ELSEVIER

Available online at www.sciencedirect.com

SciVerse ScienceDirect

Procedia Engineering 15 (2011) 2644 – 2649

Procedia Engineering

www.elsevier.com/locate/procedia

Advanced in Control Engineering and Information Science

A New Algorithm for Bearings-Only Parametric Trajectory Tracking

Jinmang Liu[ab]*, Zhonglin Wu[a], Jinlong Yang[b], Zhenxing Li[a]

[a] *The Missile Institute of Air Force Engineering University, Sanyuan, Shaanxi, 713800, China*
[b] *School of Electronic Engineering, Xidian University, Xi'an 710071, China*

Abstract

For the single sensor target tracking with bearings-only measurements, a novel trajectory invariable-information target tracking algorithm was proposed, and bearings-only target can be tracked by the parameter trajectory. For the measure frequency of sensor is high, the mathematic model of bearings-only tracking is analyzed by dividing the trajectory into many linearization parts. The tracking parameter of bearings-only target trajectory is deduced, so the bearings-only target can be tracked by the parameter trajectory. The simulation results show that the new algorithm has a favorable tracking precision.

Selection and/or peer-review under responsibility of [CEIS 2011]
Keywords: Parameter trajectory; Target tracking; Bearings-only measurement

1. Introduction

Under the complicated battlefield environment, sensors usually could not get the integrated measurement information of targets. Therefore, more and more attention has been paid to target tracking algorithms which are based on the information of unattached coordinates [1]. In reference [2], a bearings-only tracking algorithm has been presented, which is based on a circle-error approximation method.

For the sampling rate of infrared sensors reach 10~30 times per second, it can take the target motional model as a uniform rectilinear motional model, and adopt the method of line subsection approximation to

* Corresponding author. *E-mail address*: liujinmang1@163.com.

doi:10.1016/j.proeng.2011.08.497

analyze the motion of maneuvering target. By using these features, the mathematic model of bearings-only tracking is set up, which can estimate the parametric trajectory of the bearings-only target.

2. The Parametric Trajectory Model of Bearings-Only Target

It can suppose that target moves along the line with a velocity of V_1 in a two-dimensional plane. The trajectory of target is shown in figure 1.

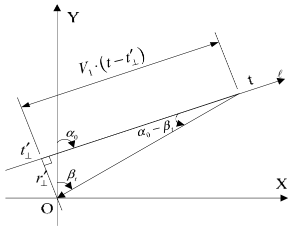


Fig. 1 Planar diagram of uniform rectilinear motion of target

As shown in Figure 1, observation station is in O point, it can suppose that target moves along the straight line ℓ with a uniform rectilinear motional form and the heading angle is α_0. At the moment of t, the azimuth is β_t and the distance between this position and vertical point of trajectory is $V(t - t_\perp)$, which V is the velocity of target. So it can estimate the calculated ideal value at any time points t_i:

$$\cos\beta_i = \frac{-r'_\perp \cos\alpha_0 + V_1(t_i - t'_\perp)\sin\alpha_0}{\sqrt{r'^2_\perp + V_1^2(t_i - t'_\perp)^2}} = \frac{-\cos\alpha_0 + \mu_1(t_i - t'_\perp)\sin\alpha_0}{\sqrt{1+\mu_1^2(t_i - t'_\perp)^2}} \triangleq \frac{b_t}{a_t} \tag{1}$$

$$\sin\beta_i = \frac{r'_\perp \sin\alpha_0 + V_1(t_i - t'_\perp)\cos\alpha_0}{\sqrt{r'^2_\perp + V_1^2(t_i - t'_\perp)^2}} = \frac{\sin\alpha_0 + \mu_1(t_i - t'_\perp)\cos\alpha_0}{\sqrt{1+\mu_1^2(t_i - t'_\perp)^2}} \triangleq \frac{c_t}{a_t} \tag{2}$$

In the equation above: $a_t^2 = 1 + \mu_1^2(t - t'_\perp)^2$, $b_t = -\cos\alpha_0 + \mu_1(t_i - t'_\perp)\sin\alpha_0$, $\mu_1 = \dfrac{V_1}{r'_\perp}$,

$c_t = \sin\alpha_0 + \mu_1(t_i - t'_\perp)\cos\alpha_0$.

By analyzing Eqs. (1) and (2), if the parameters of α_0, μ_1 and $t'_\perp$ could be determined, the motional trajectory of target will be determined uniquely, which can implement the accurate estimation of bearings-only sequences and the tracking of the motional trajectory. So it is very important to determine the values of α_0, μ_1 and $t'_\perp$.

3. The Method of Determining Parametric Trajectory of Target

3.1 The objective function of bearings-only sequences

According to Eqs. (1) and (2), it can calculate the azimuth of target at any time points with prior knowledge of α_0, μ_1 and $t'_\perp$. So the problem is to calculate the three parameters with the measurement error of bearings-only sequences.

Firstly, it supposes that $\mathbf{x} = [\mu_1 \quad t'_\perp \quad \alpha_0]^T$, so it can calculate the partial derivatives of the vector in Eq. (1), and obtaine Eq. (3):

$$\frac{\partial \sin\beta_t}{\partial \mathbf{x}} = \frac{b_t}{a_t^2}\mathbf{u} = \cos\beta_t \frac{\mathbf{u}}{a_t}\ ,\ \frac{\partial \cos\beta_t}{\partial \mathbf{x}} = -\frac{c_t}{a_t^2}\mathbf{u} = -\sin\beta_t \frac{\mathbf{u}}{a_t} \tag{3}$$

Where

$$\mathbf{u} = \begin{bmatrix} \frac{(t-t_\perp)}{a_t} & \frac{-\mu}{a_t} & a_t \end{bmatrix}^T \ \frac{\partial^2 \sin\beta_t}{\partial \mathbf{x}\partial \mathbf{x}^T} = -\frac{c_t}{a_t^3}\mathbf{u}\mathbf{u}^T - \frac{b_t}{a_t^5}\mathbf{A}\ ,\ \frac{\partial^2 \cos\beta_t}{\partial \mathbf{x}\partial \mathbf{x}^T} = -\frac{b_t}{a_t^3}\mathbf{u}\mathbf{u}^T + \frac{c_t}{a_t^5}\mathbf{A} \tag{4}$$

$$\mathbf{A} = \begin{bmatrix} 2\mu_1(t-t'_\perp)^3 & 1-\mu_1^2(t-t'_\perp)^2 & 0 \\ 1-\mu_1^2(t-t'_\perp)^2 & 2\mu_1^3(t-t'_\perp) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Then it supposes $\hat{\beta}_i$ as the actual measurement value of azimuth and n_i as Gaussian mixture measurement noise of azimuth channel. So the observation model of bearings-only sequences is shown as follow:

$$\hat{\beta}_i = \beta_i + n_i \quad , i = 1,2,\cdots n \quad , n_i \sim N(0,\sigma^2) \tag{5}$$

The objective function with circle-error approximation of bearings-only sequences $\{\hat{\beta}_i, i = 1,.....,m\}$ is shown in Eq. (6):

$$Q_m = \sum_{i=1}^{m} \left\| e^{j\hat{\beta}_i} - ke^{j\beta_i} \right\|^2 = \sum_{i=1}^{m} [(\cos\hat{\beta}_i - k\cos\beta_i)^2 + (\sin\hat{\beta}_i - k\sin\beta_i)^2] = \sum_{i=1}^{m} (1 + k^2 - 2k\cos\beta_i\cos\hat{\beta}_i - 2k\sin\beta_i\sin\hat{\beta}_i) \tag{6}$$

In Eq. (6), k is a modified product term for making unbiased estimation. It is different between the circle-error approximate method and least square method to the non-linear angle sequences, which use the non-linear form of function replace the calculation of square. This improvement can make the calculational result more reliable and stable.

3.2 The error analysis

According to the methods of the mean and variance of the $\cos\hat{\beta}_i$ and $\sin\hat{\beta}_i$, it can be deduced as follow:

$$E\cos\hat{\beta}_i = k\cos\beta_i\, , E\sin\hat{\beta}_i = k\sin\beta_i\, , k = e^{-\frac{\sigma^2}{2}}\, , E\cos(mn_i) = e^{-\frac{m^2\sigma^2}{2}} = k^{m^2}\, ,$$

$$E\sin(mn_i) = 0\, , E(\cos\hat{\beta}_i - k\cos\beta_i)^2 = \frac{1-k^2}{2}(1 - k^2\cos 2\beta_i)\ E(\sin\hat{\beta}_i - k\sin\beta_i)^2 = \frac{1-k^2}{2}(1 + k^2\cos 2\beta_i)\, .$$

Under the actual measurement situation of parameter of trajectory, the difference between the azimuth estimation and measurement value is equal to the mixture measurement noise. So the mean of Eq. (6) is derived as follow:

$$E(Q_m) = \sum_{i=1}^{m} E[1 + k^2 - 2k\cos(\hat{\beta}_i - \beta_i)] = \sum_{i=1}^{m} (1-k^2) = m(1-k^2) \approx m\sigma_n^2 \tag{7}$$

It can calculate the variance of Eq. (6) which is based on the incoherence between the measurement noises.

$$E[Q_m - E(Q_m)]^2 = 4k^2 E\{\sum_{i=1}^{m} [k - \cos(\hat{\beta}_i - \beta_i)]\}^2 = 2mk^2(1-k^2)^2 \approx 2m\sigma_n^4 \tag{8}$$

For unknown to the value of the variance of measurement noises, it can calculate the value of k . So it can calculate the partial derivatives of k in Eq. (6), suppose the value is equal to 0, and then get the result as follow:

$$k = \frac{1}{m}\sum_{i=1}^{m}\cos(\hat{\beta}_i - \beta_i) \tag{9}$$

put Eq. (9) into Eq. (6), it can get the objective function Q_{mk} as:

$$Q_{mk} = m - \frac{[\sum_{i=1}^{m}\cos(\hat{\beta}_i - \beta_i)]^2}{m} = m(1-k^2) \tag{10}$$

The objective function has the adaptive ability to the measurement error, which can get the steady effect in the calculation process.

3.3 The calculation of parametric trajectory

For Q_{mk} is a non-linear function, so it can calculate the parametric vector $\mathbf{x}$ with Newton iteration method. For the right side of Eq. (10) is a function whose independent variable is k, so it can get the partial derivatives of $\mathbf{x}$ by calculating k, and then put the partial derivative result to Eqs. (3) and (4), it can obtain the results as follow:

$$\frac{\partial k}{\partial \mathbf{x}} = \frac{1}{m}\sum_{i=1}^{m}(\cos\hat{\beta}_i \frac{\partial \cos\beta_i}{\partial \mathbf{x}} + \sin\hat{\beta}_i \frac{\partial \sin\beta_i}{\partial \mathbf{x}}) = \frac{1}{m}\sum_{i=1}^{m}[\sin(\hat{\beta}_i - \beta_i)\frac{\mathbf{u}_i}{a_i}] \triangleq \mathbf{v} \tag{11}$$

$$\frac{\partial^2 k}{\partial \mathbf{x}\partial \mathbf{x}^T} = \frac{1}{m}\sum_{i=1}^{m}(\cos\hat{\beta}_i \frac{\partial^2 \cos\beta_i}{\partial \mathbf{x}\partial \mathbf{x}^T} + \sin\hat{\beta}_i \frac{\partial \sin\beta_i}{\partial \mathbf{x}\partial \mathbf{x}^T}) = -\frac{1}{m}\sum_{i=1}^{m}[\cos(\hat{\beta}_i - \beta_i)\frac{\mathbf{u}_i\mathbf{u}_i^T}{a_i^2}] - \frac{1}{m}\sum_{i=1}^{m}[\sin(\hat{\beta}_i - \beta_i)\frac{\mathbf{A}_i}{a_i^4}] \triangleq \mathbf{B} \tag{12}$$

The partial derivative of Q_{mk} for $\mathbf{x}$ can be computed as:

$$\frac{\partial Q_{mk}}{\partial \mathbf{x}} = -2mk\frac{\partial k}{\partial \mathbf{x}} = -2mk\mathbf{v} \tag{13}$$

$$\frac{\partial^2 Q_{mk}}{\partial \mathbf{x}\partial \mathbf{x}^T} = -2m\frac{\partial k}{\partial \mathbf{x}}\frac{\partial k}{\partial \mathbf{x}^T} - 2mk\frac{\partial^2 k}{\partial \mathbf{x}\partial \mathbf{x}^T} = -2m\mathbf{v}\mathbf{v}^T - 2mk\mathbf{B} \tag{14}$$

Newton iterative equation of parameter flight path is:

$$\mathbf{x}_{n+1} = \mathbf{x}_n - \left[\frac{\partial^2 Q_{mk}}{\partial \mathbf{x}\partial \mathbf{x}^T}\right]_n^{-1} \left.\frac{\partial Q_{mk}}{\partial \mathbf{x}}\right|_n = \mathbf{x}_n - \left[\mathbf{v}\mathbf{v}^T + k\mathbf{B}\right]_n^{-1} k\mathbf{v}\big|_n \tag{15}$$

According to matrix lemma,

$$(k\mathbf{B} + \mathbf{v}\mathbf{v}^T)^{-1} = \frac{1}{k}[\mathbf{B}^{-1} - \frac{(\mathbf{B}^{-1}\mathbf{v})(\mathbf{v}^T\mathbf{B}^{-1})}{k + \mathbf{v}^T\mathbf{B}^{-1}\mathbf{v}}]$$

$$\mathbf{x}_{n+1} = \mathbf{x}_n - \left.\frac{k}{k + \mathbf{v}^T\mathbf{B}^{-1}\mathbf{v}}\mathbf{B}^{-1}\mathbf{v}\right|_n \tag{16}$$

Iterative process is halted with the iterative error reaching a given extent. Initial value should be chosen near by real parameter. At this time, back item of rightness of Eq. (12) is close to zero. If the three parameters of bearing-only are calculated, substitute it in Eq. (9), and k can be gained, then bearing variance is estimated:

$$\hat{\sigma}_n^{\ 2} = -2\ln k \tag{17}$$

Substitute k in Eq. (10), minimum error of circle-error approaching, which could be used as quantitative analysis of the result.

3.4 question debate

a) Iterative initial value selection.

In this algorithm , the initial values of three parameter $\alpha_0, \mu_1, t'_\perp$ can be fixed by Eq. (7) in [2].

b) Systematic error compensation

In the process of filtering, systematic error is unavoidable. Different noisy standard deviation is adopted, and then simulates for N times. Comparing average value with the real parameters, if the results lean to one side, it shows that systematic error exists and the offset should be subtracted. If the results fluctuate around the real parameters, it can be counted as agonic estimation.

4. Simulations

Assuming the position of sensor is $(0m, 0m)$, the speed of target is $(80m/s, 50m/s)$, initial point is $(0m, 500m)$, sampling period is $\mathrm{T} = 0.05\mathrm{s}$, flight path of the target is show in Figure 2.

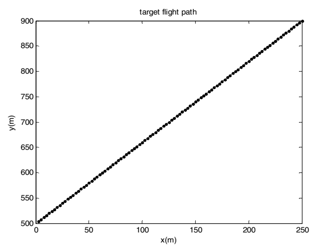


Fig. 2 target flight path figure

1) Noise standard deviation σ of bearing measurement is 3mrad, sampling times from $10:109$ .The variation of three parameter with sampling times is shown in Figures 3-5.

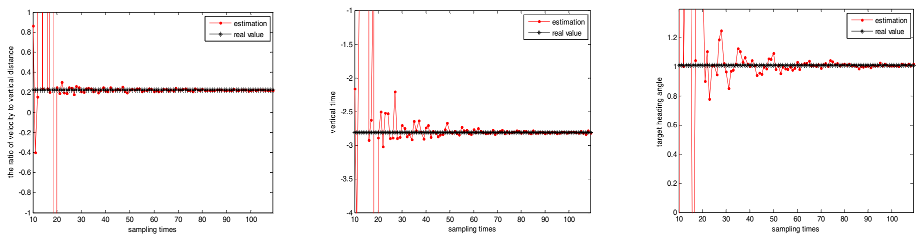
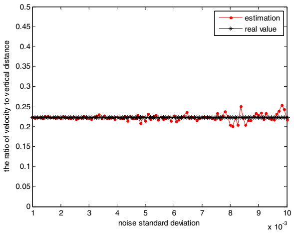
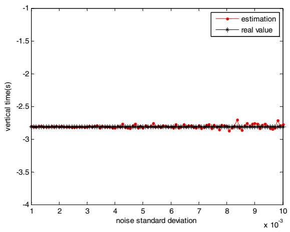


Fig3. the ratio of velocity to vertical distance Fig4. vertical time Fig5. target heading angle

The figures show that, when sampling times is fewer, the difference of estimation and real value of three parameters are enormous, however, with the increase of sampling times, estimation draws close to real value.

2) Noise standard deviation $\sigma \in (0.001, 0.01)$, divide it into 100 inter zones, and the sampling times is 100. The variation of three parameters with standard deviation is shown in Figures 6-8.





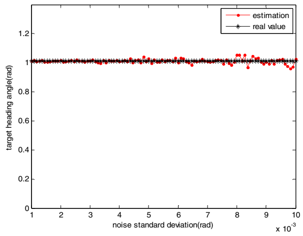


Fig6. the ratio of velocity to vertical distance Fig7. vertical time Fig8. target heading angle

From Figures 6-8, it can be seen that when noise standard deviation is huge, certain distinction exists between the estimation and real value. With the decrease of standard deviation, estimation approaches to real value.

5. Conclusions

For high observability of some sensors, targets can be viewed as uniform linear motion in short time, and sectoring linear approaching methods should be used in maneuvering targets. Because of this, the model of bearing-only based on circle-error approaching is established. The method of unattached coordinates is adopted, and parameter trajectory of bearing-only target tracking can be solved by using a set of measured bearings. In this way, bearing-only target tracking problem is successfully realized.

Acknowledgements

This paper was supported by Shannxi Provincial Natural Science Foundation of China (2010JM8013).

References

[1] Hongrui Li. Two-Step Modeling and Observability for Bearings-Only Tracking[J]. International Conference on Mechatronics and Automation,2009:691-695

[2] Liu Jinmang. A new parameterized track filtering method for single-station based bearing-only infared target[J].Journal of electronics and information technology, 2010， 32(9):2253-2257.

[3] S. Koteswara Rao.Unscented Kalman Filter With Application To Bearings-Only Passive Manoeuvring Target Tracking[J]. IEEE-International Conference on Signal processing,2008:219-224.

[4] A.N. Bishop and P.N. Pathirana. Localization of emitters via the intersection of bearing lines: A ghost elimination approach .IEEE Transaction on Vehicular Technology, To Appear 2007.

[5] Kutluyıl Doğançay Bias Compensation for the Bearings-Only Pseudo linear Target Track Estimator[J].IEEE Trans on Signal Processing,2006,54(1):59-67.

[6] Adrian N. Bishop, Brian D.O. Anderson, Baris Fidan, Pubudu N. Pathirana, Guoqiang Mao, Bearing-Only Localization Using Geometrically Constrained Optimization[J].IEEE Trans on Aerospace and Electronic System,2009,Jan,45(1):308-320.

[7] DONG Zhi-rong. Nonlinear least-square algorithms used for TMA in bearing-only system-the engineering mathematic model and algorithms[J].Information comand control system and simulation technology, 2005，27(2):4-7.