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A Nonlinearity-Retaining State Estimation for Three-Phase Distribution System

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Abstract

A three-phase distribution system state estimation algorithm is proposed in this paper based on the retaining of nonlinearity. It takes advantage of the characteristics of the radial feeder and is suitable for systems with high R/X ratios. Considering the imbalance of a three-phase distribution system, this method is suitable for the observed system which involves real-time measurements and some power and voltage metered information treated as pseudo measurements. Because the method retains the nonlinear second order terms, the Jacobian matrix is constant and the second order terms are easier to compute and computing time is faster. Theoretical analysis and digital simulations are carried out and the results show that the method is reliable, efficient and requires less memory, suits on-line application for radial distribution systems.

Keywords: distribution system, state estimation, three-phase imbalance, nonlinearity

1. Introduction

State estimation for a distribution system is to estimate the state of the distribution system reliably, accurately and completely using its measurement information. Since the distribution system has its own characteristics compared with the transmission system, the state estimation for the distribution system is very different from that for the transmission system, which is mainly embodied in the following aspects: (1) the distribution system is assumed a radial structure; (2) it also has high R/X ratios; (3) imbalanced three-phase; (4) lack of real-time measurements on the distribution system. Therefore, the state estimation method is suitable for transmission systems, such as the fast decoupled state estimation method and is liable to result in a numerical stability problem which renders it inapplicable. Since the last couple of
years, scholars at home and abroad have proposed some distinctive new methods [1]-[3] in considering the characteristics of the distribution system. Based on the research of the distribution system state estimation, a three-phase distribution system state estimation algorithm is proposed in this paper based on retaining nonlinearity in rectangular coordinates. It takes advantage of the characteristics of the radial feeder and is suitable for its wide range of the R/X ratios. Considering the imbalance of a three-phase distribution system, this method is suitable for the observed system which involves real-time measurements and pseudo measurements of power and some voltage measurements. According to the elements proposed in this paper, we programmed an application software and ran simulations for IEEE 13 and 14 bus standard systems [4]. The results showed that the algorithm has the feature of great efficiency, reliability and practicability.

2. A Algorithm of Nonlinearity-Retaining State Estimation for Three-Phase Distribution System

2.1 Basic WLS State Estimation

Given a network connection, parameter of branches and measurements system, the nonlinear measurement function can be denoted by

\[ z = h(x) + v \] (1)

Where \( z \) represents the measurements vector, \( h(x) \) is the vector computed with the state variable vector \( x \), and \( v \) is the vector of measurement errors. Given measurements for vector \( z \), the state variable vector \( \hat{x} \) is the value for which the objection function is minimized

\[ J(x) = (z - h(x))^T R^{-1} [z - h(x)] \] (2)

where \( R^{-1} \) is a matrix whose diagonal elements are measurement weights. To obtain \( \hat{x} \), we evaluate partial derivative of both sides of (2), and make it equal to zero,

\[ \frac{\partial J(x)}{\partial x} = -H^T(x) R^{-1} [Z - h(x)] \] (3)

where \( H(x) = \frac{\partial J(x)}{\partial x} \) is an \( m \times n \) Jacobian matrix derived from the measurement vector [5].

In the state estimation of a radial distribution system, voltages of root node feeders are treated as accurate values instead of an estimation in order to improve the accuracy of calculations. Since there are no electric relationships among the feeders except for the root node, each feeder’s state can be respectively estimated to reduce both computation and memory requirements.

Furthermore, power flowing on the parallel grounding branches are converted into power injected into the corresponding bus. Thus, the system can be treated as being without grounding branches. So, the number of branches of radial feeder with \( N \) nodes is \( I = N - 1 \) and the diagonal elements of the nodal admittance matrix is

\[ Y_{ii} = - \sum_{j=1, j \neq i}^{N} Y_{ij} \]
2.2 A Nonlinearity-Retaining State Estimation Using Rectangular Form

A nonlinearity-retaining method is applied to solving the power flow problem. Considering the second derivative and the constant Jacobian matrix, both of the speed of computing and convergence characteristics are improved. Let us begin with introducing the algorithms proposed in [6], [7]. Consider the equation below,

\[ y' = y(x) \]  

(4)

where \( y' \) is an \( n \times 1 \) specified vector; \( y(x) \) is an \( n \times 1 \) vector of homogeneous quadratic function; \( x \) is an \( n \times 1 \) unknown variable vector.

Equation (4) can be expressed in exact Tailor series expansion without truncation errors:

\[ y' = y(x^{(0)}) + H \Delta x + y(\Delta x) \]

(5)

which shows that the third term of the Tailor series expansion of the equation which has the algebraic equation in the form of (4) has the same form as the first term but different variables \( \Delta x \) instead of \( x^{(0)} \). Therefore, the second derivative can be computed easily.

Measurement functions can be written under rectangular coordinates as follows,

\[
\begin{bmatrix}
    Z_p \\
    Z_Q \\
    Z_V^2
\end{bmatrix} = \begin{bmatrix}
    P^{(0)} \\
    Q^{(0)} \\
    V^{2(0)}
\end{bmatrix} + \begin{bmatrix}
    K & N \\
    T & L \\
    J_a & J_b
\end{bmatrix} \begin{bmatrix}
    \Delta f \\
    \Delta e
\end{bmatrix} + \begin{bmatrix}
    s_P \\
    s_Q \\
    s_{V^2}
\end{bmatrix}
\]  

(6)

where

\[
K_{ij} = \begin{cases}
-(B_{ij}e_i - G_{ij}f_i) & j \neq i \\
\sum_{j=1}^{i-1} (G_{ij}f_j + B_{ij}e_j) - B_{ii}e_i + G_{ii}f_i & j = i 
\end{cases}
\]

\[
N_{ij} = \begin{cases}
(G_{ij}e_i + B_{ij}f_i) & j \neq i \\
\sum_{j=1}^{i-1} (G_{ij}e_j - B_{ij}f_j) + G_{ii}e_i + B_{ii}f_i & j = i 
\end{cases}
\]

\[
T_{ij} = \begin{cases}
-(G_{ij}e_i + B_{ij}f_i) & j \neq i \\
\sum_{j=1}^{i-1} (G_{ij}e_j - B_{ij}f_j) - G_{ii}e_i - B_{ii}f_i & j = i 
\end{cases}
\]

\[
J_{a_i} = \begin{cases}
0 & j \neq i \\
-2f_i & j = i
\end{cases} \quad J_{b_i} = \begin{cases}
0 & j \neq i \\
-2e_i & j = i
\end{cases}
\]

\( V^2 \) represents the vector consisting of the squares of the bus voltage magnitude to ensure the homogeneity and quadratic order.
Hereby, it is reasonable to explore the nonlinearity-retaining state estimation algorithm. In order to present easily, the single-phase system is taken as a described object and the initial voltages of each bus is assumed to be the same as the voltage of reference node,

\[ e_i^{(0)} + j f_i^{(0)} = e_s + j0 \]  (7)

where \( e_s \) denotes reference node voltage magnitude. Equation (6) can be expressed as

\[
\begin{bmatrix}
Z_p \\
Z_Q \\
Z_{V^2}
\end{bmatrix} = e_s \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - e_s \begin{bmatrix} B & -G \\ G' & B' \\ 0 & 2M_vA^T \end{bmatrix} \begin{bmatrix} \Delta f \\ \Delta q \\ \Delta v^2 \end{bmatrix} + \begin{bmatrix} s_p \\ s_Q \\ s_{V^2} \end{bmatrix} \]  (8)

where \( I \) denotes a unit column vector, \( A \) node-branch-associating matrix, and \( M_v \) is a measurement of voltage-branch-associating matrix, and elements of \( B, G, B', G' \) are the same as those of corresponding admittance matrix. Let

\[
C = \begin{bmatrix} C_p \\ C_Q \\ C_{V^2} \end{bmatrix} = \begin{bmatrix} Z_p \\ Z_Q \\ Z_{V^2} \end{bmatrix} - e_s \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} s_p \\ s_Q \\ s_{V^2} \end{bmatrix} \]  (9)

\[
\begin{aligned}
s_VALIDATE_CODE_1 = \Delta e_s \left( \frac{C_p}{e_s} \right) - \Delta f_s \left( \frac{C_Q}{e_s} \right) \\
\end{aligned}
\]

\[
\begin{aligned}
s_Q = \Delta f_s \left( \frac{C_p}{e_s} \right) + \Delta e_s \left( \frac{C_Q}{e_s} \right) \\
\end{aligned}
\]

\[
\begin{aligned}
s_{V^2} = \Delta e_s^2 + \Delta f_s^2 \\
\end{aligned}
\]  (10)

The computing of the second derivative is as presented in [7]. Namely, if active and reactive power measurements are not paired, \( C_p / e_s, C_Q / e_s \) need to be computed using following equations,

\[
\frac{C_p}{e_s} = \sum_{j=1} I_{G_j} (\Delta e_j - B_{ij} \Delta f_j) \\
\frac{C_Q}{e_s} = -\sum_{j=1} I_{G_j} (\Delta f_j + B_{ij} \Delta e_j) \]  (11)

If decompounded form is adopted, measurement (8) can be expressed as

\[
h(e, f) = e_s \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - e_s \begin{bmatrix} M_pBA^T - M_pGA^T \\ M_QGA^T - M_QBA^T \\ 0 & 2M_vA^T \end{bmatrix} \begin{bmatrix} \Delta f \\ \Delta q \\ \Delta v^2 \end{bmatrix} + \begin{bmatrix} s_p \\ s_Q \\ s_{V^2} \end{bmatrix} \]  (12)
where \( M_p \) and \( M_Q \) denote active and reactive power measurement associating matrix respectively.

The state estimation problem is to solve (3), then

\[
H^T(x)R^{-1}H(x)\begin{bmatrix} \Delta f \\ \Delta e \end{bmatrix} = H^T(x)R^{-1}C
\]  

\( A^T \) can be eliminated from \( H^T \) which is in the right side of the equation. It ensures the invariant sparseness of the matrix \( M \) and the computing time can be reduced by updating the right side of the equation. Multiplying \( M \) and \( A \) results in a large number of injecting elements. Then we get

\[
\begin{bmatrix}
BM_p^T & GM_Q^T & 0 \\
-GM_p^T & BM_Q^T & 2M_p^T
\end{bmatrix}R^{-1}\begin{bmatrix}
M_pBA^T & -M_pGA^T \\
M_QGA^T & M_QBA^T & 0 & 2M_p^TA^T
\end{bmatrix}\begin{bmatrix} \Delta f \\ \Delta e \end{bmatrix} =
\begin{bmatrix}
C_p \\
C_Q \\
C_v
\end{bmatrix}
\]

(14)

whose left side is decompounded as

\[
L \cdot L^T\begin{bmatrix} \Delta f \\ \Delta e \end{bmatrix} = \begin{bmatrix}
BM_p^T & GM_Q^T & 0 \\
-GM_p^T & BM_Q^T & 2M_p^T
\end{bmatrix}R^{-1}\begin{bmatrix}
1 & C_p \\
e_e & C_Q \\
e_e & C_v
\end{bmatrix}
\]

(15)

Thus, iteration equations consist of (9), (10), (11) and (15) are used to perform state estimation. Another important problem in this paper is as follows: Consideration of three-phase imbalance. When the three-phase model is adopted, flat starting voltages of three phases at the same bus are \( 1 + j0, -\frac{1}{2} - j\frac{\sqrt{3}}{2}, -\frac{1}{2} + j\frac{\sqrt{3}}{2} \) respectively rather than 1. On the condition of initial voltages above, it is impossible to rewrite (6) following the form of (8). To solve the problem, operator matrix \( T = diag\{1, \alpha, \alpha^2\} \) is introduced where \( \alpha = \frac{2\pi}{3} \). The voltages are rotation transformed,

\[
V'_i = TV_i
\]  

(16)

where \( V_i, V_j \) are respectively voltage vectors at bus i before and after the transformation. After that the initial value is scheduled the same as (7). Namely,

\[
e_i^{P(0)} + jf_i^{P(0)} = e_i + j0 \quad i = 1, 2, 3, \ldots
\]  

(17)

where \( P \) represents phase A, B or C. Then, elements of the three-phase Jacobian matrix are the remaining real and imaginary parts of the corresponding elements of the admittance matrix. Thus, the problem of finding the initial value of the voltage has been successfully solved and the process of deduction above still holds water. No unnecessary details will be given here.
3. Test Simulation

MATLAB taken as tool, test system is consist of simulation measurement of IEEE 13 bus and 34 bus test systems. In order to double check the reliability of the algorithm when the R/X ratio of branches is large, the parameters of the branches of both systems are adjusted specially: resistance value of unit length branch is adjusted to three times more than original value. Simulation programming is performed with C language [8]-10.

The results are shown in TABLE 1. In the table, \( n \) represents the number of iteration and \( t \) represents computing time of the CPU. Pentium 586 is the core of the computer used in the test.

In TABLE 1, each No. of statistic is 50; measurement schemes of three phases is uniform; \( J(x) \) and \( J(\hat{x}) \) denote average value of objective function derived from measurement errors and residual errors respectively; \( S_M \) and \( S_E \) denote statistic values of measurement errors and estimated errors statistics respectively; convergence criteria is \( \max_i |\Delta x_i^{(i)}| < 0.000 \) 1.p.u.

<table>
<thead>
<tr>
<th>System</th>
<th>Measurement</th>
<th>( J(x) )</th>
<th>( J(\hat{x}) )</th>
<th>( n )</th>
<th>( S_M )</th>
<th>( S_E )</th>
<th>( t ) ms</th>
</tr>
</thead>
<tbody>
<tr>
<td>IEEE13</td>
<td>21(^c)</td>
<td>101.351(^c)</td>
<td>21.455(^c)</td>
<td>2(^c)</td>
<td>1.003(^c)</td>
<td>0.783(^c)</td>
<td>29(^c)</td>
</tr>
<tr>
<td>(Unadjusted)</td>
<td>51(^c)</td>
<td>121.994(^c)</td>
<td>52.063(^c)</td>
<td>2(^c)</td>
<td>0.986(^c)</td>
<td>0.919(^c)</td>
<td>29(^c)</td>
</tr>
<tr>
<td>IEEE34</td>
<td>12(^d)</td>
<td>220.848(^d)</td>
<td>12.235(^d)</td>
<td>2(^d)</td>
<td>0.968(^d)</td>
<td>0.934(^d)</td>
<td>29(^d)</td>
</tr>
<tr>
<td>(Unadjusted)</td>
<td>102(^d)</td>
<td>285.132(^d)</td>
<td>100.029(^d)</td>
<td>2(^d)</td>
<td>1.006(^d)</td>
<td>0.922(^d)</td>
<td>29(^d)</td>
</tr>
<tr>
<td>IEEE13</td>
<td>24(^e)</td>
<td>24.62(^e)</td>
<td>26.638(^e)</td>
<td>2(^e)</td>
<td>1.058(^e)</td>
<td>0.932(^e)</td>
<td>30(^e)</td>
</tr>
<tr>
<td>(Unadjusted)</td>
<td>51(^e)</td>
<td>140.768(^e)</td>
<td>123.788(^e)</td>
<td>2(^e)</td>
<td>1.093(^e)</td>
<td>0.715(^e)</td>
<td>34(^e)</td>
</tr>
<tr>
<td>IEEE34</td>
<td>12(^f)</td>
<td>218.647(^f)</td>
<td>12.789(^f)</td>
<td>2(^f)</td>
<td>0.997(^f)</td>
<td>0.756(^f)</td>
<td>38(^f)</td>
</tr>
<tr>
<td>(Unadjusted)</td>
<td>102(^f)</td>
<td>333.473(^f)</td>
<td>104.180(^f)</td>
<td>2(^f)</td>
<td>1.000(^f)</td>
<td>0.739(^f)</td>
<td>71(^f)</td>
</tr>
</tbody>
</table>

There should be \( S_M \approx 1 \) for measurement-simulated system which can satisfy the requirements. Normal state estimation programming, objective function \( J(\hat{x}) \) derived from the measurement residual should approach measurement redundancy. By observing TABLE 1, it is found that it does not affect the programming by adjusting the r/x ratio. Voltage value measurements in programming is not only used to enhance the redundancy level of measurement system, but it is also one of the factors which decides the observation of the system. The example shows that the convergence characteristic of programming changes rather slightly when there are a large number of voltage magnitude measurements in the system.

To summarize, a state estimation algorithm with constant Jacobian matrix is proposed when considering the second order terms enlightened by the power flow algorithm retaining nonlinearity. Through rotation transforming, the method is found to solve the problem in estimating the three-phase model’s state retaining nonlinearity. Jacobian matrix treated constant, computing speed can be increased drastically. With the second order terms considered, the convergence characteristics achieve improvement.

The proposed algorithm takes advantages of the character of radial feeder without introduction of assumption of simplified \( r \) and \( x \). Considering the imbalance of a three-phase distribution system, this method is suitable for the observable system with some voltage measurements. The system is established by the real-time measurement and pseudo measurement of power. Theoretical analysis and digital simulations show that it is suitable for on-line application for radial distribution system, efficient, convergence reliable and is a valuable method which shows a great deal of promise in future practical applications.
References


Biographies

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