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# A Fuzzy Adaptive GPS/INS Integrated Navigation Algorithm

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## Abstract

The statistical properties of the measurement noise of the missile-borne navigation system would vary under different actual working conditions. To address this issue, a fuzzy logic adaptive Kalman filtering based missile-borne GPS/INS integrated navigation algorithm is proposed in this paper. By monitoring the output parameter PDOP of the GPS receiver, the fuzzy logic adaptive controller is utilized to modify the measurement noise variance of the Kalman filter. Thus the Kalman filter can be adjusted to the optimal state, which eventually improves the accuracy of the integrated navigation system. The simulation results show that the proposed algorithm has strong adaptability to the time-varying measurement noise, and outperforms the conventional Kalman filter algorithm by providing more accurate solutions.

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*Keywords:* Kalman filtering; Fuzzy adaptive algorithm; Position Dilution of Precision (PDOP); Integrated navigation

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## 1. Introduction

Nowadays the missile integrated navigation system aims to achieve high accuracy, high reliability, good autonomy, simple structure and low cost. As two commonly used navigation systems, the inertial navigation system (INS) and global positioning system (GPS) have good complementary property, which could significantly improve the accuracy and reliability of navigation systems [1, 2].

Kalman filtering is a commonly used optimization filtering tool in the integrated navigation system, which is based on the recursive linear minimum variance estimation. Generally, Kalman filters require

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that the dynamic process and noise have been identified, and the system noise and measurement noise are both white noises with zero mean values. The Kalman filter may divergent if there is model error or noise uncertainty [3]. Due to the complexity of the actual situation and the uncertainty of GPS signal interference, the statistical properties of the measurement noise of missile-borne GPS/INS integrated navigation system would change along with the actual working environment. The initial prior values do not represent the actual noise in the actual working situation. By conducting repeated experiments on the INS, the statistical properties of the measurement noise in the working situation can be obtained. However, the statistical properties of the measurement noise are unknown in reality [4].

To address the above issues, a fuzzy adaptive Kalman filtering technique is proposed in this paper. Based on the PDOP (position error coefficient) of the output parameters from the GPS receiver, the measurement noise matrix could be adjusted online with the assistance of the designed fuzzy inference system (FIS), so as to make the measurement noise matrix to approach to the true one and avoid filter divergence.

## 2. The Design of the Fuzzy Logic Adaptive Kalman Filter Controllers

### 2.1. GPS/INS integrated navigation system model

GPS/INS integrated navigation system model is provided as follows [5]

$$\begin{cases} X_k = \Phi_{k,k-1}X_{k-1} + \Gamma_{k-1}W_{k-1} \\ Z_k = H_kX_k + V_k \end{cases} \quad (1)$$

where  $X_k$  is the system state-vector,  $Z_k$  is the measurement-vector,  $\Phi_{k,k-1}$  and  $H_k$  are the known state transition matrix and observation matrix, respectively,  $\Gamma_{k-1}$  is the known model noise transition matrix,  $W_{k-1}$  and  $V_k$  are sequences of uncorrelated Gaussian random vectors, of which the mean and covariance satisfy

$$E[W_k] = 0 \quad (2)$$

$$\text{Cov}[W_k, W_j] = Q_k \delta_{kj} \quad (3)$$

$$E[V_k] = 0 \quad (4)$$

$$\text{Cov}[V_k, V_j] = R_k \delta_{kj} \quad (5)$$

$$\text{Cov}[W_i, V_j] = 0 \quad (6)$$

The Kalman filter algorithm can be described by the next set of equations [6],

(a) Time update equations:

$$\hat{X}_{k/k-1} = \Phi_{k/k-1} \hat{X}_{k-1} \quad (7)$$

$$P_{k/k-1} = \Phi_{k,k-1} P_{k-1} \Phi_{k,k-1}^T + \Gamma_{k-1} Q \Gamma_{k-1}^T \quad (8)$$

(b) Measurement update equations:

$$K_k = P_{k/k-1} H_k^T (H_k P_{k/k-1} H_k^T + R_k)^{-1} \quad (9)$$

$$\hat{X}_k = \hat{X}_{k/k-1} + K_k (Z_k - H_k \hat{X}_{k/k-1}) \quad (10)$$

$$P_k = (I - K_k H_k) P_{k/k-1} \quad (11)$$

where  $\hat{X}_k$  represents the estimation of the system state-vector  $X_k$ ,  $P_k$  is the covariance matrix of the state-estimate error, and  $K_k$  is commonly referred to as the filter gain.

In order to prevent the filter divergence, an adaptive Kalman filter using the fuzzy adaptive controller is proposed, in which the covariance matrix of the model measurement noise is adjusted online thus to make it approach to the true one, and eventually avoid the filter divergence. With the proposed approach, the noise covariance matrix of the model is set to be

$$R_k = \alpha R \quad (12)$$

where  $\alpha$  is the adjustment factor obtained from the fuzzy adaptive controller, which will be introduced in the next sub-section. Rewrite the iterative filtering algorithm in Eq. (9) as

$$K_k = P_{k/k-1} H_k^T (H_k P_{k/k-1} H_k^T + \alpha R)^{-1} \quad (13)$$

With Eq. (13), the Kalman filtering is endowed with the adaptive capability and the accuracy of the final estimation could be increased.

## 2.2. Design of the Fuzzy Logic Adaptive Controllers

In our proposed approach, the key component is how to set the value of  $\alpha$  in Eq. (12). Since the output parameter (PDOP) of the GPS receiver directly reflects its positioning accuracy, i.e. the change of GPS measurement errors, basically PDOP reflects the change of the covariance matrix of the measurement noise  $R$ . Therefore, PDOP is employed as the reference for adjusting  $\alpha$ , which is defined as follows [7]:

$$PDOP = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2} / \sigma \quad (14)$$

where  $\sigma$  is the pseudo-range error factor,  $\sigma_x$ ,  $\sigma_y$  and  $\sigma_z$  are the positioning errors in the earth coordinate system on the three axes (x-axis, y-axis, z-axis), respectively.

The Sugeno type fuzzy logic system is employed in this paper, which has been applied to many fields. The result of each rule is a linear combination of the input, and the output is the weighted linear combination of these results. Therefore, it is very easy and intuitive to implement the Sugeno type fuzzy logic system. Moreover, it is much more efficient compared to other fuzzy logic systems since it can generate complex nonlinear function with only a few language rules [8].

In general, the form of fuzzy rules for the Sugeno fuzzy logic system can be expressed as [9]:

$$R^{(i)} : \text{If } x_1 \text{ is } A_1^i \text{ and } x_2 \text{ is } A_2^i \text{ and...and } x_n \text{ is } A_n^i,$$

$$\text{Then, } y_i = C_{i0} + \sum_{k=1}^n C_{ik} \bullet x_k \quad (i=1, 2 \dots M)$$

where  $A_n^i$  is a fuzzy set,  $M$  is the rule section number,  $C_{ik}$  ( $k=0:n$ ) is the real number of the  $i^{\text{th}}$  rule. The output PDOP value from the GPS receiver acts as the input of the fuzzy controller. When the PDOP value is large, i.e. the positioning error of GPS is large; a small  $\alpha$  is set to reduce the confidence level for the measurement values. When the PDOP value is small, i.e. the GPS positioning error is relatively small; a larger  $\alpha$  is used to increase the confidence level. The final membership function is shown in Fig.1, which represents the membership of the input PDOP value in the domain of the fuzzy sets. The membership is represented by  $s_i$  ( $0 \leq s_i \leq 1$ ).

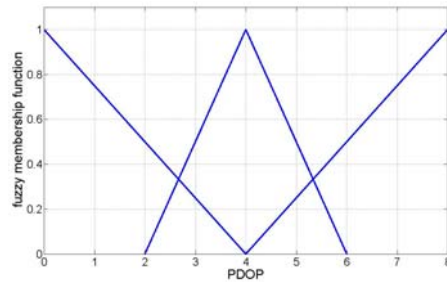


Fig.1. the membership function of PDOP

The FIS output rules to adjust the adjustment factor  $\alpha$  are as follows,

If PDOP is  $a$ , then  $\alpha = f_a(\text{PDOP})$ ;

where “PDOP” represents the output position error of the GPS receiver,  $a$  represent “less”, “equal” or “more”,  $f_a(\text{PDOP})$  is a linear function of PDOP,  $\alpha$  represents the adjustment factor of the measurement noise.

### 3. Simulation Results

In this paper, the position-velocity combination pattern is employed in the GPS/INS integrated navigation system. The 18 system state variables are selected as the attitude error angle, velocity error, position error and error of gyroscopes and accelerometers. During the simulation, the model is kept to be accurate between [0-5000]s. In [5000-10000]s, the measurement noise increases to be 3 times of the original. In [10000-18000]s, the measurement noise increased to be 6 times of the original.

The simulation is conducted by using the conventional Kalman filter algorithm and the proposed fuzzy adaptive Kalman filter algorithm introduced above, respectively. The error curves of position and velocity are shown in Fig.2. (a) and (b), respectively. The dotted lines denote the results of the conventional Kalman filter, and the solid lines represent the result of the proposed fuzzy adaptive Kalman filter.

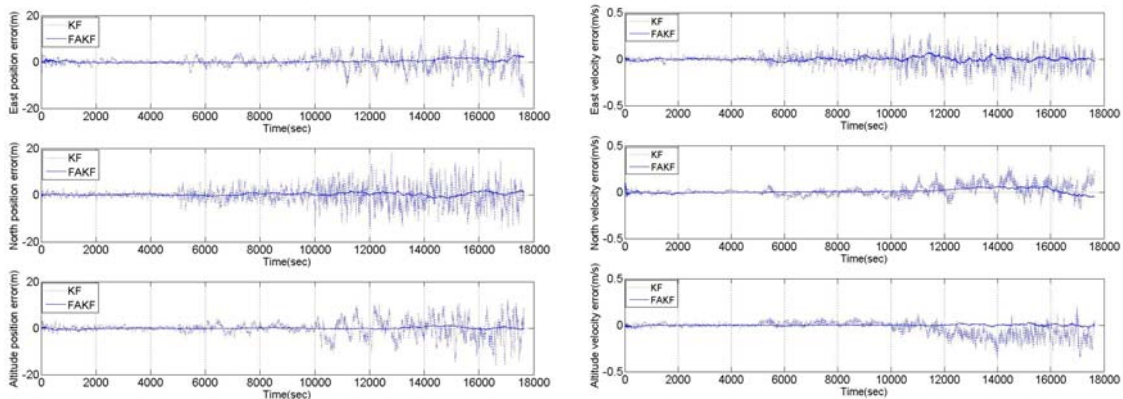


Fig.2. (a) the position error of two algorithms; (b) the velocity error of two algorithms

It can be observed from Fig.2. (a) and (b), from 5000s to 18000s, the model initial array  $R$  cannot accurately describe the statistical properties of the measurement noise due to the fact that the measurement noise has changed twice. Therefore, the error of standard Kalman filter increases

significantly due to the model error. However, for the fuzzy adaptive Kalman filter, since it can adaptively adjust the array to approach to the real measurement noise level to guarantee the accuracy of the model, the filtering error is much smaller than that of the standard Kalman filter. This can be observed from the error curves of the position and velocity in Fig.2. (a) and (b).

#### 4. Conclusions

An adaptive Kalman filter approach using the fuzzy logic controller to detect and prevent the divergence of the conventional Kalman filter is proposed in this paper. Through detecting the output PDOP value of the GPS receiver, a proper measurement noise adjustment factor could be obtained using the fuzzy logic controller, so as to make the measurement noise to approach to the true value. Eventually, the accuracy of the filter can be improved. Simulation results show that the fuzzy adaptive Kalman filter outperforms the conventional Kalman filter by providing more accurate solutions.

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