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Charged-lepton mixing and lepton flavor violation

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ABSTRACT

We present a model for calculating charged-lepton mixing matrices. These matrices are an essential ingredient for predicting lepton flavor-violating rates in the lepton number nonuniversal models recently proposed to explain anomalies in B-meson decays. The model is based on work on "constrained flavor breaking" by Appelquist, Bai and Piai relating the charged-lepton mass matrix, $\mathcal{M}_\ell$, to those for the up- and down-type quarks, $\mathcal{M}_{u,d}$. We use our recent model of lepton nonuniversality to illustrate the magnitudes of flavor-violating B-decay rates that might be expected. Decays with $\mu\tau$ final states generally have the highest rates by far.

The LHCb Collaboration has reported several features of B-meson decays involving $b \to s\ell^+\ell^-$ transitions that consistently point to a departure from the Standard Model (SM) of particle physics:

- The ratio R_K of the decay rates of $B^+ \to K^+\ell^+\ell^-$ for $\ell = \mu, e$ [1]

$$R_K \equiv \frac{\mathcal{B}(B^+ \to K^+\mu^+\mu^-)}{\mathcal{B}(B^+ \to K^+e^+e^-)} = 0.745^{+0.090}_{-0.074}\,(\mathrm{stat}) \pm 0.036\,(\mathrm{syst})\,. \quad (1)$$

  This result is a 2.6σ deficit from the standard model (SM) prediction, $R_K = 1 + \mathcal{O}(10^{-4})$ [2–4].
- The direct measurement [5],

$$\mathcal{B}(B^+ \to K^+\mu^+\mu^-)_{[1,6]} = (1.19 \pm 0.03 \pm 0.06) \times 10^{-7}\,. \quad (2)$$

  This is about 30% lower than the SM prediction, $\mathcal{B}(B^+ \to K^+\mu^+\mu^-)^{\mathrm{SM}}_{[1,6]} = (1.75^{+0.60}_{-0.29}) \times 10^{-7}$ [6–8].
- The observable P_5' in $B^0 \to K^{*0}\mu^+\mu^-$ angular distribution exhibits a deficit in two bins, quantified by LHCb as 2.9σ for each bin [9]. However, the theoretical error is debated [10–12].
- The joint CMS–LHCb measurement [13]

$$\mathcal{B}(B_s \to \mu^+\mu^-)_{exp} = (2.8^{+0.7}_{-0.6}) \times 10^{-9} = (0.76^{+0.20}_{-0.18}) \times \mathcal{B}(B_s \to \mu^+\mu^-)_{SM}\,. \quad (3)$$

  Although this is consistent with the SM prediction [14], the central value is about 25% low — as it is for R_K [15].

These three measurements were made in the low $q^2 = M^2_{\ell\ell}$ region of 1.0–6.0 GeV^2, away from charmonium resonances in the $\ell^+\ell^-$ spectrum.

The R_K-measurement suggests lepton nonuniversality (LNU) occurs in $b \to s\ell^+\ell^-$ transitions; the other measurements are consistent in magnitude and sign. It is no wonder, then, that they have inspired a number of LNU models of new physics (NP) above the electroweak energy scale, involving the exchange of multi-TeV particles [16–27,15,28–40].

LNU interactions at high energy are accompanied by LFV interactions unless the leptons involved are chosen to be mass eigenstates [15]. Such a choice is an act of fine tuning in the absence of a dynamical or symmetry mechanism justifying it.[1] Further, since charged leptons (and quarks) are massless at Λ_{LNU}, far above the weak scale, it is difficult to understand the motivation or need for flavor-invariant Yukawa couplings there. If the anomalies reported by LHCb hold up, LFV decays such as $B \to K^{(*)}\mu e$ and $B \to K^{(*)}\mu\tau$ should occur at rates much larger than in the SM due to tiny neutrino masses alone. The purpose of this paper is to present a model for estimating these and other LFV rates implied by new LNU interactions.

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[1] Attempts in this direction are in Refs. [32,38].

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LHCb data suggest that LNU is affecting muons but not electrons. To describe this, a simple model was adopted in Ref. [15] (hereafter referred to as GGL) in which a heavy Z' boson couples only to third-generation quarks and leptons, namely,

$$\mathcal{H}_{NP} = G\,\bar{b}'_L\gamma^\lambda b'_L\,\bar{\tau}'_L\gamma_\lambda\tau'_L\,. \tag{4}$$

This chiral structure is consistent with B-decay data which is well fit if the SM and NP contributions to the $b \to s\ell^+\ell^-$ interaction are a product of left-handed currents (LL) [22–24,41]. In Eq. (4), $G = g^2_{Z'}/M^2_{Z'} = 1/\Lambda^2_{NP} \ll G_F$ is a new Fermi constant. The primed fields refer to the gauge basis, the one in which the charged weak currents are generation-universal.[2] They are related to mass-eigenstate (unprimed) fields by unitary matrices U^d_L and U^ℓ_L:

$$b'_L \equiv d'_{L3} = \sum_{i=1}^{3} U^d_{L3i}\,d_{Li}\,, \quad \tau'_L \equiv \ell'_{L3} = \sum_{i=1}^{3} U^\ell_{L3i}\,\ell_{Li}\,. \tag{5}$$

The interaction responsible for the discrepancies in R_K, $B^+ \to K^+\mu^+\mu^-$, $B_s \to \mu^+\mu^-$ and P'_5 is then

$$\begin{aligned}\mathcal{H}_{NP}(\bar{b} \to \bar{s}\mu^+\mu^-) \\ = G\left[U^{d*}_{L33}U^d_{L32}|U^\ell_{L32}|^2\,\bar{b}_L\gamma^\lambda s_L\,\bar{\mu}_L\gamma_\lambda\mu_L + \text{H.c.}\right].\end{aligned} \tag{6}$$

In GGL we said that the hierarchy of the Cabibbo–Kobayashi–Maskawa (CKM) matrix $V_{CKM} = U^{u\dagger}_L U^d_L$ for quarks and the apparent preference of the new physics for muons over electrons suggest that $|U^{d,\ell}_{L31}|^2 \ll |U^{d,\ell}_{L32}|^2 \ll |U^{d,\ell}_{L33}|^2 \simeq 1$. Then, in order that this Hamiltonian deplete the SM contribution, we assumed $GU^{d*}_{L33}U^d_{L32} < 0$. This sign choice is correct if $U^d_L \simeq V_{CKM}$. In truth, however, we know little about U^d_L and U^ℓ_L other than $V_{CKM} = U^{u\dagger}_L U^d_L$ and that U^ℓ_L plays a similar part in the less well-measured PMNS matrix, V_{PMNS} [43].

Experimentalists need better targets. A recent paper by Boucenna, Valle and Vicente [35] made a first stab at this by assuming $U^\ell_L = V_{PMNS}$. In our opinion, $V_{PMNS} = U^{\nu\dagger}_L U^\ell_L$ seems likely to be strongly influenced by the unknown neutrino mixing matrix. Ref. [34] discussed lepton flavor mixing in the context of LNU due to a leptoquark interaction [22]. While we will present results for the Z'-induced $\mathcal{H}_{NP}$, our scheme for charged-lepton mixing is independent of the dynamical nature of LNU and LFV.

The model we propose for calculating charged-lepton mixing matrices, $U^\ell_{L,R}$, is based on a recent paper on "constrained flavor breaking" by Appelquist, Bai and Piai (ABP) [44]. They assumed a global constrained flavor symmetry group, $SU(3)^3$, broken by just two Yukawa spurions. This implies one equation among the three Yukawa matrices, Y_u, Y_d, Y_ℓ. They are related to the quark and charged-lepton mass matrices $\mathcal{M}_a$ in

$$\begin{aligned}&\mathcal{H}_{mass} \\ &= \sum_{i,j=1}^{3}\left[\bar{u}'_{Li}\mathcal{M}_{u\,ij}u'_{Rj} + \bar{d}'_{Li}\mathcal{M}_{d\,ij}d'_{Rj} + \bar{\ell}'_{Li}\mathcal{M}_{\ell\,ij}\ell'_{Rj} + H.c.\right]\end{aligned} \tag{7}$$

by $\mathcal{M}_a = Y_a v/\sqrt{2}$ where v is the Higgs vacuum expectation value. The matrices $U^a_{L,R}$ bring these to real, diagonal, positive form:

$$\widehat{\mathcal{M}}_a \equiv \mathcal{M}_{a,\,diag} = U^{a\dagger}_L\mathcal{M}_a U^a_R \qquad (a = u, d, \ell)\,. \tag{8}$$

[2] This interaction has been extended in Ref. [27] to the $SU(3) \otimes SU(2) \otimes U(1)$-invariant form it must have if $\Lambda_{NP} \gg \Lambda_{EW}$ [42], and used to provide a simultaneous explanation for R_K and $R(D^{(*)})$. A consistent gauge model must also be anomaly-free. These extensions are not needed in this paper.

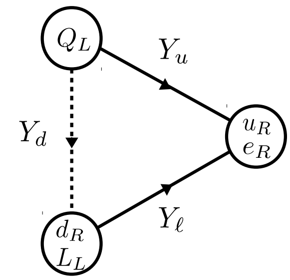

Fig. 1. Moose diagram for the ABP model of constrained flavor breaking, Ref. [44]. The solid links are the input Yukawa matrices chosen by ABP (our case A), the dashed link is then predicted.

The ABP model is based on the moose diagram of Fig. 1. Requiring (a) that the quark doublet and singlet fields, Q_L, u_R and d_R, must be assigned to different $SU(3)$'s (to have realistic masses and V_{CKM}); (b) that L_L and e_R likewise be assigned to different $SU(3)$'s; and (c) that L_L and e_R be assigned to $SU(3)$ groups other than Q_L's (to avoid $\widehat{\mathcal{M}}_\ell \propto \widehat{\mathcal{M}}_u$ or $\widehat{\mathcal{M}}_d$), leaves six possibilities [44]. The one chosen by ABP is depicted Fig. 1. Having taken Y_u and Y_ℓ independent, Y_d is predicted up to a constant, η:

$$Y_d = \eta Y_u Y^\dagger_\ell\,. \tag{9}$$

Equivalently, $\mathcal{M}_d = \widehat{\eta}\mathcal{M}_u\mathcal{M}^\dagger_\ell$.

As ABP were interested only in masses of the charged leptons, not their mixing, they did not consider assignments differing by an interchange of L_L and e_R which just interchanges Y_ℓ and $Y^\dagger_\ell$. But, this swaps $\mathcal{M}_\ell$ and $\mathcal{M}^\dagger_\ell$ and, hence, U^ℓ_L with U^ℓ_R, resulting in different mixing factors for LNU/LFV processes. ABP rejected choosing Y_u and Y_d as the flavor-breaking spurions because that implies $Y_\ell = \eta Y^\dagger_d Y_u$ and, hence, unrealistic charged-lepton masses. We agree. They also rejected the possibility $Y_u = \eta Y_d Y_\ell$, arguing that it has difficulty obtaining a large enough top Yukawa coupling. But η is a free parameter of unknown origin, and there is considerable freedom in choosing the textures for the Y_a, so we will consider this case. We will see it is closely related to the case ABP considered. Thus, we consider four cases. Written as a relation from which $\mathcal{M}_\ell$ is determined, they are (ignoring the dimensionful η-factor):

$$\mathcal{M}_\ell = \begin{cases} (\mathcal{M}^{-1}_u\mathcal{M}_d)^\dagger & \text{(A)} \\ \mathcal{M}^{-1}_u\mathcal{M}_d & \text{(B)} \\ \mathcal{M}^{-1}_d\mathcal{M}_u & \text{(C)} \\ (\mathcal{M}^{-1}_d\mathcal{M}_u)^\dagger & \text{(D)} \end{cases} \tag{10}$$

By a special choice of quark bases, neither the gauge nor mass bases defined above, ABP obtained the mass matrix and, hence, mass ratios for the charged leptons from Eq. (9) in terms of the known quark masses and CKM matrix elements [43]. No assumption of particular quark mass textures was required. While their results are not in agreement with data, they are not all that bad, so there is promise in their approach. But, the matrices diagonalizing their lepton mass matrix are not transformations from the gauge to the mass basis, and thus cannot be used for turning $\mathcal{H}_{NP}$ in Eq. (4) into predictions of LFV rates. For that, we need specific textures for $\mathcal{M}_{u,d}$ in the gauge basis, ones that provide a reasonable account of the quark masses and V_{CKM}.

Fortunately, quark mass textures good enough for our purpose exist; see, e.g., Ref. [45]. We use ones developed in connection with a scenario for solving the strong-CP problem in QCD [46]. In this scenario, the phases in $\mathcal{M}_{u,d}$ are rational multiples of π so that they easily satisfy

$$\bar{\theta} \equiv \arg\det\mathcal{M}_q = \arg\det\mathcal{M}_u + \arg\det\mathcal{M}_d = 0\,. \tag{11}$$

The mass matrices are $\mathcal{M}_{q=u,d} = U_L^q \widehat{\mathcal{M}}_q U_R^{q\dagger}$ where we use $\overline{\text{MS}}$ quark masses renormalized at the top-quark pole mass $M_t = 173.5\,\text{GeV}$, with eigenvalues $\widehat{\mathcal{M}}_u = \text{diag}(0.00126, 0.611, 163.5)\,\text{GeV}$ and $\widehat{\mathcal{M}}_d = \text{diag}(0.00264, 0.0522, 2.72)\,\text{GeV}$.[3] Then:

$$\mathcal{M}_u = \begin{pmatrix} (0,0) & (0.01038, -2\pi/3) & (0,0) \\ (0.1325, 0) & (0.5964, 0) & (0,0) \\ (0,0) & (0,0) & (163.5, 0) \end{pmatrix}, \tag{12}$$

$$\mathcal{M}_d = \begin{pmatrix} (0,0) & (0.01112, 0) & (0.01322, 0) \\ (0.01013, \pi/3) & (0.05012, 0) & (0.1127, \pi/3) \\ (0,0) & (0,0) & (2.721, \pi/3) \end{pmatrix}. \tag{13}$$

The notation is $(|\mathcal{M}_{q,\,ij}|, \arg(\mathcal{M}_{q,\,ij}))$. The motivation for these mass textures is explained in Appendix B of Ref. [46]. Note that, since quark masses are multiplicatively and *universally* renormalized above M_t, the lepton mass textures in Eq. (10) are insensitive to QCD running from M_t to Λ_{NP}. The CKM matrix obtained by diagonalizing $\mathcal{M}_{u,d}$, removing its unphysical phases and casting it in standard form [47], is

$$\begin{aligned} V_{CKM} &= U_L^{u\dagger} U_L^d \\ &= \begin{pmatrix} (0.976, 0) & (0.216, 0) & (0.0045, -0.978) \\ (0.216, \pi) & (0.976, 0) & (0.0415, 0) \\ (0.0075, -0.516) & (0.410, 3.161) & (0.999, 0) \end{pmatrix}. \end{aligned} \tag{14}$$

This reproduces measured CKM matrix entries to within a few per cent, except for V_{ub} and V_{td} which are within 20% [48].

The $U_{L,R}^\ell$ are obtained (up to a diagonal matrix of pure phases) by diagonalizing

$$\mathcal{M}_{\ell,\,LL}^2 = \mathcal{M}_\ell \mathcal{M}_\ell^\dagger \text{ and } \mathcal{M}_{\ell,\,RR}^2 = \mathcal{M}_\ell^\dagger \mathcal{M}_\ell\,. \tag{15}$$

For cases A and C (and for cases B and D)

$$\mathcal{M}_{\ell,\,LL}^2(C) = \mathcal{M}_{\ell,\,LL}^{-2}(A)\,. \tag{16}$$

Therefore, the (dimensionless) eigenvalues of $\mathcal{M}_{\ell,\,LL}^2(C)$ are the inverses of those of $\mathcal{M}_{\ell,\,LL}^2(A)$, i.e.,

$$(m_\tau^2, m_\mu^2, m_e^2)_C = (m_e^{-2}, m_\mu^{-2}, m_\tau^{-2})_A\,, \tag{17}$$

and $U_{L,R}^\ell(C)$ are the same as $U_{L,R}^\ell(A)$ *with their first and third columns interchanged.*

The mass-squared matrices for case A are

$$\begin{aligned} &\mathcal{M}_{\ell,\,LL}^2(A) \\ &= \begin{pmatrix} (0.1218, 0) & (3.191, 2.50) & (3.413, 2.57) \\ (3.191, -2.50) & (83.84, 0) & (89.69, 0.0716) \\ (3.413, -2.57) & (89.69, -0.0716) & (95.98, 0) \end{pmatrix}, \end{aligned} \tag{18}$$

$$\begin{aligned} &\mathcal{M}_{\ell,\,RR}^2(A) \\ &= \begin{pmatrix} (171.3, 0) & (38.40, -3.08) & (0.1589, -1.96) \\ (38.40, 3.08) & (8.617, 0) & (0.03660, 1.11) \\ (0.1589, 1.96) & (0.03660, -1.11) & (2.77\times 10^{-4}, 0) \end{pmatrix}. \end{aligned} \tag{19}$$

The predicted and measured ratios of the lepton masses are

$$\begin{aligned} &m_e/m_\tau = 1.53\times 10^{-4}\,,\ m_\mu/m_\tau = 0.00802 \qquad \text{(cases A, B)}\,; \\ &m_e/m_\tau = 1.53\times 10^{-4}\,,\ m_\mu/m_\tau = 0.0191 \qquad \text{cases C, D)}\,; \\ &m_e/m_\tau = 2.88\times 10^{-4}\,,\ m_\mu/m_\tau = 0.0595 \qquad \text{(Ref. [43])}\,. \end{aligned} \tag{20}$$

The predicted ratios are not great, but they do exhibit a qualitatively correct hierarchy. Different quark mass textures will lead to different ratios.[4]

The lepton mixing matrices for this case are

$$\begin{aligned} &U_L^\ell(A) = \\ &\begin{pmatrix} (0.9808, 1.325) & (0.1935, -1.945) & (0.02597, 3.026) \\ (0.1515, 1.065) & (0.7149, 0.8359) & (0.6826, 0.5264) \\ (0.1231, -2.372) & (0.6719, -2.369) & (0.7304, 0.4548) \end{pmatrix}, \end{aligned} \tag{21}$$

$$\begin{aligned} &U_R^\ell(A) = \\ &\begin{pmatrix} (0.02203, 2.337) & (0.2176, -0.7759) & (0.9758, -0.4548) \\ (0.1016, 2.303) & (0.9705, -0.8359) & (0.2187, 2.627) \\ (0.9946, -1.325) & (0.1039, -1.321) & (0.9062\times 10^{-3}, 1.502) \end{pmatrix}, \end{aligned} \tag{22}$$

where phases have been chosen to make $\widehat{\mathcal{M}}_\ell$ real and positive (see Eq. (15)). The columns of $U_L^\ell(A)$ are the orthonormal eigenvectors $\nu_{e_L}, \nu_{\mu_L}, \nu_{\tau_L}$ of $\mathcal{M}_{\ell,LL}^2(A)$ with rows labeled by e', μ', τ', and similarly for $U_R^\ell(A)$. For the hermitian conjugate case, with $\mathcal{M}_\ell(B) = \mathcal{M}_u^{-1}\mathcal{M}_d$, the mixing matrix $U_L^\ell(B) = U_R^\ell(A)$. The number of physical phases in U_L^ℓ depends on the nature of the neutrino sector, whether Dirac or Majorana. These phases may induce new sources of CP violation in decay, but only by interfering with SM amplitudes. Since LFV processes have at most tiny SM amplitudes, their rates involve only absolute values of $U_{L,R}^\ell$ elements.

These mixing matrices, or ones developed from other quark-mass textures and the ABP ansatz, can be used to predict LNU and LFV rates in any NP model of these processes. For our $\mathcal{H}_{NP}$, Eq. (4), the elements of interest in U_L^ℓ are the third row, $\nu_{\tau_L'}$. In particular, the amplitudes for $B \to K^{(*)}\ell_i^+\ell_j^-$ and $B_s \to \ell_i^+\ell_j^-$ involve $U_{L3i}^{\ell *} U_{3j}^\ell$. In case A, $|U_{L,32}^\ell| \simeq |U_{L,33}^\ell| \simeq 1/\sqrt{2} \gg |U_{L,31}^\ell|$, contrary to our naive expectation that these matrices have a CKM-like hierarchy [15]. Even more surprising $|U_{R,31}^\ell| \gg |U_{R,32}^\ell| \gg |U_{R,33}^\ell|$. Note that this means that the $U_{L,3i}^\ell$ of case D are CKM-like, as naively expected. These features are a consequence of the block-diagonal $\mathcal{M}_{\ell,LL}^2$ and $\mathcal{M}_{\ell,RR}^2$, the latter exhibiting an extreme example of level-crossing. In turn, these trace back to the textures of $\mathcal{M}_u$ and $\mathcal{M}_d$ (and the ABP ansatz). $\mathcal{M}_u$ is $(2\times 2)\oplus(1\times 1)$ block-diagonal and employs a see-saw to make $m_u \ll m_c$ without an $\mathcal{O}(m_u)$ matrix element. Approximately the same structure in $\mathcal{M}_d$ plus $U_L^u \simeq 1$ lead to the famous relation $\tan\theta_C \cong \theta_{12} \simeq \sqrt{m_d/m_s}$ in V_{CKM}.

Finally, we apply these results to our model "third-generation" Hamiltonian, Eq. (4), and evaluate branching ratios for the LFV processes $B \to K^{(*)}\ell_i^\pm\ell_j^\mp$ and $B_s \to \ell_i^+\ell_j^-$. Since these rates will be proportional to $|U_{L3i}^{\ell *} U_{L3j}^\ell|^2$, both lepton charge assignments may be combined. The first order of business is to note that cases B and C are excluded in our model. In those cases, $|U_{L31}^\ell|^2$ is not much smaller than $|U_{L32}^\ell|^2$, implying $R_K \gtrsim 1$.

[3] We also use $m_b(m_b) = 4.18$ GeV, $m_c(m_c) = 1.275\,\text{GeV}$, $m_s(2\,\text{GeV}) = 95$ MeV, $m_d(2\,\text{GeV}) = 4.8$ MeV and $m_u(2\text{ GeV}) = 2.3$ MeV [43].

[4] We have reproduced the results of Ref. [44] using their $\mathcal{M}_\ell$. The magnitudes of the corresponding $U_{L,R}^\ell$ matrix elements are similar to those in Eqs. (21), (22).

Table 1
Branching ratios for LFV decays of B-mesons and $B_s \to \tau^+\tau^-$ from Eqs. (21), (22), (23), (24), using the central values of ρ_{NP}, of $\mathcal{B}(B^+ \to K^+\mu^+\mu^-) \simeq (4.29 \pm 0.22) \times 10^{-7}$ [1] and of $\mathcal{B}(B_s \to \mu^+\mu^-) = (2.8^{+0.7}_{-0.6}) \times 10^{-9}$ [13]. All decays are corrected for phase space (see text). Branching ratio limits are from Refs. [49,43].

Case	$B^+ \to K^+\mu^\pm\tau^\mp$	$B^+ \to K^+e^\pm\tau^\mp$	$B^+ \to K^+e^\pm\mu^\mp$	
A	1.14×10^{-8}	3.84×10^{-10}	0.52×10^{-9}	
D	0.89×10^{-6}	0.67×10^{-10}	1.17×10^{-12}	
Exp.	$< 4.8 \times 10^{-5}$	$< 3.0 \times 10^{-5}$	$< 9.1 \times 10^{-8}$	
Case	$B_s \to \mu^\pm\tau^\mp$	$B_s \to e^\pm\tau^\mp$	$B_s \to e^\pm\mu^\mp$	$B_s \to \tau^+\tau^-$
A	1.37×10^{-8}	4.57×10^{-10}	1.73×10^{-12}	5.61×10^{-7}
D	1.06×10^{-6}	0.80×10^{-10}	3.91×10^{-15}	0.76×10^{-4}
Exp.	–	–	$< 1.1 \times 10^{-8}$	–

For $\ell_i \neq \ell_j$, our model implies (summing over both lepton charge modes)

$$\frac{\mathcal{B}(B \to K^{(*)}\ell_i^\pm\ell_j^\mp)}{\mathcal{B}(B \to K^{(*)}\mu^+\mu^-)} \simeq 2\rho_{NP}^2 \left|\frac{U^{\ell *}_{L3i}U^\ell_{L3j}}{|U^\ell_{L32}|^2}\right|^2, \tag{23}$$

$$\frac{\mathcal{B}(B_s \to \ell_i^\pm\ell_j^\mp)}{\mathcal{B}(B_s \to \mu^+\mu^-)} \simeq 2\rho_{NP}^2 \left|\frac{U^{\ell *}_{L3i}U^\ell_{L3j}}{|U^\ell_{L32}|^2}\right|^2 \times \left[\frac{m_i^2 + m_j^2 - (m_i^2 - m_j^2)^2/M_{B_s}^2}{2m_\mu^2}\right]\left(\frac{2p_\ell}{M_{B_s}}\right). \tag{24}$$

Here [15],

$$\rho_{NP} = \frac{\frac{G}{2}\, U^{d*}_{L33}U^d_{L32}|U^\ell_{L32}|^2}{-\frac{4G_F}{\sqrt{2}}V^*_{tb}V_{ts}\frac{\alpha_{EM}(m_b)}{4\pi}C_9^e + \frac{G}{2}\, U^{d*}_{L33}U^d_{L32}|U^\ell_{L32}|^2} = -0.136\,, \tag{25}$$

where $V = V_{CKM}$, C_9^e is the Wilson coefficient for the operator O_9 in $\bar{b} \to \bar{s}e^+e^-$, $U^{d*}_{L33}U^d_{L32} \cong V^*_{tb}V_{ts}$ in the quark-mass model of Ref. [46], and p_ℓ is the momentum of the outgoing lepton in the B_s rest frame. The value of ρ_{NP} is obtained from the global-fit result $C_{9,NP} \simeq -12\% C_{9,SM}$ [24], rather than from R_K alone. This ρ_{NP} applies to axial-vector amplitudes as well because the SM interaction renormalized at m_b is pure LL to a good approximation.

From Eq. (25) and the calculated $U_L^{d,\ell}$ matrices, one can estimate the G-coupling strength. For cases A and D one has $G \simeq 4.3 \times 10^{-8}\,\mathrm{GeV}^{-2}$ and $1.8 \times 10^{-6}\,\mathrm{GeV}^{-2}$. These imply the approximate upper bounds $\Lambda_{NP} = 1/\sqrt{G} = 4.8$ TeV and 745 GeV, respectively. These mass scales seem low for a Z', but it must be remembered that it couples primarily to the third generation.

There are two approximations in Eq. (23) as applied to $B \to K\ell^+\ell^-$ ratios. The denominator is best-measured for $B^+ \to K^+\mu^+\mu^-$; it is $\mathcal{B}(B^+ \to K^+\mu^+\mu^-) = (4.29 \pm 0.22) \times 10^{-7}$ integrated over the full q^2-range, 0–22 GeV^2 [5]. This integration extrapolates over most of the charmonium region, $q^2 = 8$–$15\,\mathrm{GeV}^2$, ignoring the presence of the narrow resonances. Charmonium resonances do not, of course, influence the numerators, but we do not know whether LFV searches will include this region. Second, and potentially more important, LFV modes to τ's have smaller phase space than those with $\mu^+\mu^-$. For these semileptonic decays, this effect is accounted for using the results of Ref. [36]. Employing our own calculations, we have corrected for phase space the $B_s \to \ell\ell'$ decay rates involving τ's.

Our results are shown in Table 1. As was to be expected for our third-generation $\mathcal{H}_{NP}$, modes involving $\mu\tau$ have the largest rates followed by $e\tau$ with rates smaller by one or more orders of magnitude. Rates for the experimentally easier $e\mu$ modes are very small and may be beyond reach in the near future; the one exception in our model is $B^+ \to K^+e\mu$ in case A. The large $B_s \to \mu\tau$ rate predictions are not yet excluded. The best public limits on these LFV modes are also listed in Table 1 [49,43].

In conclusion, it is natural to ask how general are our results; are they to be expected in other NP models of the B-decay anomalies or in other schemes for calculating the mixing matrices? Particularly, is the relative importance of $B \to X\mu\tau$ that we found likely to be a common feature of such models? It is hard to be sure, of course, but we do believe it is. As we emphasized of our $\mathcal{H}_{NP}$, the LHCb data strongly points to the third generation, or at most just the second and third generations, as the seat of lepton nonuniversality. Further, the hierarchy of charged-lepton masses — not unlike that for the quarks — suggests block-diagonal mass matrices and, therefore, mixing matrices somewhere along the line from our original CKM-like expectation to the ones we found in Eqs. (21), (22) from the ABP ansatz.

These expectations can be compared with those obtained within other proposed flavor models. Our prediction of a generic enhancement over the SM rate of decay modes involving the third generation is also advertised in the class of models discussed in Ref. [38], although they have unobservable LFV by construction. The only model allowing for a direct comparison is Ref. [35]. For the $B \to K$ transitions to either $e\tau$ or $e\mu$, the branching-ratio ranges predicted in our cases A and D encompass those predicted in their model. In the $\mu\tau$ case our predictions are above theirs in both A and D cases, although they also predict a relative enhancement of this channel with respect to the other LFV modes.

Therefore, in addition to more firmly establishing the apparent lepton nonuniversality in B decays, it is important that LHCb and other experiments mount searches for lepton flavor violation, with special attention to improving significantly the limits on $\mu\tau$ and even $e\tau$ decay modes.

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