec{\sigma}\times\vec{B})_\varphi\,d\varphi - \frac{1}{8}\oint S^0\sigma_i e^i{}_\mu(x)\,dx^\mu\\
&= -2\pi\lambda_m\sigma^3 - \pi\chi\sigma^2.
\end{aligned} \tag{7}$$

The phase shift (7) is known as the analogue effect of the He–McKellar–Wilkens effect [27] given in the presence of a topological defect [32]. Of course, magnetic charges do not exist in the nature, thus, several discussions about the topological nature of the quantum phase of permanent electric dipoles have been done in the last decade [55]. In order to reproduce the same field configuration of the He–McKellar–Wilkens setup [27], several experiments have been proposed [56–58]. Based on the experiment of Ref. [58], where it was considered a long ferromagnetic wire electrically charged whose magnetization is parallel to the wire direction and where the magnitude of magnetization changes linearly along the wire, geometric quantum phases for a neutral particle with permanent magnetic and electric dipole moments were studied in [59]. In the same way done in Ref. [59], we can consider the radial magnetic field obtained in the experiment of Ref. [58], and obtain the phase shift (7).

At this moment, let us consider the presence of a uniform electric field $\vec{E}=\mathbf{E}_0\hat{z}$ parallel to the z axis of the dislocation (4). In this case, the Schrödinger–Pauli equation becomes

$$i\frac{\partial\psi}{\partial t}=\frac{1}{2m}\left(\vec{p}-\frac{1}{8}S^0\vec{\sigma}\right)^2\psi+d\vec{\sigma}\cdot\vec{E}\psi. \tag{8}$$

Hence, by using the Dirac phase factor [60], we can write the solution of the Schrödinger–Pauli equation (8) in the form: $\psi=e^{i\phi}\psi_0$, where ψ_0 is the solution of the following equation:

$$i\frac{\partial\psi_0}{\partial t}=\frac{\hat{p}^2}{2m}\psi_0. \tag{9}$$

Thus, the phase shift acquired by the wave function of the neutral particle with permanent electric dipole moment in the presence of a dislocation is

$$\begin{aligned}\phi_{\text{SAB}}&=-d\int_0^\tau\vec{\sigma}\cdot\vec{E}\,dt-\frac{1}{8}\oint S^0\sigma_i e^i{}_\mu(x)\,dx^\mu\\&=-d\mathbf{E}_0\tau\sigma^3-\pi\chi\sigma^2.\end{aligned} \tag{10}$$

The phase shift (10) corresponds to the analogue effect of the Scalar Aharonov–Bohm effect for a neutral particle with a permanent electric dipole moment. We must note that, by taking $\chi=0$ into (10), the phase shift corresponds to the Scalar Aharonov–Bohm effect for a neutral particle obtained by Anandan in [40]. We can see that the geometric phase (10) is a non-abelian phase and it does not depend on the velocity of the neutral particle, that is, it is a nondispersive phase [61,62]. Moreover, we must note that we have obtained the non-abelian geometric phase (10) without making the adiabatic approximation, which constitutes a phase shift like the Aharonov–Anandan quantum phase [17].

From now on, we discuss a way of implementing one-qubit quantum gates based on the quantum holonomies provided by the non-abelian geometric phases ϕ_{HMW} given in (7), and ϕ_{SAB} given in (10). The holonomy associated with the analogue effect of the He–McKellar–Wilkens effect is given by

$$U_1(\omega,\chi)=e^{i\phi_{\text{HMW}}}=e^{-i\omega\sigma^3-i\chi\pi\sigma^2}, \tag{11}$$

where we have defined the parameter $\omega=2\pi\,d\lambda_m$. The holonomic quantum computation based on the He–McKellar–Wilkens effect can be achieved by using ω and χ as control parameters. The control parameter ω is related to the intensity of the radial magnetic field, thus, any choice of the values of ω corresponds to changing the intensity of the magnetic field. On the other hand, the control parameter χ is related to the strength of the dislocation. In general, one can measure the strength of the dislocation in a crystalline solid by using crystallography techniques in the laboratory. Hence, the parameter χ can be considered a control parameter in the sense that we can verify the strength of the dislocation before building the interferometry experiment. The parameter χ is related to the Burgers vector $\vec{b}$ by $\chi=b/2\pi$. In crystalline solids, the intensity of the Burgers vector is of order of some interatomic distances. In this way, b is of order of Angstroms, and the contribution of the defect to the topological phase is small, thus, the paths around of the defect must be repeated many times in order to obtain a significant contribution to this holonomy.

Furthermore, the holonomy associated with the analogue effect of the Scalar Aharonov–Bohm effect for neutral particles is given by

$$U_2(\lambda,\chi)=e^{i\phi_{\text{SAB}}}=e^{-i\lambda\sigma^3-i\chi\pi\sigma^2}, \tag{12}$$

where we have defined another parameter $\lambda=d\mathbf{E}_0\tau$. In this case, the holonomic quantum computation based on the Scalar Aharonov–Bohm effect for neutral particles can be achieved by considering λ and χ as control parameters. Now, we have that the control parameter λ is related to the intensity of the electric field.

Now, we wish to show that we can make any arbitrary rotation on the permanent electric dipole moment of the neutral particle by using the holonomy (12) associated with analogue effect of the Scalar Aharonov–Bohm effect for neutral particles. First of all, we consider the logical states being the projections of the electric dipole moment on the z-axis of the dislocation, that is, $|0_L\rangle=|d_+\rangle$ and $|1_L\rangle=|d_-\rangle$. The states $|d_+\rangle$ and $|d_-\rangle$ correspond to the projections of the electric dipole moment parallel and antiparallel to the z-axis of the dislocation. Thus, by taking the values of the control parameters λ and χ of the holonomy (12) in the range $0<\lambda<1$ and $0<\chi<1$, we can apply the Zassenhaus formula $e^{A+B}=e^Ae^Be^{-\frac{1}{2}[A,B]}\cdots$ (where A and B are matrices) on the holonomy (12). In this way, we have

$$U_2(\lambda,\chi)\approx e^{-i\lambda\sigma^3}e^{-i\chi\pi\sigma^2}e^{-i\lambda\pi\chi\sigma^1}. \tag{13}$$

By using the definition of function of a matrix, $e^A=\sum_{i=0}^{\infty}\frac{A^n}{n!}$, we can write the quantum holonomy (13) in the form:

$$U_2(\lambda,\chi)\approx\alpha_0 I+\alpha_1 i\sigma^1-\alpha_2 i\sigma^2+\alpha_3 i\sigma^3, \tag{14}$$

where the parameters α_i are

$$\begin{aligned}\alpha_0&=\cos\lambda\cos\pi\chi\cos\lambda\pi\chi+\sin\lambda\sin\pi\chi\sin\lambda\pi\chi;\\\alpha_1&=\sin\lambda\sin\pi\chi\cos\lambda\pi\chi-\cos\lambda\cos\pi\chi\sin\lambda\pi\chi;\\\alpha_2&=\cos\lambda\sin\pi\chi\cos\lambda\pi\chi+\sin\lambda\cos\pi\chi\sin\lambda\pi\chi;\\\alpha_3&=\cos\lambda\sin\pi\chi\sin\lambda\pi\chi-\sin\lambda\cos\pi\chi\cos\lambda\pi\chi.\end{aligned} \tag{15}$$

Hence, we have shown that we can make any arbitrary rotation on the electric dipole moment of the neutral particle by applying the quantum holonomy (14) on the logical states $|0_L\rangle=|d_+\rangle$ and $|1_L\rangle=|d_-\rangle$, which constitutes a universal set of one-qubit quantum gates [63]. Thus, we have seen that we can implement the holonomic quantum computation based on the analogue effect of the Scalar Aharonov–Bohm effect for neutral particles without making the adiabatic approximation.

Comparing with the model proposed in Ref. [34] for the holonomic quantum computation based on the Aharonov–Casher setup, where the quantum computation is performed when a particle placed at site b encircles the other particle placed at site a, and the logical states are defined as $|0\rangle=|\mu(a)q(b)\rangle$ and $|1\rangle=|q(a)\mu(b)\rangle$, we can see that we have proposed a simpler model than the model of Ref. [34] because the logical states are given by the projections of the permanent electric dipole moment.

The same procedure can be made to achieve the holonomic quantum computation based on the analogue effect of the He–McKellar–Wilkens effect. In this case, we change λ to ω in the expression (14), and we obtain the quantum holonomy associated with the analogue effect of the He–McKellar–Wilkens effect. Since the logical states are the same of the last case, thus, we also have a universal set of one-qubits quantum gates.

In conclusion, we have obtained the analogue effect of the Scalar Aharonov–Bohm effect for a neutral particle with a permanent electric dipole moment in the presence of a dislocation, and we have shown a new proposal for implementing one-qubit quantum gates based on the analogue effects of the Scalar Aharonov–Bohm effect and the He–McKellar–Wilkens effect. We have seen that the presence of a dislocation produces a new contribution to the geometric phases which allow us to make any arbitrary rotation on the logical states given by the projections of the electric dipole moment on the z-axis of the dislocation. Any rotation on one-qubit can be performed by using the intensity of the field and the parameter related to the strength of the dislocation as control parameters. We should note that in the Holonomic Quantum Computation proposed in Ref. [4], the quantum gates are realized in an abstract parameter space, where we define transversing paths in an energetically degenerate subspace of this abstract parameter space. In our proposal for implementing one-qubits quantum gates, the topological phases (7) and (10) are generated in a physical space spanned by the states of the electric dipole moment. Hence, the quantum holonomies associated with the topological phases (7) and (10) do not need any path ordering in order to implement one-qubit quantum gates. Since the Scalar Aharonov–Bohm effect for a neutral particle with permanent magnetic dipole moment was observed in [64], the Scalar Aharonov–Bohm effect in the presence of a dislocation seems to be physically acceptable for implementing the holonomic quantum computation. Furthermore, the quantum holonomies associated with the He–McKellar–Wilkens effect add a new theoretical discussion in the field of the holonomic quantum computation.

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