Europ. J. Combinatorics (1993) **14**, 95–102

Some Combinatorial Aspects of Time-stamp Systems

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The aim of this paper is to outline a combinatorial structure appearing in distributed computing, namely a directed graph in which a certain family of subsets with k vertices have a successor. It has been proved that the number of vertices of such a graph is at least $2^k - 1$ and an effective construction has been given which needs $k2^{k-1}$ vertices. This problem is issued from some questions related to the labeling of processes in a system for determining the order in which they were created. By modifying some requirements on the distributed system, we show that there arise other combinatorial structures leading to the construction of solutions the size of which becomes a linear function of the input.

1. INTRODUCTION

In this paper we address some combinatorial problems arising from questions in distributed computing. Before setting the combinatorial structures, let us first describe in detail the problem of time-stamping. In a system, we consider two kinds of events, namely the creation and the death of processes. We assume that two such events cannot occur simultaneously. A global 'scheduler' assigns a time-stamp to any process at the moment of its creation, according to the actual set of time-stamps assigned to the living processes. Such a time-stamp cannot be modified during the lifetime of the process. The aim of these time-stamps is the following: any external observer of the system who looks at any two processes must be able, solely by considering their two time-stamps, to determine the order of their creation. Note that a simple solution consists in using the values of a counter as time-stamps, and incrementing it whenever a process is created. Then, the observer can use the natural order on integers for determining which one among any two processes was created first. Such a solution needs an unbounded set of time-stamps.

In [9], Israëli and Li proved that, when the number of living processes is assumed to be bounded by an integer k, a time-stamp system with a finite number of elements may be used. In this case, when a process P disappears, its time-stamp becomes vacant and can be used by the scheduler for a newly created process. However, before using this time-stamp, the scheduler has to wait until all the processes more recent than P have disappeared.

Israëli and Li associate with this problem a directed graph $G = (X, E)$. The vertices of G are the time-stamps and the arcs encode the precedence relation among them. Thus, the initial problem consists now in constructing a directed graph satisfying the following condition: for any sequence of vertices $x_1, x_2, \ldots, x_p$ with $p < k$ such that for any $i < j$, $(x_i, x_j) \in E$, there exists a vertex y such that $(x_i, y) \in E$, $1 \leq i \leq p$. Hence, the scheduler chooses such a vertex y as a new time-stamp when $x_1, x_2, \ldots, x_p$ correspond to the time-stamps of the living processes. The notation and a summary of the previously obtained results is given in Section 2.

In other sections, we consider two new problems consisting in building restricted time-stamp systems and we provide solutions with a set the size of which is a linear function of the maximal number k of living processes. In Section 3, we consider the case in which only one of the p older processes can disappear. We call these systems *p-restricted* time stamp systems. Using lexicographic product on graphs we obtain a p-restricted time-stamp system with $p2^{p-1}(2k - 2p + 1)$ elements. The second restriction is obtained by weakening the information asked from the system. It is assumed

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that there are always exactly k living processes (immediately after the death of any process another one is created), and only the determination of the latest created process is required, given the set of labels of all living processes. We call these systems *weak* time-stamp systems. The determination of weak time-stamp systems has already been considered [12] and a solution with k^2 time-stamps was given. We improve this result by proposing a weak time-stamp system with $2k - 1$ elements, and prove this construction to be optimal. Our construction makes use of a matching from the family of $(k-1)$-subsets of $\{1, \ldots, 2k-1\}$ onto the family of its k-subsets. This matching has been considered by many authors [1, 3, 10, 14].

2. Time-stamps

In this section, we give the definitions and some combinatorial results on time-stamp systems, most of them being due to Israëli and Li. Let us begin with notation.

A *directed graph* is defined as a finite set X of *vertices* together with a set of *arcs* which is a subset E of $X \times X$. If (x, y) is an arc, the vertex y is said to dominate x. The set of all dominators of a vertex x is denoted by $\Gamma_G(x)$:

$$\Gamma_G(x) = \{y \mid (x, y) \in E\}.$$

For a subset $Y \subset X$, $\Gamma_G(Y)$ denotes the set of vertices which are dominators of all the elements of Y:

$$\Gamma_G(Y) = \bigcap_{y \in Y} \Gamma_G(y).$$

Throughout the paper we only consider loopless and antisymmetric graphs. They satisfy

$$\forall x, y \in X, \quad (x, x) \notin E \qquad \text{and} \qquad (x, y) \in E \Rightarrow (y, x) \notin E.$$

A sequence $(y_1, y_2, \ldots, y_p)$ of vertices is an *ordered sequence* if for any $1 \leq i < j \leq p$, y_j is a dominator of y_i.

Definition 2.1. A time-stamp system of order k is a directed graph, in which any ordered sequence having fewer than k elements has a dominator.

In such a graph, any vertex belongs to an ordered sequence of cardinality k. A related notion was considered by many authors after Erdös [7, 8, 13]; namely, that of a tournament (i.e. a directed graph in which for any pair of vertices $\{x, y\}$ one is the dominator of the other) satisfying the so-called property $S(p)$. For such a tournament, any subset of cardinality p has a dominator. Hence, any tournament with property $S(p)$ is a time-stamp system of order $p + 1$, but the converse is not true. An example of a tournament which is a time-stamp system of order 4 and which does not satisfy $S(3)$ is given below. The lower bounds found for the number of vertices a tournament must have in order to satisfy $S(p)$, are hence not valid for time-stamp systems; however, similar constructions hold.

For any graph $G = (X, E)$ and any vertex x, denote by G_x the graph the vertex set of which is $\Gamma_G(x)$ and the edge set of which is $E \cap (\Gamma_G(x) \times \Gamma_G(x))$. We obtain:

Proposition 2.2. *If G is a time-stamp system of order k, then for any x in X, G_x is a time-stamp system of order $k - 1$.*

Proof. If $(y_1, y_2, \ldots, y_p)$ is an ordered sequence in G_x then $(x, y_1, y_2, \ldots, y_p)$ is an ordered sequence in G. If $p < k - 1$, since G is a time-stamp system of order k, the sequence $(x, y_1, \ldots, y_p)$ has a dominator which is in $\Gamma_G(x)$. □

Corollary 2.3. *The number of vertices of a time-stamp system of order k is not less than* $2^k - 1$.

Proof. We use induction on k. For $k = 0, 1$ there is nothing to prove. The first non-trivial case is $k = 2$, and the smallest time-stamp system of order 2 is the circuit C_3 with 3 vertices. Let G be a time-stamp system of order $k + 1$ having n vertices. By the induction hypothesis and by Proposition 2.2 each of the G_x's has not less than $2^k - 1$ vertices. Hence the number of arcs $|E|$ of G satisfies $|E| \geq n(2^k - 1)$. Since G is antisymmetric and loopless $|E| \leq [n(n-1)]/2$ and the result follows. □

Note that the converse of Proposition 2.2 holds:

Proposition 2.4. *Let G be an antisymmetric graph such that for any vertex x, G_x is a time-stamp system of order $k - 1$. Then G is a time-stamp system of order k.*

Proof. Let $(x_1, x_2, \ldots, x_l)$, $l < k$ be an ordered sequence in G. Then $(x_2, \ldots, x_l)$ is an ordered sequence in G_{x_1}. By the hypothesis it has a dominator x in G_{x_1}, and x is a dominator of $(x_1, x_2, \ldots, x_l)$. □

The following classical notion in graph theory is useful for building time-stamp systems.

Definition 2.5. Let $G = (X, E)$ and $H = (Y, F)$ be two directed graphs. The lexicographic product $G \otimes H$ has vertex set $X \times Y$ and its set of arcs is given by

$$(x', y') \in \Gamma_{G \otimes H}(x, y) \text{ iff } (x, x') \in E \text{ or } (x = x' \text{ and } (y, y') \in F).$$

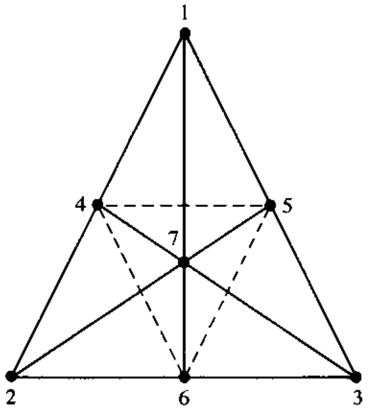
Proposition 2.6. *If G and H are time-stamp systems of respective order k and l, then $G \otimes H$ is a time-stamp system of order $k + l - 1$.*

Proof. Let $(u_1, u_2, \ldots, u_m)$ be an ordered sequence in $G \otimes H$, such that $m < k + l - 1$. Let $u_i = (x_i, y_i)$; then the sequence of x_i's is an ordered sequence in G. Note that the x_i's are not necessarily distinct. If the number of distinct x_i's is less than k, they have a dominator x in X and for any $y \in Y$, (x, y) is a dominator of $(u_1, u_2, \ldots, u_m)$. If the number of distinct x_i's is not less than k, then the number of those equal to x_m is less than l. Let $(y_j, \ldots, y_m)$ be such that $x_j = x_m$ and $x_{j-1} \neq x_m$. This sequence is an ordered sequence in H with less than l elements, thus it has a dominator y and (x_m, y) is a dominator of $(u_1, u_2, \ldots, u_m)$. □

From this proposition follows a method for the construction of time-stamp systems of an arbitrary order. Using the graph C_3, it is possible to obtain a time-stamp system of order k with 3^{k-1} vertices [9]. Other time-stamp systems are known; for little values of k the smallest ones are given by the tournaments satisfying $S(p)$ and for greater values by a construction due to Zielonka [16]. We recall here these results, one of them being that the Fano plane of order 7 gives a time-stamp system of order 3. Consider the dominators of vertex i as a line L_i of this plane. Since a time-stamp system is loopless and antisymmetric, the lines have to be numbered in such a way that

$$i \notin L_i \quad \text{and} \quad j \in L_i \Rightarrow i \notin L_j.$$

This can be done for the Fano plane; the numbering is given in Figure 1.



$L_1 = \{4, 5, 6\}$ $L_5 = \{2, 3, 6\}$

$L_2 = \{1, 6, 7\}$ $L_6 = \{3, 4, 7\}$

$L_3 = \{1, 2, 4\}$ $L_7 = \{1, 3, 5\}$

$L_4 = \{2, 5, 7\}$

FIGURE 1. The Fano plane.

The corresponding graph F_7 is the smallest time-stamp system of order 3: it has 7 vertices and is given by $\Gamma_{F_7}(i) = L_i$. E. Szekeres and G. Szekeres [13] constructed a tournament satisfying property $S(3)$ with 19 vertices. Note that $C_3 \otimes C_3 \otimes C_3$ is a time-stamp system of order 4 which is a tournament but which does not satisfy $S(3)$. The following construction, due to Zielonka [16], yields a time-stamp system of order k with $k2^{k-1}$ vertices; for $k \geq 9$ no time-stamp system with a smaller number of vertices is known.

Consider the subset X_k of $\{1 \cdots k\} \times \{0, 1\}^k$ consisting of elements $(\alpha, x_1, \ldots, x_k)$ such that $x_\alpha = 0$, as a set of vertices of a graph $G = (X_k, E_k)$, and let E_k be such that

$$(\beta, y_1, \ldots, y_k) \in \Gamma_G(\alpha, x_1, \ldots, x_k) \text{ if } (\alpha > \beta \text{ and } x_\beta \neq y_\alpha) \text{ or } (\alpha < \beta \text{ and } x_\beta = y_\alpha).$$

PROPOSITION 2.7. *G is a time-stamp system of order k having $k2^{k-1}$ vertices.*

PROOF. Obviously the number of vertices of G is $k2^{k-1}$. Since there are no arcs between two vertices with the same first component, any ordered sequence U of G must have vertices in which all the first components are distinct. Now, if U has less than k elements then at least one $\alpha \in \{1, 2, \ldots, k\}$ is available for the first component of a dominator x of U. To finish the proof, it is necessary to define the other components $x_1, \ldots, x_k$ of x. Of course, $x_\alpha = 0$; to obtain x_β, if there exists an element $y \in U$ with β as the first component, take

$$x_\beta = y_\alpha \quad \text{if } \beta < \alpha \qquad \text{and} \qquad x_\beta = 1 - y_\alpha \quad \text{if } \alpha < \beta.$$

If no such element exists, take $x_\beta = 0$. □

REMARK. There is no time-stamp system of order 4 with 15 vertices: it is not difficult to see that any such graph would be a tournament in which each vertex would have exactly 7 dominators, any pair of vertices 3 dominators and any triplet only 1 dominator. Brown and Reid [2] have shown that it is not possible to construct such tournaments. A time-stamp system of order 4 with 16 vertices has recently been constructed by Tromp [15].

3. RESTRICTED TIME-STAMP SYSTEMS

Consider the family $\mathfrak{F}$ of ordered sequences having k elements in a time-stamp system of order k; then, for any U and any $u_i \in U$, there exists $v \in X$ such that $U \backslash \{u_i\} \cup \{v\} \in \mathfrak{F}$.

Returning to the labeling of processes, this means that when k processes are living and one dies, a time-stamp can be given to a new process. Let us restrict the set of processes which may die to the older ones, introducing the following notion.

Definition 3.1. A p-restricted time-stamp system of order k is a loopless antisymmetric graph such that there exists a family $\mathfrak{F}$ of ordered sequences with k elements satisfying the following:
(3.1) for any vertex $x \in G$, $\exists U \in \mathfrak{F}$ such that $x \in U$;
(3.2) if $U = (u_1, \ldots, u_k)$ and if $i \leqslant p$, $\exists v$ such that $(u_1, \ldots, u_{i-1}, u_{i+1}, \ldots, v) \in \mathfrak{F}$.

We will first build a 1-restricted time-stamp system of order k, then we will show that for a fixed p there exists a p-restricted time-stamp system of order k with a number of vertices which is a linear function of k.

Definition 3.2. Let G_k be the graph with vertex set $\{1, \ldots, 2k-1\}$ and for each vertex i, let $\Gamma_{G_k}(i) = \{i+1, i+2, \ldots, i+k-1\}$ where the sums are taken mod$(2k-1)$.

This graph is a tournament and, moreover, each vertex is the dominator of $k-1$ vertices and has $k-1$ dominators. It is not so difficult to verify the following:

Proposition 3.3. *G_k is a 1-restricted time-stamp system of order k.*

Proof. Consider the family $\mathfrak{F}$ of all ordered sequences having k elements. Any $U \in \mathfrak{F}$ has the form $(i, i+1, \ldots, i+k-1)$, where the sums are taken mod$(2k-1)$. Since we are only checking the 1-restricted property, it is sufficient to find a dominator for $(i+1, \ldots, i+k-1)$, which is $i+k$. □

The graph G_k allows us to build p-restricted time-stamp systems for any arbitrary integer p, since we have:

Proposition 3.4. *Let $H = (X, E)$ be a time-stamp system of order p. Then $H \otimes G_k$ is a p-restricted time-stamp system of order $k+p-1$.*

Proof. Let us first give some notation. Let Y_k denote the set of vertices of G_k and for any ordered sequence $V = (v_1, \ldots, v_m)$ in $H \otimes G_k$, where $v_i = (x_i, y_i)$, let $\alpha(V)$ be the subset of X consisting of the first components of the v_i's, and let $\beta(V)$ be the subset of Y_k consisting of the second components of the elements the first component of which is equal to x_m:

$$\alpha(V) = \{x_i \mid u_i = (x_i, y_i)\}, \qquad \beta(V) = \{y_i \mid x_i = x_m\}.$$

Let $\mathfrak{F}$ be the family of ordered sequences V having $k+p-1$ elements and such that:
(i) $\text{card}(\alpha(V)) \leqslant p$;
(ii) $\beta(V) = y, y+1, \ldots, y+i \text{ mod}(2k-1), \qquad i<k.$
Thus, $\beta(V)$ consists of consecutive elements in Y_k.

We first prove that a vertex (x, y) of $H \otimes G_k$ belongs to at least one element of $\mathfrak{F}$. Consider an ordered sequence U in H of order p and containing x as its last element,

$$U = (x_1, x_2, \ldots, x_{p-1}, x_p = x).$$

Then, the following sequence v is an element of $\mathfrak{F}$:

$$(v_1 = (x_1, y_1), v_2 = (x_2, y_2), \ldots, v_p = (x, y_p), v_{p+1} = (x, y_p+1), \ldots, v_{p+k-1} = (x, y_p+k-1))$$

where the y_i's $(i = 1, \ldots, p)$ are arbitrarily taken in Y_k.

Now let $V=(v_1, v_2, \ldots, v_m)$ be an element of $\mathfrak{F}$ where $m=p+k-1$, and consider $v_i \in V$, $i \leqslant p$. Since $\beta(V)$ is an ordered sequence in G_k we have $\operatorname{card}(\beta(V)) \leqslant k$: hence either $v_i=(x_i, y_i)$ is such that $x_i \neq x_m$ or $i=p$ and $(x_j, y_j)=(x_m, y_i+j-i)$ for $j=i+1, \ldots, m$.

If $x_i=x_m$ or if $\beta(V)$ has fewer than k elements, let $v=(x_m, y_m)$. Then

$$(v_1, v_2, \ldots, v_{i-1}, v_{i+1}, \ldots, v_m, v)$$

is an element of $\mathfrak{F}$.

If $x_i \neq x_m$ and $\beta(V)$ has k elements, then $\alpha(V)$ has no more than $m-k+1=p$ elements and $\alpha(V)\backslash\{v_i\}$ has fewer than p elements. Since H is a time-stamp system of order p there exists $x \in X$ such that $\alpha(V)\backslash\{x_i\} \cup \{x\}$ is an ordered sequence in H, with x as the last element. Let $v=(x, y)$, where y is any element of Y_k. Then $(v_1, v_2, \ldots, v_{i-1}, v_{i+1}, \ldots, v_m, v)$ is an element of $\mathfrak{F}$. □

Corollary 3.5. *There exists a p-restricted time-stamp system of order k with* $p2^{p-1}(2k-2p+1)$ *vertices.*

4. Weak Time-stamp Systems

A time-stamp system allows us to compare any pair of stamps. In many applications this strong request may be weakened to the determination of the latest created process when the whole set of the k living processes (namely, their time-stamps) is known. This informal requirement may be made precise by the following definition.

Let X be a finite set and let $\mathfrak{F}$ be a family of k-subsets of X. Then $\mathfrak{F}'$ denotes the family of $(k-1)$-subsets Y' of X such that $\exists Y \in \mathfrak{F}$, $Y' \subset Y$.

Definition 4.1. A weak time-stamp system of order k on the family $\mathfrak{F}$ is given by two mappings α and β,

$$\alpha: \mathfrak{F} \to X, \qquad \beta: \mathfrak{F}' \to X,$$

satisfying:

$$\forall x \in X, \exists Y \in \mathfrak{F}, \qquad x \in Y; \tag{1}$$

$$\alpha(Y) \in Y \qquad \text{and} \qquad \beta(Y') \notin Y'; \tag{2}$$

$$\alpha(Y' \cup \beta(Y')) = \beta(Y') \qquad \text{and} \qquad \beta(Y\backslash\alpha(Y)) = \alpha(Y). \tag{3}$$

Note that the two parts of (3) are equivalent, provided that $\forall Y \in \mathfrak{F}$, $\exists Y' \in \mathfrak{F}'$ such that $Y=Y' \cup \beta(Y')$ and $\forall Y' \in \mathfrak{F}'$, $\exists Y \in \mathfrak{F}$ such that $Y'=Y\backslash\alpha(Y)$.

In the context of processes, $\alpha(Y)$ is the latest created time-stamp where Y is a set of k living processes, and $\beta(Y')$ is the time-stamp which has to be assigned to a new process when the set of living processes is Y'.

According to this definition, it is assumed that there are always k or $k-1$ living processes. If the determination of the latest created process is required for any set of less than k processes, then we are led to a situation more or less similar to that of ordinary time-stamp systems. To verify this fact it suffices to consider the algorithm allowing us to compare any pair of time-stamps contained in the same element Y of $\mathfrak{F}$ by deleting iteratively the last element of Y until one of the two time-stamps to be compared to is found.

The following proposition allows us to build weak time-stamp systems:

Proposition 4.2. *There exists a weak time-stamp system on* $\langle X, \mathfrak{F}\rangle$ *if* (1) *is satisfied and there exists a bijection* λ *of* $\mathfrak{F}$ *onto* $\mathfrak{F}'$ *such that*

$$\forall Y \in \mathfrak{F}, \qquad \lambda(Y) \subset Y. \tag{4}$$

PROOF. Let $\mathfrak{F}$ be a family of k-subsets of X satisfying (1), and let λ be a bijection of $\mathfrak{F}$ onto $\mathfrak{F}'$ satisfying (4). Define α and β by:

$$\alpha(Y) = Y \backslash \lambda(Y), \qquad \beta(Y') = \lambda^{-1}(Y') \backslash Y'.$$

Clearly, the definitions of α and β imply (2). The verification of (3) is straightforward:

$$\alpha(Y' \cup \beta(Y')) = \alpha(\lambda^{-1}(Y')) = \lambda^{-1}(Y') \backslash \lambda(\lambda^{-1}(Y')) = \beta(Y').$$

Conversely, let $(X, \mathfrak{F}, \alpha, \beta)$ be a weak time-stamp system and consider λ defined by $\lambda(Y) = Y \backslash \alpha(Y)$, it is easy to verify that λ' defined by $\lambda'(Y') = Y' \cup \beta(Y')$ is the inverse of λ. □

COROLLARY 4.3. *For any weak time-stamp system* $(X, \mathfrak{F}, \alpha, \beta)$, $|X| \geqslant 2k - 1$.

PROOF. Consider the bipartite graph the vertex set of which is $\mathfrak{F} \cup \mathfrak{F}'$, and the edge set of which is given by the pairs $\{Y, Y'\}$ satisfying $Y' \subset Y$. In this graph, every element $Y \in \mathfrak{F}$ has valency k and any element $Y' \in \mathfrak{F}'$ has valency at most $|X| - k + 1$. Thus, if m denotes the cardinality of $\mathfrak{F}$, we obtain

$$km \leqslant m(|X| - k + 1)$$

and the result follows. □

PROPOSITION 4.4. *For any k, there exists a weak time-stamp system of order k with* $2k - 1$ *elements. Moreover, the computation of the mappings* α and β *can be done with a number of operations which is a linear function of k.*

PROOF. Let $X = \{1, \ldots, 2k - 1\}$ and let $\mathfrak{F}$ be the family of all k-subsets of X. Then $\mathfrak{F}'$ is the family of $(k - 1)$-subsets of X. The existence of a matching from $\mathfrak{F}'$ onto $\mathfrak{F}$ is a classical result of combinatorial theory. It may be obtained as a consequence of Hall's theorem, also known as the 'marriage theorem'. The following algorithms allow the computation of $\alpha(Y)$ and $\beta(Y')$; they use a last-in/first-out stack S:

```
Algorithm 1: determination of α(Y)
. for i := 1 step 1 until 2k − 1 do
.   begin
.     if i ∉ Y then push(S, i)
.     else if notempty(S)
.       then pop(S) else x := i
.   end;
. α(Y) := x
```

```
Algorithm 2: determination of β(Y')
. for i := 1 step 1 until 2k − 1 do
.   begin
.     if i ∉ Y' then push(S, i)
.     else if notempty(S) then pop(S)
.   end;
. while notempty(S)
.   do begin x := top(S); pop(S)
.   end;
. β(Y') := x
```

□

These algorithms can be found in [10, Exercise 1, p. 567]; they are attributed there to De Bruijn *et al.* [3]. Aigner [1] proposed another algorithm using lexicographic order on the k-subsets of $\{1, 2, \ldots, 2k - 1\}$, and Trehel [14] has proved that these two algorithms give the same matching.

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Received 31 January 1992 and accepted 9 September 1992

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