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Procedia Engineering 28 (2012) 586 – 593

**Procedia
Engineering**

www.elsevier.com/locate/procedia

2012 International Conference on Modern Hydraulic Engineering

Game Analysis in the Construction Claim Negotiations

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Abstract

The claim negotiations are very important to both sides which have attended the engineering project construction. On one hand, the contractors want to cut down loss through negotiations and protect their own legitimate interests, on the other hand, the owners want to shift risks by it and reduce the project construction cost. The strategy and tactics of both sides are one kind of typical game form in the claim negotiations. This article firstly uses alternating offers model to describe the claim negotiations between the owner's and contractor's bargaining. By estimating the contractor's possibility distribute of retention value and the conditional probability of contractor's bidding price given under the assumption, the prior beliefs are revised with the Bayesian principle and counter-offer strategy is adjusted. Finally, the paper analyzes and uses strategies which have been considered the time value of money of each other's bargaining to enable the claim event be solved effectively, of course enhancing the engineer project's construction-efficiency.

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Keywords: Game analysis; Construction claims; Alternate bid; Time value of money

1. Introduction

Engineering change and claims in construction projects are difficult to avoid. Usually because of changes and claims involved with funds and progress, claim negotiations are always very difficult. It is a typical game form for both side of the project, but it generally has a certain distinction from Two-person Zero-sum Game. Because the parties participating in the game are not diametrically opposed and better, faster way to complete the task of the construction projects is their shared purpose. Therefore, analyzing and studying strategy and tactics of both sides of construction claim negotiations can make contributions

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to an effective solution to contract disputes and improve the efficiency of the construction project.

2. Time-limited alternating-offer game model

The game in construction claim negotiations generally belongs to "Limited Two-person Zero-sum Game". This paper is based on the " Limited Two-person Zero-sum Game " model and actual situation and solution requirements of the engineering contract claim, which has adopted new paradigm to describe alternating-offer game for owner and contractor and finally applying Bayesian Rule^[1] to correct beliefs to obtain a balanced solution.

2.1. Finite Two-person Zero-sum Game

Finite Two-person Zero-sum Game is known as "matrix game." The model is as follows: Let the player I have m pure strategies $\alpha_1, \alpha_2, \dots, \alpha_m$, with the S_1 represents the set: $S_1 = \{ \alpha_1, \alpha_2, \dots, \alpha_m \}$; and use S_2 to stand the player II with the same pure strategy set: $S_2 = \{ \beta_1, \beta_2, \dots, \beta_n \}$. The player I select α_i from S_1 , while the player II chose β_j from the S_2 , from which measures constitute a Board of a situation (α_i, β_j) . As shown in Table 1.

Table 1. Matrix game

	β_1	β_2	...	β_j	...	β_n
α_1	α_{11}	α_{12}	...	α_{1j}	...	α_{1n}
α_2	α_{21}	α_{22}	...	α_{2j}	...	α_{2n}
...
α_i	α_{i1}	α_{i2}	...	α_{ij}	...	α_{in}
...
α_m	α_{m1}	α_{m2}	...	α_{mj}	...	α_{mn}

There are total number of $m \times n$ situations. If one of the player I wins a matrix $A = (\alpha_{ij})$, then we denote this response as G and write $G = \{S_1, S_2, A\}$.

2.2. Improved time-limited alternating-offer game model

In general two-person Zero-Sum Game, two participants are in a diametrically opposed position and one of whom is from the loss of another person. That is gains and losses of two-player zero-sum^[2]. In construction claim negotiations, due to the specialty of construction, both participants are not mutually exclusive goals (better and faster to accomplish the construction task is the common goal) and are inappropriate to use the maximum and minimum (min-max) method or the general linear programming solution to solve it. The article lists the alternating-offer bargaining sheet and use Bayesian Rule to correct beliefs to obtain a balanced solution.

2.2.1 Alternating-offer pattern shown in Table 2.

(6) Estimating the risk

According to Zeuthen model[3], the maximum risk tolerance of the owner and the contractor is

$$(P_{o\max}, P_{c\max}): P_{o\max} = \frac{u'_{oo} - u'_{oc}}{u'_{oo} - u_o(c)} \text{ and } P_{c\max} = \frac{u'_{cc} - u'_{co}}{u'_{cc} - u_c(c)} \quad (7)$$

Of which: $P_{o\max}$ — the maximum risk tolerance of the owner;

$P_{c\max}$ — The maximum risk tolerance of the contractor;

u'_{cc} — The contractor's given offer in the round t , the effectiveness of the contractor;

u'_{oo} — The owner's given offer in the round t , the utility of the owner ;

u'_{co} — The owner's given offer in the round t , the effectiveness of the contractor;

u'_{oc} — The contractor's given offer in the round t , the utility of the owner;

$u_c(c)$ — The contractor's effectiveness in conflicts; and assuming $u_c(c)=0$ in the game ;

$u_o(c)$ — The owner's utility in conflicts; and assuming $u_o(c)=0$ in the game.

(7) Calculating the concession rate

If the owner's $P_{o\max}$ is lower than the contractor's, the owner will get the larger loss in the claim and will make concessions. The concession degree will make the contractor's maximum risk tolerance $P_{c\max}$ less than or equal to its minimum value $P_{o\max}$ ^[4]. The concession rate calculated as follows:

$$P_{c\max} = \frac{u'_{cc} - u'_{co}}{u'_{cc} - u_c(c)} \quad (8)$$

(8) M rounds of bargaining

After m rounds of bargaining, with one side's offer close to or identical to the other side's retention and both accept it. The owner's payment is $\pi_o = u(-e_m)$ and the contractor's win is $\pi_c = u(e_m)$.

(9) Finding Pareto optimal

Calculating the sum of both effects: $u = u_o + u_c$ strives for d_u/d_x . If u has an increasing trend and also a decreasing trend within the scope of the claim, there may be exists a maximum value, when in the vicinity of the final decision claim offer shall get the maximum value, achieving the Pareto optimal.

3. Positive analysis

A watercourse prevention project which was started at the beginning of December and planned the completion time was the end of May at the following year. When carrying out it, due to stock-ground's lack of source material, the mining blasting obstruction from the around villagers, the Spring Festival holiday, snowfall's leading to high-voltage cable drop and environmental protection and other reasons, resulting in discontinuity laying-off to postpone the project and originally planned project were forced to start at the high-water level which had been intended to begin construction at dry season, resulting in productivity slowdown. Therefore the contractor claimed with the following three reasons:

(1)The owner provided insufficient material source and too thick cover layer and the number of workable block stone couldn't reach engineering requirements and because of the substratum's brittle, the acquisition rate of mining blocks after burst was not high. According to the relevant provisions of the contract, the contractor was entitled to get the compensation of period extension and adding cost.

(2)In another yard stock ground, constructors and vehicles got continuous interference by the surrounding villagers. Therefore the work efficiency was affected, intermittent return to work after the project, resulting in the extension of the project.

(3)The suspension order of the supervising engineer, the original project should be started in January or February delayed until March or April to construct. And the water-level's rise caused the difficulty of

riprap which increased the duration of extension.

When the contractor calculated the losses, the total cost of compensation was 10 million Yuan.

After receiving the claim report, the owner and supervising engineer negotiated about the responsibility of the contractor and the undertaken risk. The actual loss was estimated at 500 million Yuan, which meant the owner’s retention value was $S_o=500$ million. In addition, the owner evaluated the contractor’s distribution of retention value and the conditional probability of the given price under the assumption based on the previous negotiation strategy of the contractor respectively shown in Table3 and Table4:

Table3: The owner’s estimating on the probability of retention of the contractor

Assume	R1 300	R2 400	R3 500	R4 600	R5 800	R6 1000
Probability $P(R_i)$	0.05	0.20	0.40	0.20	0.10	0.05

Note: Adjusting the data in the table according to project case study (unit: million Yuan). Of which: R_i ——The incomplete beliefs sets of the owner with the contractor’s retention value, such as $R_1=300, R_2=400$ etc. ($i=1,2\dots n$);

$P(R_i)$ ——The estimating probability of the assuming set $\{R_i\}$, such as $P(R_1)=0.05, P(R_2)=0.20$ etc. ($i=1,2\dots n$), $\sum P(R_i)=1$.

Table4: The conditional probability of the contractor’s offer under the assumption of the given owner

$P(e / R_i)$	Possible events						
	e_1 1000	e_2 800	e_3 600	e_4 600	e_5 500	e_6 400	e_7 300
300	0.05	0.10	0.15	0.20	0.25	0.20	0.05
400	0.10	0.15	0.20	0.25	0.20	0.10	0.00
500	0.10	0.20	0.30	0.30	0.10	0.00	0.00
600	0.30	0.20	0.20	0.30	0.00	0.00	0.00
800	0.40	0.60	0.00	0.00	0.00	0.00	0.00
1000	1.00	0.00	0.00	0.00	0.00	0.00	0.00

3.1. The negotiation process

The first phase is the initial bargain. Contractor's first offer is 10 million Yuan. Taking into account of the current circumstances and the importance of the claim, the owner abated the price as 300 million.

In the second stage:

(1)The owner revised his beliefs in the probability of retention (R_i) based on the contractor's bid and his own existing knowledge. The Bayesian Rule for calculating the probability is :

$$P(R_i / e) = \frac{P(R_i)P(e / R_i)}{\sum_{k=1}^6 P(e / R_k)P(R_k)} = \frac{P(R_i)P(e / R_i)}{P(e)} = 0.235$$

(2)The estimated value of the contractor's retention is

$$e_c = \sum R_i P(R_i) = 300 \times 0.05 + 400 \times 0.20 + 500 \times 0.40 + 600 \times 0.20 + 800 \times 0.10 + 1000 \times 0.05 = 545$$

After knowing the price, the owner would amend the contractor's retention value as:

$$e_s = \sum R_i P(R_i / e) = 300 \times 0.012 + 400 \times 0.094 + 500 \times 0.188 + 600 \times 0.282 + 800 \times 0.188 + 1000 \times 0.235 = 689.8$$

(3) Solving the owner’s effect function and representing it as $u_o = kx + b$, which indicated x as claims of quotation or counter offer. Assuming the retention value (e_o)’s utility value (\bar{u}_o) as 0.75 ($0 < \bar{u}_o < 1$). The effect function of the two key points had been award that the minimum value was (300, 1) and the

retention value was (500, 0.75). The owner’s effect function is: $u_o = 1 - (1 - \bar{u}_o) \times \frac{x - e_{\min}}{e_o - e_{\min}} = 1.375 - 1.25 \times 10^{-3} x$

(4) The effect’s function of the owner’s estimation on the contractor. The contractor’s effect function could be expressed as $u_c = kx + b$ and according to the owner’s belief revision and known two points of this function, that’s the maximum value (1000, 1) and retention value (689.8, 0.6). Setting the contractor’s retention utility value as 0.6, therefore, the owner predicted the contractor’s effect function is:

$$u_c = 1 - (1 - \bar{u}_c) \times \frac{e_{\max} - x}{e_{\max} - e_c} = -0.289 + 1.3 \times 10^{-3} x$$

(5) Solving the joint effect function. From the known effect functions, the link between the owner and contractor is as follows:

$$u_o = (1 - u_c) \frac{(e_{\max} - e_c) \times (1 - \bar{u}_o)}{(e_o - e_{\min}) \times (1 - \bar{u}_c)} + 1 - (1 - \bar{u}_o) \times \frac{e_{\max} - e_{\min}}{e_o - e_{\min}} = 1.097 - 0.962u_c$$

(6) Estimating the risk. According to Zeuthen model, the maximum risk tolerance of the owner and the contractor are:

$$P_{o \max} = \frac{u_{oo}^1 - u_{oc}^1}{u_{oo}^1 - u_o(c)} = \frac{1 - (1.375 - 1.25 \times 10^{-3} \times 1000)}{1 - 0} = \frac{1 - 0.125}{1} = 0.875$$

$$P_{c \max} = \frac{u_{cc}^1 - u_{co}^1}{u_{cc}^1 - u_c(c)} = \frac{1 - (-0.289 + 1.3 \times 10^{-3} \times 300)}{1 - 0} = \frac{1 - 0.101}{1} = 0.899$$

Because the owner’s $P_{o \max} = 0.875$ was lower than the contractor’s $P_{c \max} = 0.899$, which meant the owner’s risk-bearing capacity was less and the owner could get larger loss in the claim would make concessions while the contractor would adhere to the original offer.

(7) Calculating the concession rate

$$P_{c \max} = \frac{u_{cc}^1 - u_{co}^1}{u_{cc}^1 - u_c(c)} = \frac{1 - u_{co}}{1 - 0} = 0.875 \Rightarrow u_{ce} = 0.125; u_c = -0.289 + 1.3 \times 10^{-3} x = 0.125 \Rightarrow x = 318.46$$

Therefore, on the second stage the owner’s counter-offer was 3.1846 million Yuan or slightly higher than the number. Setting the Counter-offer of the owner on the second round was \$ 3.2 million. Accordingly, the contractor also estimated the owner’s retention value and adjusted the value based on the owner’s first counter-offer, calculating the maximum risk tolerance and determining the level of compromise.

On the third stage, analyze and quote according to the same procedure of the second phase. After six rounds of bargaining, the contractor’s offer was 5.23 million Yuan and the owner’s retention value was 5.215 million Yuan. The two numbers were close and both sides accepted 5.22 million Yuan. The game ended. The owner paid for: $\pi_o = -522$ million Yuan and the contractor won $\pi_c = 522$ million Yuan.

3.2. Finding the Pareto Optimal

As we have had the owner’s and the contractor’s effect functions on the second stage:

$$u_o = 1 - (1 - \bar{u}_o) \times \frac{x - e_{\min}}{e_o - e_{\min}} = 1 - \frac{x - 300}{4 \times (e_o - 300)} \quad \text{and} \quad u_c = 1 - (1 - \bar{u}_c) \times \frac{e_{\max} - x}{e_{\max} - e_c} = 1 - 0.4 \times \frac{1000 - x}{1000 - e_c} \tag{9}$$

The joint utility of both sides is:
$$u = u_o + u_c = 2 - \frac{0.25x - 75}{e_o - 300} - \frac{400 - 0.4x}{1000 - e_c} \tag{10}$$

$$\frac{du}{dx} = -\frac{0.25}{e_o - 300} + \frac{0.4}{1000 - e_c} = \frac{0.4e_o + 0.25e_c - 370}{(1000 - e_c)(e_o - 300)} \tag{11}$$

According to the suppositions $500 \leq e_o < 1000, 300 < e_c < 1000$ and with no decreasing progressively of e_o , we could get: $0.4e_o + 0.25e_c - 370 \geq 0.4 \times 500 + 0.25e_c - 370 > 200 + 0.25 \times 500 - 370 = -45$ (12)

$$0.4e_o + 0.25e_c - 370 < 0.4 \times 1000 + 0.25 \times 1000 - 370 = 280 \tag{13}$$

The joint utility function of both sides in the claim price section within the existing increasing trend and another declining, the maximum value was 5.28 million, The two parties' accepting value (5.22 million Yuan) was close the maximum which basically achieved the Pareto optimal.

4. Bargaining with the consideration of time value of money

Because of the characteristics of the construction, the time value of money is particularly important to the contractor. Stahl and Rubinstein had improved the alternating-offer game model [5]. In the model, there is no restricted boundary for the frequency of offering, but each participant is provided with time-cost, while the payment depends on the acceptance of offer and the number of rounds required. If the contract stipulates the claim is not resolved within three months after the contractor's declaration, the owner shall pay the deferred interest of the claim and interest rate is identical with the same period the Central Bank's which represents as i_0 . Due to less timely accessing to money of claims, the contractor have to take loans to solve financial shortage, or give up investing other profitable investment opportunities and its maximum loss is i_c . It takes a month for the contractor to quote the price and the owner to counter-offer. The owner's discount factor is $\delta_o = \frac{1}{(1+i_0)}$ and the contractor's is $\delta_c = \frac{1}{(1+i_c)}$. According to the definition of the discount rate, we get $i_0 \leq i_c$ and so $\delta_o \geq \delta_c$. If the owner accepted the contractor's offer in the m period, the owner win $\pi_o = -e_m$ and the contractor get $\pi_c = \delta_c^m e_m$, which predicates the nature of time value of money has affected both sides' decision-making. On the third period, let the contractor's offer as e_3 and if the owner accept it, then the owner get $-e_3$ and the contractor's win would be $\delta_c^3 e_3$. If the owner do not accept it and counter-offer it to e_4 , having $\delta_o e_4 < e_3$ (given $\delta_o e_4 > e_3$, the owner would rather accept e_3) and if the contractor would accept it, the contractor win $\delta_c^4 e_4$. Because $\delta_c^4 e_4 = \delta_c^3 \delta_c e_4 \leq \delta_c^3 \delta_o e_4 < \delta_c^3 e_3$, the payment received at the fourth round of the contractor is less than the third round, so the contractor will adjust the offer of the third round, making $\delta_o e_4 > e_3$. The owner predict the response of the contractor and will bargain as $e_2 \geq e_3$ in the second round, making the contractor win $\pi_c = \delta_c^2 e_2 \geq \delta_c^2 e_3 > \delta_c^3 e_3$. If the contractor predicts the result, he would make the owner pay for $-e_1 \geq -e_2$ in the initial offer, that's $e_1 < e_2$. So the best strategy for the contractor is to quote the actual loss price at the first offer according to the practice to solve the problem as soon as possible to obtain claim price. Various stages of the bid prices are shown in Table 5.

Table 5. The offer in the claim game

Phases	Owners to pay	The contractor's payment	Bidder
The first round	$-e_1$	$\delta_c e_1$	Contractor
The second round	$-e_2$	$\delta_c^2 e_2$	Owner
The third round	$-e_3$	$\delta_c^3 e_3$	Contractor
The fourth round	$-e_4$	$\delta_c^4 e_4$	Owner

Therefore, taking into account of the time value of money in the claim game will shorten the time to make the negotiations and claims faster to be solved.

5. Conclusion

In construction claim negotiations, Bargaining Model and Bayesian Rule are used to analyze the claim negotiation process dynamically, which more reasonably solves the problems existed in the claims and makes the amount of the claim approach to the actual loss as nearly as possible. Taking into account of the time value of money is favorable to solve the claim events reasonably and quickly, and also make benefits to improve the whole country's project management level.

References

- [1] Zhang Weiyong. Game Theory and Information Economics, Shanghai: Shanghai People's Publishing House [M], 2002
- [2] Quan Xiantang et al. Analysis of the Economic Game Theory, Beijing: Mechanical Industry Press [M], 2003
- [3] Liu Xihua et al. Exaggerating the Risk Loss and Fraud Claims Game Problems. Systems Engineering Theory Methods [J], 2004 (3): 229 ~ 233
- [4] Roger. B. Myerson. Game Theory— Conflict Analysis. Beijing: China Economic Publishing House [M], 2001
- [5] Z. Ren et al. Learning in multi-agent systems: a case study of construction claims negotiation. Advanced engineering informatics, Volume 16, Issue 4, October 2002