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Supergravity background of the λ -deformed $\text{AdS}_3 \times S^3$ supercoset

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Abstract

We construct the solution of type IIB supergravity describing the integrable λ -deformation of the $\text{AdS}_3 \times S^3$ supercoset. While the geometry corresponding to the deformation of the bosonic coset has been found in the past, our background is more natural for studying superstrings, and several interesting features distinguish our solution from its bosonic counterpart. We also report progress towards constructing the λ -deformation of the $\text{AdS}_5 \times S^5$ supercoset.

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1. Introduction

Integrability is a remarkable property, which has led to a very impressive progress in understanding of string theory over the last two decades (see [1] for review). While initially integrability was discovered for isolated models, such as strings on $\text{AdS}_p \times S^q$ [2], later larger classes of integrable backgrounds have been constructed by introducing deformations parameterized by continuous variables. The first example of such family, known as beta deformation [3], has been found long time ago [4], but recently two new powerful tools for constructing integrable string theories have emerged. One of them originated from studies of the Yang–Baxter sigma models [5–7], and it culminated in construction of new integrable string theories, which

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became known as η -deformations [8–12]. The second approach originated from the desire to relate two classes of solvable sigma models, the Wess–Zumino–Witten [13] and the Principal Chiral [14] models, and it culminated in the discovery of a one-parameter family of integrable conformal field theories, which has WZW and PCM as its endpoints [15–17].¹ This connection becomes especially interesting when the PCM point represents a string theory on $\text{AdS}_p \times S^q$ space, and the corresponding families, which became known as λ -deformations, have been subjects of recent investigations [19–22]. A close connection between the η and λ deformations has been demonstrated in [20]. In this article we study the λ -deformation for $\text{AdS}_3 \times S^3$ and $\text{AdS}_5 \times S^5$.

While the metrics for the λ -deformation of $\text{AdS}_p \times S^q$ have been constructed in [17,19], the issue of the fluxes supporting these geometries has not been fully resolved. Although the metric for the deformation can be uniquely constructed starting from the corresponding coset, there are two distinct prescriptions for the dilaton: one is based on a bosonic coset [17], and the other one uses its supersymmetric version [16]. In the first case the deformations for all $\text{AdS}_p \times S^q$ have been constructed in a series of papers [17,19], while in the second case, which is more natural for describing superstrings, only the result for $\text{AdS}_2 \times S^2$ is known [22]. In this article we construct the geometry describing the λ -deformed $\text{AdS}_3 \times S^3$ supercoset and report progress towards finding the deformed $\text{AdS}_5 \times S^5$ solution.

This paper has the following organization. In section 2 we review the procedure for constructing the λ -deformation, which will be used in the rest of the paper. In section 3 we use this procedure to construct the metric and the dilaton for the deformed $\text{AdS}_3 \times S^3$, but unfortunately construction of Ramond–Ramond fluxes requires a separate analysis. In section 3.3 we determine these fluxes by solving supergravity equations, and in sections 3.4–3.5 we find some interesting connections between the new background and solutions which exist in the literature. Section 4 reports progress towards constructing the λ -deformation for super-coset describing strings on $\text{AdS}_5 \times S^5$. Specifically, we determine the metric and the dilaton, but unfortunately we were not able to compute the Ramond–Ramond fluxes. The λ -deformation of $\text{AdS}_2 \times S^2$ constructed in [22] is reviewed in Appendix A, and its comparison with higher dimensional cases is performed throughout the article.

2. Brief review of the λ -deformation

We begin with reviewing the procedure for constructing the NS–NS fields for the λ -deformed cosets. Such deformation belongs to a general class of two-dimensional integrable systems with equations of motion in the form

$$\begin{aligned} \partial_\mu I^\mu &= 0, \\ \partial_\mu I_\nu - \partial_\nu I_\mu + [I_\mu, I_\nu] &= 0, \end{aligned} \quad (2.1)$$

where currents I_μ take values in a semi-simple Lie algebra. Integrability of this system can be demonstrated by writing it as a zero-curvature condition for a linear problem²:

$$\begin{aligned} \mathcal{D}_\mu \Psi &= 0, \quad \mathcal{D}_\mu(\Lambda) = \partial_\mu + \frac{\Lambda^2}{\Lambda^2 - 1} I_\mu + \frac{\Lambda}{\Lambda^2 - 1} \epsilon_{\mu\rho} I^\rho, \\ [\mathcal{D}_\mu(\Lambda), \mathcal{D}_\nu(\Lambda)] &= 0. \end{aligned} \quad (2.2)$$

¹ See [18] for earlier work in this direction.

² We denote that spectral parameter by Λ instead of the conventional λ to avoid confusion with a variable governing the deformation.

Two well-known examples of the integrable systems described by equations (2.1) are the Principal Chiral Model (PCM) [14] and the Wess–Zumino–Witten model [13] for a group G :

$$S_{PCM}(\tilde{g}) = -\frac{\kappa^2}{\pi} \int \text{Tr}(\tilde{g}^{-1} \partial_+ \tilde{g} \tilde{g}^{-1} \partial_- \tilde{g}), \quad I_\mu = \tilde{g}^{-1} \partial_\mu \tilde{g}, \quad (2.3)$$

$$S_{WZW}(g) = -\frac{k}{2\pi} \int \text{Tr}(g^{-1} \partial_+ g g^{-1} \partial_- g) + \frac{ik}{6\pi} \int_B \text{Tr}(g^{-1} dg)^3, \quad I_\mu = g^{-1} \partial_\mu g, \quad (2.4)$$

and the λ -deformation interpolates between these systems. This deformation utilizes two important symmetries of (2.3) and (2.4): the global $G_L \times G_R$ symmetry of the PCM and the $G_{L,cur} \times G_{R,cur}$ symmetry of the current algebra of the WZW.

λ -deformation for groups. Let us review the construction introduced in [15], which allows one to interpolate between the systems (2.3) and (2.4) while preserving integrability. To find such λ deformation, one adds the PCM and WZW models (2.3), (2.4) for the same group G and gauges the $G_L \times G_{diag,cur}$ subgroup of global symmetries. This is accomplished by modifying the derivative in the PCM as

$$\partial_\pm \tilde{g} \rightarrow D_\pm \tilde{g} = \partial_\pm \tilde{g} - A_\pm \tilde{g}, \quad (2.5)$$

and by gauging the resulting WZW model. Integrating out the gauge fields A_\pm , one arrives at the final action [15]³

$$S(g) = S_{WZW}(g) + \frac{1}{\pi} \frac{k^2}{k + \kappa^2} \int J_+^a (1 - \lambda^2 D)_{ab}^{-1} J_-^b, \quad \lambda^2 = \frac{k}{k + \kappa^2}, \quad 0 \leq \lambda \leq 1, \quad (2.6)$$

$$D_{ab} = \text{Tr}(t_a g^{-1} t_b g), \quad J_\pm^a = -i \text{Tr}(t^a \partial_\pm g g^{-1}): \quad J_+^a = R_\mu^a \partial_+ X^\mu, \quad J_-^a = L_\mu^a \partial_- X^\mu.$$

Deformation (2.6) interpolates between the PCM ($\lambda = 1$) and the WZW model ($\lambda = 0$) while preserving integrability [15].

To extract the gravitational background describing the deformation, one rewrites (2.6) as

$$S(g) = S_{WZW}(g) + \frac{k^2}{\pi} \int (R^T M^{-1} L)_{\mu\nu} \partial_+ X^\mu \partial_- X^\nu, \quad M = (k + \kappa^2)(1 - \lambda^2 D), \quad (2.7)$$

and compares the result with the action of the sigma model

$$S = \frac{1}{2} \int (G + B)_{\mu\nu} \partial_+ X^\mu \partial_- X^\nu. \quad (2.8)$$

This leads to the metric and to the Kalb–Ramond field:

$$ds^2 = \frac{k}{2\pi} L^T L + \frac{k^2}{2\pi} L^T (DM^{-1} + [M^{-1}]^T D^T) L \quad (2.9)$$

$$B = \frac{1}{1 - \lambda^4} \left(B_0 + \frac{\lambda^2}{2} L^T \left[(D^T - \lambda^2)^{-1} - (D - \lambda^2)^{-1} \right] \wedge L \right),$$

where B_0 is a Kalb–Ramond field of an undeformed WZW model with the field strength

$$H_0 = -\frac{1}{6} f_{abc} L^a \wedge L^b \wedge L^c. \quad (2.10)$$

³ We follow the conventions of [20,22], and the deformation parameter λ used in [15,17] is equal to λ_{cur}^2 .

Recalling the definition of M and the relation $D^T D = 1$, one can rewrite the metric in terms of convenient frames:

$$ds^2 = e^a e^a, \quad e^a = \sqrt{k(1 - \lambda^4)}(D - \lambda^2)_{ab}^{-1} L^b. \tag{2.11}$$

Expressions for D and L are given in (2.6).

Dilaton for the λ -deformation. Although extraction of the metric and the Kalb–Ramond field for the lambda deformation is rather straightforward, the procedure for calculating the dilaton is controversial. The original proposal of [15] suggested the expression

$$e^{-2\Phi_B} = e^{-2\Phi_0} k^{\dim G} \det(\lambda^{-2} - D), \tag{2.12}$$

which can be written as

$$e^{2\Phi_B} = \frac{1}{\det[(Ad_f - \lambda^{-2})|_{\hat{f}}]}, \tag{2.13}$$

where the determinant is taken in the algebra. In [16] it was argued that for supergroups and supercosets an alternative expression is more appropriate:

$$e^{2\Phi} = \frac{1}{\text{sdet}[(Ad_f - \lambda^{-2})|_{\hat{f}}]}. \tag{2.14}$$

Here the superdeterminant is computed in the full superalgebra \hat{f} . The difference between (2.13) and (2.14) originates from difference in the gauge fields which have been integrated out.

Recalling that an element of a superalgebra can be written as

$$\mathcal{M} = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array} \right], \tag{2.15}$$

where (A, D) are even and (B, C) are odd blocks [23,24], the expression (2.14) becomes

$$e^{2\Phi} = \frac{\det[(Ad_f - \lambda^{-2})|_{\hat{f}_1 \oplus \hat{f}_3}]}{\det[(Ad_f - \lambda^{-2})|_{\hat{f}_0 \oplus \hat{f}_2}]} \tag{2.16}$$

Here \hat{f}_0 and \hat{f}_2 refer to the even subspaces A and D , while \hat{f}_1 and \hat{f}_3 refer to the odd subspaces B and C . In this article we will refer to (2.13) (which is equal to the denominator of (2.16)) as the *bosonic prescription*, and the numerator of (2.16) would be called the *fermionic contribution* to the dilaton.

λ -deformation for cosets. The extension of the λ -deformation to cosets G/H is presented in [17]. Separating the generators T^A of G into T^a corresponding to $H \subset G$ and T^α corresponding to the coset G/H , one finds the metric

$$ds^2 = e^\alpha e^\alpha, \quad e^\alpha = -\sqrt{\frac{k(1 - \lambda^4)}{2\lambda^4}}(M^{-1})^{\alpha B} L^B, \tag{2.17}$$

$$M_{AB} = \left[\begin{array}{cc} (D - 1)_{ab} & D_{a\beta} \\ D_{\alpha b} & (D - \lambda^{-2} 1)_{\alpha\beta} \end{array} \right], \quad D_{AB} = \text{Tr}(T_A g^{-1} T_B g),$$

$$L^A = -i \text{Tr}(g^{-1} dg T^A), \quad \text{Tr}(T_A T_B) = \delta_{AB}.$$

The expression for the dilaton is given by the generalizations of (2.13) and (2.16) [17,16,20]:

$$e^{2\Phi_B} = \frac{1}{\det[(Ad_f - 1 - (\lambda^{-2} - 1)P_\lambda)]}, \tag{2.18}$$

$$e^{2\Phi} = \frac{\det[(Ad_f - 1 - (\lambda^{-2} - 1)P_\lambda)|_{\hat{f}_1 \oplus \hat{f}_3}]}{\det[(Ad_f - 1 - (\lambda^{-2} - 1)P_\lambda)|_{\hat{f}_0 \oplus \hat{f}_2}]} \tag{2.19}$$

Here P_λ is a projector which separates the generators of H and the coset G/H , and it has the form [16]

$$P_\lambda = P_2 + \frac{\lambda}{\lambda + 1}[P_1 - \lambda P_3], \quad P_1 + P_3 = 1. \tag{2.20}$$

Here P_2 is the projector in the bosonic sector, which can be written as

$$P_2 = \begin{bmatrix} 0_{ab} & 0_{a\beta} \\ 0_{\alpha b} & 1_{\alpha\beta} \end{bmatrix}. \tag{2.21}$$

The action of fermionic projectors P_1 and P_3 is evaluated on a case-by-case basis, and we will address this question in the sections 3 and 4. Notice that P_2 has already appeared in the matrix M defined in (2.17):

$$M_{AB} = D_{AB} - 1 - (\lambda^{-2} - 1)P_2 = Ad_f - 1 - (\lambda^{-2} - 1)P_2. \tag{2.22}$$

We conclude this discussion with reviewing a very interesting observation made in [19]: factorization of the λ -dependence in the determinant of M_{AB} . This technical simplification becomes especially useful in the $AdS_5 \times S^5$ case, where one has to deal with large matrices. Following [19], we write M_{AB} as a product of two block-triangular matrices:

$$M = \begin{bmatrix} \mathbf{A} & 0 \\ \mathbf{C} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{A}^{-1}\mathbf{B} \\ 0 & \mathbf{P} \end{bmatrix}. \tag{2.23}$$

As demonstrated in [19], matrix \mathbf{P} has eigenvalues $\lambda^{-2} \pm 1$, so the coordinate dependence of the bosonic dilaton (2.18) comes from $\det \mathbf{A}$. We find that direct evaluation of the determinant of M is easier than construction of \mathbf{P} , but our final results confirm that the coordinate dependence of $\det M$ is inherited from $\det \mathbf{A}$.

3. Deformation of $AdS_3 \times S^3$

Let us apply the procedure reviewed in the last section to $AdS_3 \times S^3$. The bosonic part of the sigma model is described by a product of two cosets

$$\frac{SU(2) \times SU(2)}{SU(2)_{diag}} \times \frac{SU(1, 1) \times SU(1, 1)}{SU(1, 1)_{diag}}, \tag{3.1}$$

and the full string theory is described by a super-coset [25]⁴

$$\frac{PSU(1, 1|2)^2}{SU(1, 1) \times SU(2)}. \tag{3.2}$$

⁴ Various aspects of integrability of string on this background are further discussed in [26].

In section 3.1 we construct the metric and the bosonic contribution to the dilaton for the cosets (3.1), (3.2). While this will give the full answer for (3.1), the dilaton for the supercoset (3.2) also receives a fermionic contribution, which will be evaluated in section 3.2. In section 3.3 we construct the Ramond–Ramond fluxes supporting the λ -deformed supercoset (3.2), and properties of the new geometries are discussed in sections 3.4 and 3.5.

3.1. *Metric and the bosonic dilaton*

The metric is constructed using the bosonic coset (3.1), then S^3 and AdS_3 decouple, and they can be studied separately. We begin with analyzing the sphere, and deformation of AdS_3 can be found by performing an analytic continuation.

Deformation of the sphere. To describe the coset $\frac{SU(2)_l \times SU(2)_r}{SU(2)_{diag}}$, we use the algebraic parameterization introduced in [17]:

$$g_l = \begin{bmatrix} \alpha_0 + i\alpha_3 & \alpha_2 + i\alpha_1 \\ -\alpha_2 + i\alpha_1 & \alpha_0 - i\alpha_3 \end{bmatrix}, \quad g_r = \begin{bmatrix} \beta_0 + i\beta_3 & \beta_2 + i\beta_1 \\ -\beta_2 + i\beta_1 & \beta_0 - i\beta_3 \end{bmatrix}, \tag{3.3}$$

where variables α_k, β_k are subject to the determinant constraints

$$\sum (\alpha_k)^2 = 1, \quad \sum (\beta_k)^2 = 1. \tag{3.4}$$

Gauging of the diagonal part of $SU(2)_l \times SU(2)_r$ makes the description (3.3) redundant, and to remove the unphysical degrees of freedom we impose a convenient gauge, which was also used in [17]. Acting on g_l as $g_l \rightarrow h^{-1}g_l h$, we can set $\alpha_2 = \alpha_3 = 0$, then the remaining $U(1)$ transformations $h = \exp[ix\sigma_1]$ can be used to set $\beta_3 = 0$:

$$\alpha_2 = \alpha_3 = \beta_3 = 0. \tag{3.5}$$

Following [17] we introduce a convenient coordinate γ and solve the constraints (3.4) to express all remaining components of g_1 and g_2 in terms of $(\alpha_0, \beta_0, \gamma)$:

$$\beta_1 \equiv \frac{\gamma}{\sqrt{1 - \alpha_0^2}}, \quad \alpha_1 = \sqrt{1 - \alpha_0^2}, \quad \beta_2 = \sqrt{1 - \beta_0^2 - \frac{\gamma^2}{1 - \alpha_0^2}}. \tag{3.6}$$

To simplify notation, we will drop the subscripts of α_0 and β_0 .

The elements of $SU(2)_l \times SU(2)_r$ can be represented as block-diagonal 4×4 matrices:

$$g = \begin{pmatrix} g_l & 0 \\ 0 & g_r \end{pmatrix}, \quad g^\dagger g = I, \tag{3.7}$$

then the generators corresponding to the subgroup H and to the coset G/H can be written in terms of the Pauli matrices.⁵

$$H = SU(2)_{diag} : T_a = \frac{1}{2} \begin{bmatrix} \sigma_a & 0 \\ 0 & \sigma_a \end{bmatrix}, \quad a = 1, 2, 3; \\ G/H = \frac{SU(2)_l \times SU(2)_r}{SU(2)_{diag}} : T_\alpha = \frac{1}{2} \begin{bmatrix} \sigma_{\alpha-3} & 0 \\ 0 & -\sigma_{\alpha-3} \end{bmatrix}, \quad \alpha = 4, 5, 6. \tag{3.8}$$

⁵ Recall that construction (2.17) is based on normalized generators, and factor 1/2 in (3.8) ensures that $\text{Tr}(T_A T_B) = \delta_{AB}$.

Substitution of (3.7)–(3.8), where g_l, g_r are given by (3.3), (3.5), into the defining relations (2.17)⁶ leads to the metric [17]

$$\begin{aligned}
 ds^2 &= \frac{k}{2(1-\lambda^4)\Lambda} \Delta_{\mu\nu} dx^\mu dx^\nu, & \Lambda &= (1-\alpha^2)(1-\beta^2) - \gamma^2, \\
 \Delta_{\alpha\alpha} &= 4(1+\lambda^2)^2 - \beta^2(3+\lambda^2)(1+3\lambda^2), & \Delta_{\alpha\gamma} &= -\beta(1-\lambda^2)^2 \\
 \Delta_{\beta\beta} &= 4(1+\lambda^2)^2 - \alpha^2(3+\lambda^2)(1+3\lambda^2), & \Delta_{\beta\gamma} &= -\alpha(1-\lambda^2)^2 \\
 \Delta_{\gamma\gamma} &= (1-\lambda^2)^2, & \Delta_{\alpha\beta} &= \alpha\beta(1-\lambda^2)^2 + 4\gamma(1+\lambda^2)^2.
 \end{aligned}
 \tag{3.9}$$

Deformation of AdS₃. The deformation of the AdS₃ is constructed by performing an analytic continuation of (3.9). The defining relation for $g \in SU(1, 1)_l \times SU(1, 1)_r$ is

$$g = \begin{bmatrix} g_l & 0 \\ 0 & g_r \end{bmatrix}, \quad g^\dagger \Sigma_4 g = \Sigma_4, \quad \Sigma_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}, \tag{3.10}$$

and it can be enforced by starting with an element of $SU(2)_l \times SU(2)_r$, renaming the coordinates as

$$\alpha \rightarrow \tilde{\alpha}, \quad \beta \rightarrow \tilde{\beta}, \quad \gamma \rightarrow \tilde{\gamma}, \quad k \rightarrow -k, \tag{3.11}$$

and changing their range from

$$0 < \alpha^2 < 1, \quad 0 < \beta^2 < 1, \quad \gamma^2 < (1-\alpha^2)(1-\beta^2) \tag{3.12}$$

to

$$1 < \tilde{\alpha}^2, \quad 1 < \tilde{\beta}^2, \quad \tilde{\gamma}^2 < (\tilde{\alpha}^2 - 1)(\tilde{\beta}^2 - 1). \tag{3.13}$$

To view this transition as a proper analytic continuation, one can introduce alternative coordinates (a, b, γ) as

$$a^2 = 1 - \alpha^2, \quad b^2 = 1 - \beta^2. \tag{3.14}$$

Then transition from (3.12) to (3.13) amounts to a continuation from real to imaginary (a, b) . This changes the signature from $(+++)$ to $(-+-)$, and by changing the sign of k we recover $(++-)$.

Analytic continuation (3.11) along with the replacement $k \rightarrow -k$ gives the metric for the λ -deformed AdS_3

$$\begin{aligned}
 d\tilde{s}^2 &= \frac{k}{2(1-\lambda^4)\tilde{\Lambda}} \tilde{\Delta}_{\mu\nu} dx^\mu dx^\nu, & \tilde{\Lambda} &= (\tilde{\alpha}^2 - 1)(\tilde{\beta}^2 - 1) - \tilde{\gamma}^2, \\
 \tilde{\Delta}_{\tilde{\alpha}\tilde{\alpha}} &= -4(1+\lambda^2)^2 + \tilde{\beta}^2(3+\lambda^2)(1+3\lambda^2), & \tilde{\Delta}_{\tilde{\alpha}\tilde{\gamma}} &= \tilde{\beta}(1-\lambda^2)^2 \\
 \tilde{\Delta}_{\tilde{\beta}\tilde{\beta}} &= -4(1+\lambda^2)^2 + \tilde{\alpha}^2(3+\lambda^2)(1+3\lambda^2), & \tilde{\Delta}_{\tilde{\beta}\tilde{\gamma}} &= \tilde{\alpha}(1-\lambda^2)^2 \\
 \tilde{\Delta}_{\tilde{\gamma}\tilde{\gamma}} &= -(1-\lambda^2)^2, & \tilde{\Delta}_{\tilde{\alpha}\tilde{\beta}} &= -\tilde{\alpha}\tilde{\beta}(1-\lambda^2)^2 - 4\tilde{\gamma}(1+\lambda^2)^2.
 \end{aligned}
 \tag{3.15}$$

⁶ Recall the ranges of indices in (2.17): $a = \{1, 2, 3\}$, $\alpha = \{3, 5, 6\}$, $B = \{1, \dots, 6\}$.

Dilaton and RR fields for the bosonic coset. The deformation of $\text{AdS}_3 \times \text{S}^3$ constructed in [17] is described by the metric $\{(3.9), (3.15)\}$ and the dilaton corresponding to the bosonic prescription (2.18):

$$e^{-2\Phi_B} = \frac{2\Lambda(1-\lambda^2)^2(1+\lambda^2)}{\lambda^6} \frac{2\tilde{\Lambda}(1-\lambda^2)^2(1+\lambda^2)}{\lambda^6} = e^{-2\Phi_0} \Lambda \tilde{\Lambda}. \tag{3.16}$$

This article also listed the corresponding Ramond–Ramond fields:

$$C_2 = \frac{4k\lambda}{1-\lambda^2} \left[\tilde{\beta}\beta d\tilde{\alpha} \wedge d\alpha + 2\tilde{\beta}\alpha d\tilde{\alpha} \wedge d\beta - \tilde{\beta}d\tilde{\alpha} \wedge d\gamma + \tilde{\alpha}\alpha d\tilde{\beta} \wedge d\beta - \alpha d\tilde{\gamma} \wedge d\beta \right]. \tag{3.17}$$

However, as argued in [16,20,22], the dilaton (2.19) for the supercoset is more natural for describing superstrings, and in the next subsection we will find the appropriate expression and construct the corresponding Ramond–Ramond fluxes.

3.2. Fermionic contribution to the dilaton

In this subsection we will construct the dilaton for the supercoset (3.2) using the prescription (2.19). Before focusing on (3.2), we will outline the procedure for applying (2.19) to a supermatrix (2.15) constructed from extending an algebra of the bosonic coset $(G_1/H_1) \times (G_2/H_2)$.

A supersymmetric extension of an algebra $\mathfrak{g}_1 \times \mathfrak{g}_2$ has the form

$$\mathcal{M} = \begin{bmatrix} g_1 & f_{12} \\ f_{21} & g_2 \end{bmatrix}, \quad g_1 \in \mathfrak{g}_1, \quad g_2 \in \mathfrak{g}_2, \tag{3.18}$$

and to find the supercoset, we should fix the gauge corresponding to subalgebras $\mathfrak{h}_1, \mathfrak{h}_2$ and evaluate the relevant projectors P_λ . This can be done in five steps:

1. Find an automorphism J_1 of algebra \mathfrak{g}_1 which leaves invariant only the elements of \mathfrak{h}_1 . In other words, $g \in \mathfrak{g}_1$ satisfies the condition

$$J_1^{-1} g J_1 = g \tag{3.19}$$

if and only if $g \in \mathfrak{h}_1$. Automorphism J_2 in \mathfrak{g}_2 is defined in a similar way.

2. Construct an automorphism of the super-algebra as

$$\mathcal{P} = \begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix}, \tag{3.20}$$

and project out the elements \mathcal{M} which are left invariant under such automorphism⁷:

$$\mathcal{P}^{-1} \mathcal{M} \mathcal{P} = \mathcal{M}. \tag{3.21}$$

For bosonic generators this reduces to (3.19) and its counterpart for \mathfrak{g}_2 , while the projections for the fermionic matrices are

$$J_1^{-1} f_{12} J_2 = f_{12}, \quad J_2^{-1} f_{21} J_1 = f_{21}. \tag{3.22}$$

⁷ Sometimes this condition requires a modification: as we will see in section 4, in the case of $\text{AdS}_5 \times \text{S}^5$ it must be replaced by $\mathcal{P}^{-1} \mathcal{M} \mathcal{P} = \mathcal{M}^T$.

3. Construct the projector P_2 acting on bosonic generators by requiring that $[1 - P_2]$ kills the same elements as (3.19) and its counterpart with J_2 . Such P_2 projects on the bosonic part of the supercoset.
4. Construct projector P_3 acting on fermionic generators by requiring that P_3 keeps the same elements as (3.22). The fermionic projector complementary to P_3 is $P_1 = 1 - P_3$.
5. Construct the projector P_λ using the definition (2.20). Substitution of this expression into (2.19) or (2.18) and evaluation of the resulting determinant gives the dilaton for the (super) coset.

To apply this procedure to the $AdS_3 \times S^3$ coset (3.2), we observe that \mathfrak{g}_1 represents the algebra of (3.7),

$$g \in \mathfrak{g}_1 : \quad g = \begin{pmatrix} g_l & 0 \\ 0 & g_r \end{pmatrix}, \quad g_l \in \mathfrak{su}(2), \quad g_r \in \mathfrak{su}(2), \tag{3.23}$$

while the elements of $\mathfrak{h}_1 = \mathfrak{su}(2)_{diag}$ have the form

$$\begin{bmatrix} g & 0 \\ 0 & g \end{bmatrix}, \quad g \in \mathfrak{su}(2). \tag{3.24}$$

This leads to two options for the automorphism J_1 :

$$J_1 = \pm \begin{bmatrix} 0 & 1_{2 \times 2} \\ 1_{2 \times 2} & 0 \end{bmatrix}. \tag{3.25}$$

Expression for J_2 is constructed in a similar way, and putting these results together, we find two options for the automorphism \mathcal{P} :

$$\mathcal{P} = \begin{bmatrix} 0 & 1_{2 \times 2} & 0 & 0 \\ 1_{2 \times 2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1_{2 \times 2} \\ 0 & 0 & 1_{2 \times 2} & 0 \end{bmatrix} \quad \text{or} \quad \mathcal{P} = \begin{bmatrix} 0 & 1_{2 \times 2} & 0 & 0 \\ 1_{2 \times 2} & 0 & 0 & 0 \\ 0 & 0 & 0 & -1_{2 \times 2} \\ 0 & 0 & -1_{2 \times 2} & 0 \end{bmatrix}. \tag{3.26}$$

The fermionic generators of $PSU(1, 1|2) \times PSU(1, 1|2)$ appearing in (3.18) obey the relation

$$f_{12} = -i \Sigma_4 (f_{21})^\dagger \tag{3.27}$$

with Σ_4 given in (3.10), and projection (3.21) leads to further constraints. It is convenient to decouple f_{12} and f_{21} by working with holomorphic and anti-holomorphic coordinates. Relations (3.22) isolate 4 + 4 components of f_{12} and f_{21} killed by P_1 , while P_3 kills the complementary 4 + 4 components.⁸ Extraction of (P_1, P_2, P_3) , construction of P_λ via (2.20), and evaluation of superdeterminant (2.19) gives the same dilaton for both choices (3.26):

$$e^\Phi = Q e^{\Phi_B}, \quad e^{\Phi_B} = \frac{1}{\sqrt{[(1 - \alpha^2)(1 - \beta^2) - \gamma^2][(\tilde{\alpha}^2 - 1)(\tilde{\beta}^2 - 1) - \tilde{\gamma}^2]}},$$

$$Q = (1 - \lambda^2)^4 \left[\gamma + \tilde{\gamma} - \frac{4\lambda(1 + \lambda^2)}{(1 - \lambda^2)^2} (\alpha\tilde{\beta} + \tilde{\alpha}\beta) + \frac{\lambda^4 + 6\lambda^2 + 1}{(\lambda^2 - 1)^2} (\alpha\beta + \tilde{\alpha}\tilde{\beta}) \right]^2. \tag{3.28}$$

⁸ Recall that even though f_{12} and f_{21} are represented by 4×4 matrices, each of these objects has only 8 nonzero components. The details are discussed in the Appendix B, here we just refer to the explicit form of the $\mathfrak{psu}(1, 1|2) \times \mathfrak{psu}(1, 1|2)$ matrix (B.9), which clearly exhibits the non-vanishing elements.

We conclude this section by analyzing the symmetries of the metric $\{(3.9), (3.15)\}$ and the dilaton (3.28), which will be used for constructing the Ramond–Ramond fluxes. First, it is clear that neither the metric nor the dilaton has continuous symmetries, but all NS–NS fluxes are invariant under several discrete transformations:

$$\begin{aligned} S_1 : \alpha &\leftrightarrow \beta, & \tilde{\alpha} &\leftrightarrow \tilde{\beta}; \\ S_2 : \alpha &\leftrightarrow \tilde{\alpha}, & \beta &\leftrightarrow \tilde{\beta}, \quad \gamma \leftrightarrow \tilde{\gamma}, \quad k \leftrightarrow (-k). \end{aligned} \tag{3.29}$$

These symmetries will be used in the next section to select a natural solution for the RR field C_2 .

3.3. Ramond–Ramond fluxes

Although the Ramond–Ramond fluxes for the lambda-deformed backgrounds can be extracted from the fermionic part of the sigma model, such problem is notoriously complicated [22]. When similar deformation were analyzed in the past, the RR fluxes were obtained by solving supergravity equations [9,10,22], and in this section we will follow the same route. We will demonstrate that under very weak assumptions, supergravity gives the unique expression for all fluxes.

Since the undeformed $\text{AdS}_3 \times S^3$ geometry is supported by the Ramond–Ramond three-form, we assume that the situation will remain the same after the deformation, so the relevant part of action for the type IIB supergravity reads

$$S = \int d^6x \sqrt{-g} \left[e^{2\Phi} (R + 4(\partial\Phi)^2) - \frac{1}{12} F_{mnp} F^{mnp} \right]. \tag{3.30}$$

This leads to the equations of motion

$$\nabla^2 e^{-2\Phi} = 0, \tag{3.31}$$

$$\nabla_m F^{mnk} = 0, \tag{3.32}$$

$$e^{-2\Phi} (R_{mn} + 2\nabla_m \nabla_n \Phi) = \frac{1}{4} \left(F_{mpq} F_n{}^{pq} - \frac{1}{6} g_{mn} F_{spq} F^{spq} \right) \tag{3.33}$$

and the first one is solved by metric (3.9), (3.15) and the dilaton (3.28).

To construct an expression for C_2 , we observe that the left-hand side of the Einstein’s equation (3.33) has the structure

$$\frac{P}{Q^2}, \tag{3.34}$$

where Q is given by (3.28), and P is a polynomial in $(\alpha, \beta, \gamma, \tilde{\alpha}, \tilde{\beta}, \tilde{\gamma})$. This suggests a natural ansatz for C_2 :

$$C_2 = \frac{1}{Q} \tilde{C}_{\mu\nu} dx^\mu \wedge dx^\nu, \tag{3.35}$$

where all $\tilde{C}_{\mu\nu}$ are polynomials of degree two⁹ in $(\alpha, \beta, \gamma, \tilde{\alpha}, \tilde{\beta}, \tilde{\gamma})$. This ansatz leaves

$$\frac{6 \times 5}{2} \times \left[1 + 6 + 6 + \frac{6 \times 5}{2} \right] = 420 \tag{3.36}$$

⁹ The degree comes from counting powers in the left-hand side of the Einstein’s equations.

undetermined coefficients. We then found the most general solution for $\tilde{C}_{\mu\nu}$ following these steps:

1. Solving equations (3.32)–(3.33) for $\lambda = 0$, when the metric and the dilaton are relatively simple, we reduced the number of undetermined coefficients to 43.
2. Solving equations (3.32)–(3.33) in the first order in λ , we reduced the number of undetermined coefficients in the *zeroth* order to 42.
3. Eliminating the gauge freedom, we demonstrated that the solution at the zeroth order in λ is *unique* up to a gauge transformation.

Once uniqueness of the solution for $\lambda = 0$ is demonstrated, we can choose a convenient gauge which respects the discrete symmetries (3.29):

$$\begin{aligned}
 C_{\alpha\tilde{\alpha}} &= \frac{k}{Q} \left[2 - (\beta^2 + \tilde{\beta}^2) \right], & C_{\beta\tilde{\beta}} &= -\frac{k}{Q} \left[2 - (\alpha^2 + \tilde{\alpha}^2) \right], & (3.37) \\
 C_{\alpha\tilde{\beta}} &= -C_{\beta\tilde{\alpha}} = k \frac{\tilde{\gamma} - \gamma}{Q}, & C_{\alpha\tilde{\gamma}} &= -\frac{k\tilde{\beta}}{Q}, & C_{\beta\tilde{\gamma}} &= \frac{k\tilde{\alpha}}{Q}, & C_{\gamma\tilde{\alpha}} &= -\frac{k\beta}{Q}, & C_{\gamma\tilde{\beta}} &= \frac{k\alpha}{Q}.
 \end{aligned}$$

This solution is odd under S_1 and S_2 . The uniqueness of the solution in the zeroth order in λ guarantees that, up to a gauge transformation, there is a unique gauge potential C_2 , at least in the perturbative expansion in powers of λ . Making a guess consistent with symmetries (3.29), we arrive at the final solution

$$\begin{aligned}
 C_{\alpha\tilde{\alpha}} &= \frac{\hat{k}}{Q} \left[2 + c_1\beta\tilde{\beta} - c_3(\beta^2 + \tilde{\beta}^2) \right], & C_{\alpha\tilde{\beta}} &= -C_{\beta\tilde{\alpha}} = \frac{\hat{k}}{Q}(\tilde{\gamma} - \gamma), \\
 C_{\alpha\tilde{\gamma}} &= \frac{\hat{k}}{Q}[c_2\beta - \tilde{\beta}] & C_{\beta\tilde{\beta}} &= -\frac{\hat{k}}{Q} \left[2 + c_1\alpha\tilde{\alpha} - c_3(\alpha^2 + \tilde{\alpha}^2) \right], & C_{\beta\tilde{\gamma}} &= -\frac{\hat{k}}{Q}[c_2\alpha - \tilde{\alpha}], \\
 C_{\gamma\tilde{\alpha}} &= -\frac{\hat{k}}{Q}[\beta - c_2\tilde{\beta}], & C_{\gamma\tilde{\beta}} &= \frac{\hat{k}}{Q}[\alpha - c_2\tilde{\alpha}] & (3.38) \\
 c_1 &= 2c_2c_3, & c_2 &= \frac{2\lambda}{1 + \lambda^2}, & c_3 &= \frac{\lambda^4 + 6\lambda^2 + 1}{(\lambda^2 - 1)^2}, & \hat{k} &= \frac{k(1 + \lambda^2)}{1 - \lambda^2}.
 \end{aligned}$$

Notice that, unlike the solution (3.17) with the “bosonic dilaton”, the field (3.38) has a complicated lambda dependence, and the situation is similar in the $\text{AdS}_2 \times S^2$ case, which is reviewed in the Appendix A. In particular, while the field (3.17) vanishes at the WZW point ($\lambda = 0$), our solution for the supercoset (3.38) goes to a nontrivial limit, and, as we will see in section 4 and in the Appendix A, the same phenomenon persists for $\text{AdS}_2 \times S^2$ and $\text{AdS}_5 \times S^5$.

To summarize, the λ -deformed version of $\text{AdS}_3 \times S^3$ is described by the metric (3.9), (3.15), the dilaton (3.28), and the Ramond–Ramond two-form (3.38). In the next subsection we will analyze some special cases of this geometry.

3.4. Special cases

The solution (3.9), (3.15), (3.28), (3.38) simplifies in several special cases, and we will briefly discuss these interesting limits.

The gauged WZW model is obtained by setting $\lambda = 0$:

$$\begin{aligned}
 ds^2 &= \frac{k}{2\Lambda} \left[4(1 - \beta^2)d\alpha^2 + 4(1 - \alpha^2)d\beta^2 + 8\gamma d\alpha d\beta + (d\gamma - \beta d\alpha - \alpha d\beta)^2 \right] + \\
 &\quad + \frac{k}{2\Lambda} \left[4(\tilde{\beta}^2 - 1)d\tilde{\alpha}^2 + 4(\tilde{\alpha}^2 - 1)d\tilde{\beta}^2 - 8\tilde{\gamma}d\tilde{\alpha}d\tilde{\beta} + (d\tilde{\gamma} - \tilde{\beta}d\tilde{\alpha} - \tilde{\alpha}d\tilde{\beta})^2 \right], \\
 \Lambda &= (1 - \alpha^2)(1 - \beta^2) - \gamma^2, \quad \tilde{\Lambda} = (\tilde{\alpha}^2 - 1)(\tilde{\beta}^2 - 1) - \tilde{\gamma}^2, \\
 e^\Phi &= \frac{Q}{\sqrt{\Lambda\tilde{\Lambda}}} e^{\Phi_B}, \quad Q = \left[\gamma + \tilde{\gamma} + \alpha\beta + \tilde{\alpha}\tilde{\beta} \right]^2, \\
 C_2 &= \frac{k}{Q} \left[(\tilde{\alpha}d\beta - \tilde{\beta}d\alpha) \wedge (d\tilde{\gamma} + \tilde{\alpha}d\tilde{\beta} + \tilde{\beta}d\tilde{\alpha}) - (\alpha d\tilde{\beta} - \beta d\tilde{\alpha}) \wedge (d\gamma + \alpha d\beta + \beta d\alpha) \right. \\
 &\quad \left. + (\tilde{\gamma} - \gamma + \tilde{\alpha}\tilde{\beta} - \alpha\beta)(d\alpha \wedge d\tilde{\beta} - d\beta \wedge d\tilde{\alpha}) + 2(d\alpha \wedge d\tilde{\alpha} - d\beta \wedge d\tilde{\beta}) \right].
 \end{aligned}
 \tag{3.39}$$

This should be contrasted with bosonic gWZW, which has the dilaton

$$e^\Phi = \frac{1}{\sqrt{\Lambda\tilde{\Lambda}}} \tag{3.40}$$

and vanishing C_2 (see (3.17)). A similar contrast is encountered in the $\text{AdS}_2 \times \text{S}^2$ and $\text{AdS}_5 \times \text{S}^5$ cases, which discussed in section 4 and in the Appendix A.

Note that the metric (3.39) for the $SO(4)/SO(3)$ gWZW model has been discussed in [27, 11], where the element of the coset was defined as

$$\begin{aligned}
 g &= g_1(\varphi)g_2(\theta)g_3(2t)g_2(\theta)g_1(\varphi), \\
 g_k(\alpha) &= \exp(\alpha T_{k,k+1}), \quad (T_{k,k+1})^j_i = \delta_{k,i}\delta^j_{k+1} - \delta_{k+1,i}\delta^j_k, \quad k = 1, 2, 3.
 \end{aligned}
 \tag{3.41}$$

The coordinates used in (3.39) are related to the Euler angles (3.41) as

$$\begin{aligned}
 \alpha &= \cos \varphi \cos t \cos \theta + \sin \varphi \sin t, \\
 \beta &= \cos \varphi \cos t \cos \theta - \sin \varphi \sin t, \\
 \gamma &= -\cos^2 \varphi \sin^2 t + \cos^2 t (\cos^2 \theta \sin^2 \varphi + \sin^2 \theta).
 \end{aligned}
 \tag{3.42}$$

Another interesting limit is obtained by setting $\lambda = 1$. However, this limit should be approached with a great care since denominators contain $(\lambda^2 - 1)$. We will follow the procedure discussed in [20] adopting it to our coordinates. To arrive at a sensible limit, we rescale the coordinates on the sphere as

$$\alpha \propto \frac{1}{\varepsilon}, \quad \beta \propto \frac{1}{\varepsilon}, \quad \gamma \propto \frac{1}{\varepsilon^2} \tag{3.43}$$

and send ε to zero. This gives the metric of the η -deformed S^3 [9], and to see this, we introduce the standard coordinates (r, ϕ, φ) by

$$\begin{aligned}
 \alpha &= \frac{1}{\varepsilon} \frac{e^{i(\varphi+\phi)}r}{(1 + \lambda^2)\sqrt{2(1 - r^2)}}, \quad \beta = \frac{1}{\varepsilon} \frac{e^{i(\varphi-\phi)}r}{(1 + \lambda^2)\sqrt{2(1 - r^2)}}, \\
 \gamma &= \frac{1}{\varepsilon^2} \frac{e^{2i\varphi}(2(1 + \lambda^2)^2 - (1 - \lambda^2)^2r^2)}{2(1 - \lambda^4)^2(1 - r^2)}, \quad \varepsilon \rightarrow 0.
 \end{aligned}
 \tag{3.44}$$

Performing a similar change of variables on AdS₃ along with an analytic continuation

$$\phi \rightarrow \psi, \quad \varphi \rightarrow t, \quad r \rightarrow i\rho, \quad k \rightarrow -k, \tag{3.45}$$

and sending ε to zero, we arrive at the metric and the dilaton

$$\begin{aligned} ds^2 &= \frac{h}{2} \left(\frac{1}{1-\kappa^2 r^2} \left[(1-r^2)d\varphi^2 + \frac{dr^2}{1-r^2} \right] + r^2 d\phi^2 \right. \\ &\quad \left. + \frac{1}{1+\kappa^2 \rho^2} \left[-(1+\rho^2)dt^2 + \frac{d\rho^2}{1+\rho^2} \right] + \rho^2 d\psi^2 \right), \tag{3.46} \\ e^\Phi &= (1+\tilde{\lambda}^2)^4 \frac{\left[2(1-\tilde{\lambda}^2)S^2 \cos(\varphi-t) - 4\tilde{\lambda}\rho r S \cos(\phi-\psi) \right]^2}{S^2 \sqrt{(1-\tilde{\lambda}^2)^2 + (1+\tilde{\lambda}^2)^2 \rho^2} \sqrt{(1-\tilde{\lambda}^2)^2 - (1+\tilde{\lambda}^2)^2 r^2}}, \\ S &\equiv \sqrt{(1+\rho^2)(1-r^2)}, \quad \kappa = \frac{1+\tilde{\lambda}^2}{1-\tilde{\lambda}^2}, \quad h = \frac{(1-\tilde{\lambda}^2)^2}{k(1+\tilde{\lambda}^2)}, \quad \tilde{\lambda} = i\lambda. \end{aligned}$$

This geometry describes the η -deformed AdS₃ × S³ [9], and similar relations between λ - and η -deformations have been explored in [20].

3.5. Alternative parameterizations

In subsections 3.1–3.3 we derived the full supergravity solutions corresponding to the λ -deformed supercoset, but the metric for this geometry has already appeared in the literature [17,20]. We used the parameterization of [17], and in this subsection we will discuss the relation with the coordinates used in [20] and discuss one more parameterization which becomes useful for comparing AdS₃ × S³ and AdS₅ × S⁵ solutions.

To find the relation between our parameterization and the coordinates used in [20], we observe that the action by $H = SU(2)_{diag}$ changes components of g_l and g_r in (3.3), but three expressions remain invariant:

$$\vec{\alpha}^2 \equiv \sum_{i=1}^3 \alpha_i \alpha_i, \quad \vec{\beta}^2 \equiv \sum_{i=1}^3 \beta_i \beta_i, \quad \vec{\alpha} \cdot \vec{\beta} \equiv \sum_{i=1}^3 \alpha_i \beta_i. \tag{3.47}$$

Although the gauge used in [20] was different from ours, we can find the map between two sets of coordinates by matching the expressions (3.47) in two descriptions. The authors of [20] used parameterization in terms of the Euler’s angles:

$$g^{trig} = \exp[i\varphi\sigma_3 \oplus (-\sigma_3)] \exp[i\zeta\sigma_1 \oplus \sigma_1] \exp[i\phi\sigma_3 \oplus \sigma_3]. \tag{3.48}$$

Evaluating the invariants (3.47) for parameterizations (3.5)–(3.6) and (3.48), and comparing the results, we arrive at the map¹⁰

$$\alpha = \cos(\varphi + \phi) \cos \zeta, \quad \beta = \cos(\varphi - \phi) \cos \zeta, \quad \gamma = \cos 2\varphi - \frac{\cos 2\varphi + \cos 2\phi}{2} \cos^2 \zeta. \tag{3.49}$$

¹⁰ Recall that to simplify notation we introduced $\alpha = \alpha_0$ and $\beta = \beta_0$, and all our results were written in these variables.

Another interesting coordinate system comes from parameterizing the coset $SO(4)/SO(3)$ in terms of a three-dimensional vector X and an anti-symmetric 3×3 matrix A [29,19]. Such parameterization of $SO(n + 1)/SO(n)$ will be used in the next section for studying the deformed $AdS_5 \times S^5$, so it is important to introduce similar coordinates in the present case to make comparisons. The detailed discussion of parameterization and the gauge fixing is presented in section 4.1, here we just write the result¹¹:

$$g = \begin{bmatrix} 1 & 0 \\ 0 & (1 + A)(1 - A)^{-1} \end{bmatrix} \begin{bmatrix} b - 1 & bX_i \\ -bX_i & \delta_i^j - bX_i X^j \end{bmatrix}, \tag{3.50}$$

$$A = \begin{bmatrix} 0 & a & 0 \\ -a & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad b = \frac{2}{1 + (Y_1)^2 + (Y_2)^2}, \quad \vec{X} = \{Y_1, 0, Y_2\}.$$

The parameterizations (3.50) and (3.3), (3.7) correspond to different representations of $SO(4)$, so to relate them we should compare quantities which don't depend on the representation. We have already encountered such an object before:

$$D_{AB} = \text{Tr}(T_{Ag}^{-1}T_Bg). \tag{3.51}$$

To establish the map between generators, we recall that the subgroup $H = SU(2)_{diag}$ corresponds to

$$T_H^{SU(2) \times SU(2)} = \frac{1}{2} \begin{bmatrix} x_a \sigma_a & 0 \\ 0 & x_a \sigma_a \end{bmatrix}, \quad T_H^{SO(4)} = \frac{i}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & x_3 & -x_2 \\ 0 & -x_3 & 0 & x_1 \\ 0 & x_2 & -x_1 & 0 \end{bmatrix} \tag{3.52}$$

and the coset generators correspond to

$$T_{coset}^{SU(2) \times SU(2)} = \frac{1}{2} \begin{bmatrix} y_a \sigma_a & 0 \\ 0 & -y_a \sigma_a \end{bmatrix}, \quad T_{coset}^{SO(4)} = \frac{i}{\sqrt{2}} \begin{bmatrix} 0 & y_1 & y_2 & y_3 \\ -y_1 & 0 & 0 & 0 \\ -y_2 & 0 & 0 & 0 \\ -y_3 & 0 & 0 & 0 \end{bmatrix} \tag{3.53}$$

Evaluating (3.51) for (3.50) and {(3.3), (3.7)}, using appropriate generators, and matching the results, we arrive at the map

$$\alpha = \frac{1 - aY_2}{\sqrt{1 + a^2Y}}, \quad \beta = \frac{1 + aY_2}{\sqrt{1 + a^2Y}}, \quad \gamma = -\frac{Y_1^2 + a^2(Y_1^2 - 1) + Y_2^2}{(1 + a^2)Y}, \tag{3.54}$$

$$Y^2 = 1 + (Y_1)^2 + (Y_2)^2$$

and its inverse

$$a = -\frac{\sqrt{2(1 + \gamma^2) - \alpha^2 - \beta^2}}{\alpha + \beta}, \quad Y_2 = \frac{\alpha - \beta}{\sqrt{2(1 + \gamma) - \alpha^2 - \beta^2}},$$

$$Y_1 = -\sqrt{\frac{2[(1 - \alpha^2)(1 - \beta^2) - \gamma^2]}{[1 + \alpha\beta + \gamma][2(1 + \gamma) - (\alpha^2 + \beta^2)]}}. \tag{3.55}$$

¹¹ We use variables Y_1 and Y_2 in (3.50) to make comparison with $AdS_5 \times S^5$ case easier: the variable Y_1 is a counterpart of X_1 , and Y_2 is a counterpart of X_5 in (4.3).

The AdS coordinates are obtained by the replacement

$$Y_1 \rightarrow i\tilde{Y}_1, \quad Y_2 \rightarrow Y_2, \quad a \rightarrow \tilde{a}. \tag{3.56}$$

In coordinates $(Y_1, Y_2, a, \tilde{Y}_1, \tilde{Y}_2, \tilde{a})$ the dilaton becomes

$$e^\Phi = Q e^{\Phi_B}, \quad e^{\Phi_B} = \frac{\sqrt{1+a^2}Y\sqrt{1+\tilde{a}^2}\tilde{Y}}{16a\tilde{a}Y_1\tilde{Y}_1}, \quad Y^2 = 1 + Y_1^2 + Y_2^2, \quad \tilde{Y}^2 = 1 - \tilde{Y}_1^2 + \tilde{Y}_2^2, \\ Q = (1 - \lambda^2)^4 \left[-\frac{Y_1^2 + a^2(Y_1^2 - 1) + Y_2^2}{(1 + a^2)Y^2} + \frac{\tilde{Y}_1^2 + \tilde{a}^2(\tilde{Y}_1^2 + 1) + \tilde{Y}_2^2}{(1 + \tilde{a}^2)\tilde{Y}^2} \right. \\ \left. - \frac{8\lambda(1 + \lambda^2)}{(1 - \lambda^2)^2} \frac{1 - a\tilde{a}Y_2\tilde{Y}_2}{\sqrt{1+a^2}\sqrt{1+\tilde{a}^2}Y\tilde{Y}} \right. \\ \left. + \frac{\lambda^4 + 6\lambda^2 + 1}{(\lambda^2 - 1)^2} \left(\frac{1 - a^2Y_2^2}{(1 + a^2)Y^2} + \frac{1 - \tilde{a}^2\tilde{Y}_2^2}{(1 + \tilde{a}^2)\tilde{Y}^2} \right) \right]^2. \tag{3.57}$$

In particular, for the gauged WZW model ($\lambda = 0$) we find

$$Q = 4 \left[\frac{X^2 + \tilde{X}^2 - X^2\tilde{X}^2}{X^2\tilde{X}^2} \right]^2. \tag{3.58}$$

Notice that this expression does not depend on coordinates a and \tilde{a} , and the same phenomenon is encountered in the $\text{AdS}_5 \times \text{S}^5$ case, see the last factor in (4.27).

4. Towards the deformation of $\text{AdS}_5 \times \text{S}^5$

In this section we apply the procedure described in section 2 to construct the λ -deformed $\text{AdS}_5 \times \text{S}^5$ supercoset. Our final result includes the metric and the dilaton, but since the latter looks rather complicated, we were not able to solve the equations for the Ramond–Ramond fluxes.

Superstrings on $\text{AdS}_5 \times \text{S}^5$ are described by a sigma model on the supercoset [28]

$$\frac{PSU(2, 2|4)}{SO(4, 1) \times SO(5)}. \tag{4.1}$$

The corresponding superalgebra is represented by 4×4 matrices, and an explicit parameterization is presented in the Appendix B. The bosonic part of the supercoset (4.1) is given by

$$\frac{SU(2, 2)}{SO(4, 1)} \times \frac{SU(4)}{SO(5)} = \frac{SO(4, 2)}{SO(4, 1)} \times \frac{SO(6)}{SO(5)}, \tag{4.2}$$

and, as in the $\text{AdS}_3 \times \text{S}^3$ case, the two subgroups decouple in the metric (2.9) and in the bosonic contribution to the dilaton (2.18). While these objects have been computed in [19], to evaluate the fermionic contribution to the dilaton we will have to use a different parameterization, so we begin with specifying our coordinates, finding the metric and the bosonic dilaton for them, and comparing the results with [19]. The fermionic contribution to the dilaton will be evaluated in section 4.2.

4.1. *Metric and the bosonic dilaton*

To apply the procedure outlined in section 2, we need an explicit form of the coset (4.2). The most natural way to parameterize the sphere $S^5 = SO(6)/SO(5)$ is to use the Euler angles, and such description has been used in [19], but unfortunately these coordinates make the evaluation of the fermionic contribution to the dilaton nearly impossible. Thus we use the alternative coordinates introduced in [29,19], in which all expressions remain algebraic.¹²

Specifically, we write the element of $SO(6)$ as

$$g = \begin{bmatrix} 1 & 0 \\ 0 & h_m^n \end{bmatrix} \begin{bmatrix} b-1 & bX^j \\ -bX_i & \delta_i^j - bX_i X^j \end{bmatrix}, \quad b = \frac{2}{1 + X^m X_m}, \tag{4.3}$$

where X^i is a five-dimensional vector and h_m^n is an element of $SO(5)$. The defining condition for $SO(5)$, $h^T h = I$, can be solved by writing h in terms of an anti-symmetric matrix A as

$$h_m^n = [(1 + A)(1 - A)^{-1}]_m^n. \tag{4.4}$$

The $SO(5)$ rotations act on A and X as

$$A \rightarrow \Lambda A \Lambda^{-1}, \quad X \rightarrow \Lambda X. \tag{4.5}$$

To fix this gauge freedom, we follow the procedure discussed in [29]: first we rotate A to a block form¹³:

$$A = \begin{pmatrix} 0 & a & 0 & 0 & 0 \\ -a & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & b & 0 \\ 0 & 0 & -b & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{4.6}$$

and then we use the remaining $[SO(2)]^2$ rotations to set $X_2 = X_4 = 0$.

The $\mathfrak{so}(6)$ algebra has 15 generators, first ten of them form $\mathfrak{so}(5)$, while the last five correspond to the coset. Specifically, in our parameterization, the coset generators are¹⁴

$$(T_\alpha)_{mn} = -\frac{i}{\sqrt{2}} [\delta_{m1} \delta_{n(\alpha-9)} - \delta_{n1} \delta_{m(\alpha-9)}] \quad \alpha = 11, \dots 15. \tag{4.7}$$

Application of the procedure (2.17) leads to the bosonic contribution to the dilaton (2.18)

$$e^{-2\Phi_B} = \frac{1024a^2b^2(a^2 - b^2)^2 X_1^2 X_3^2}{(1 + a^2)^3 (1 + b^2)^3 X^2} \frac{(1 - \lambda^2)^3 (1 + \lambda^2)^2}{\lambda^{10}}, \tag{4.8}$$

where we defined

$$X^2 \equiv 1 + X_1^2 + X_3^2 + X_5^2.$$

¹² It appears that the authors of [19] used the same coordinates while computing the metric and rewrote the final answers in terms of the Euler's angles. We find the algebraic coordinates more convenient.

¹³ Notice that there is a slight difference in gauge fixing between $SO(n)/SO(n - 1)$ for odd and even n : matrix A has $[(n - 1)/2]$ independent components, and there are $[n/2]$ independent X .

¹⁴ Recall that throughout this article we use hermitian generator, so the element of a group is constructed as $g = \exp[iTx]$.

Note that the lambda dependence factorizes in (4.8), and this is a general feature of the bosonic dilaton, as discussed in the end of section 2. Specifically, in the present case, matrix \mathbf{P} defined in (2.23) has the form

$$\mathbf{P} = \begin{bmatrix} W(a) & 0 & 0 \\ 0 & W(b) & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \text{where} \quad W(x) \equiv (1+x^2)^{-1} \begin{bmatrix} 1 & -x \\ -x & -1 \end{bmatrix}. \tag{4.9}$$

This matrix has eigenvalues $\lambda^{-2} \pm 1$ and

$$\det \mathbf{P} = \frac{(1-\lambda^2)^3(1+\lambda^2)^2}{\lambda^{10}}. \tag{4.10}$$

The metric for λ -deformation is constructed using (2.17), and the result reads

$$ds_{(\lambda)}^2 = \sum_{\alpha} (e_{(\lambda)}^{\alpha})^2, \quad e_{(\lambda)}^{\alpha} = \frac{\sqrt{k(1-\lambda^4)}}{2\lambda^2} [\mathbf{P}^{-1}]^{\alpha}_{\beta} e_{(0)}^{\beta}, \tag{4.11}$$

where $e_{(0)}^{\beta}$ refer to the frames describing the gauged WZW model ($\lambda = 0$):

$$\begin{aligned} e_{(0)}^6 &= \frac{a^2(1+b^2)X_3^2 + (a^2-b^2)X_5^2}{a(1+a^2)(a^2-b^2)X_1} da + \frac{(1+a^2)bX_1}{(a^2-b^2)(1+b^2)} db \\ &\quad + \frac{1}{X^2} \left[-(X^2 - X_1^2)dX_1 + X_1X_3dX_3 + X_1X_5dX_5 \right], \\ e_{(0)}^7 &= \frac{da}{X_1(1+a^2)}, \quad e_{(0)}^9 = \frac{db}{X_3(1+b^2)}, \\ e_{(0)}^{10} &= -\frac{X_5da}{a(1+a^2)} - \frac{X_5db}{b(1+b^2)} + \frac{1}{X^2} \left[X_1X_5dX_1 + X_3X_5dX_3 - (X^2 - X_5^2)dX_5 \right], \\ e_{(0)}^8 &= -\frac{a(1+b^2)X_3da}{(1+a^2)(a^2-b^2)} - \frac{(1+a^2)b^2X_1^2 - (a^2-b^2)X_5^2}{b(1+b^2)(a^2-b^2)X_3} db \\ &\quad + \frac{1}{X^2} \left[X_1X_3dX_1 - (X^2 - X_3^2)dX_3 + X_3X_5dX_5 \right]. \end{aligned} \tag{4.12}$$

The AdS_5 counterparts of the metric and the dilaton are obtained by an analytic continuation

$$X_1 \rightarrow iX_1, \quad X_3 \rightarrow iX_3, \quad k \rightarrow -k, \tag{4.13}$$

and the corresponding frames are denoted by $e_{(0)}^1, \dots, e_{(0)}^5$.

4.2. Fermionic dilaton: general discussion

Although the $SO(6)/SO(5)$ representation (4.3) of the five-dimensional sphere is very intuitive, the construction of the supercoset (4.1) requires embedding of $SO(6)$ into $SU(4)$ and identifying the fermionic degrees of freedom corresponding to the supercoset. We begin with finding the $SU(4)$ matrices in parameterization (4.3).

The $SU(4)$ matrices g describe a representation of $SO(6)$, which acts on anti-symmetric 4×4 matrices A as

$$A \rightarrow gAg^T. \tag{4.14}$$

Specifically, starting with the fundamental representation of $SO(6)$ acting on six-dimensional vectors $(x_1, x_2, x_3, y_1, y_2, y_3)$, one can construct matrix A as

$$A = \begin{bmatrix} 0 & x_3 - iy_3 & -x_2 + iy_2 & x_1 + iy_1 \\ -x_3 + iy_3 & 0 & x_1 - iy_1 & x_2 + iy_2 \\ x_2 - iy_2 & -x_1 + iy_1 & 0 & x_3 + iy_3 \\ -x_1 - iy_1 & -x_2 - iy_2 & -x_3 - iy_3 & 0 \end{bmatrix}. \tag{4.15}$$

The generators of $SU(4)$ are hermitian 4×4 matrices, and to proceed with the coset construction, we need to identify the elements t_α corresponding to the generators (4.7). Comparing the action T_α on $(x_1, x_2, x_3, y_1, y_2, y_3)$ and the action of $g \in \mathfrak{su}(4)$ on (4.15), we find

$$T^\alpha c_\alpha = \frac{1}{2} \begin{bmatrix} c_{13} & c_{14} + ic_{11} & c_{15} + ic_{12} & 0 \\ c_{14} - ic_{11} & -c_{13} & 0 & -c_{15} - ic_{12} \\ c_{15} - ic_{12} & 0 & -c_{13} & c_{14} + ic_{11} \\ 0 & ic_{12} - c_{15} & c_{14} - ic_{11} & c_{13} \end{bmatrix}. \tag{4.16}$$

All generators of $SU(4)$, including (4.16), are hermitian, while generators of $SU(2, 2)$ satisfy the modified hermiticity relation

$$(T_A)^\dagger = \Sigma T_A \Sigma, \quad \Sigma = \begin{bmatrix} 0 & \sigma_3 \\ \sigma_3 & 0 \end{bmatrix}. \tag{4.17}$$

For example, the counterparts of the coset generators (4.16) are obtained by an analytic continuation

$$c_{11} \rightarrow i\tilde{c}_{11}, \quad c_{12} \rightarrow i\tilde{c}_{12}, \quad c_{13} \rightarrow i\tilde{c}_{13}, \quad c_{14} \rightarrow i\tilde{c}_{14}, \quad c_{15} \rightarrow \tilde{c}_{15}. \tag{4.18}$$

To proceed we need to construct an automorphism J_1 which satisfies (3.19) for all generators $g \in \mathfrak{su}(4)$ with the exception of (4.16). While it is easy to find this J_1 for 6×6 matrices and coset generators (4.7) (specifically, $J_1 = \pm \text{diag}(-1, 1, 1, 1, 1)$), such matrix does not exist in the four-dimensional representation of $\mathfrak{so}(6)$, and the closest analog of (3.19) is

$$J_1^{-1} g J_1 = g^T, \quad J_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}. \tag{4.19}$$

This means that condition (3.21) will be modified as

$$\mathcal{P}^{-1} \mathcal{M} \mathcal{P} = \mathcal{M}^T, \tag{4.20}$$

and such grading is a familiar feature of $PSU(2, 2|4)$ (see, for example, [23] for a detailed discussion). In our parameterization,

$$\mathcal{P} = \begin{bmatrix} J_1 & 0 \\ 0 & J_1 \end{bmatrix}, \tag{4.21}$$

and the detailed discussion of fermions projected out by (4.21) and relation to other conventions used in the literature is presented in the Appendix B. Here we only mention that if 8×8 supercoset matrix is written as

$$\mathcal{M} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}, \quad B \equiv \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix},$$

$$C \equiv \begin{bmatrix} c_1 & c_2 \\ c_3 & c_4 \end{bmatrix} = -i \begin{bmatrix} b_3^\dagger \sigma_3 & b_1^\dagger \sigma_3 \\ b_4^\dagger \sigma_3 & b_2^\dagger \sigma_3 \end{bmatrix}, \tag{4.22}$$

then projector P_3 entering P_λ (2.20) selects the components satisfying an additional relation (B.14):

$$C = \begin{bmatrix} -[\sigma_1 b_4 \sigma_1]^T & [\sigma_1 b_2 \sigma_1]^T \\ [\sigma_1 b_3 \sigma_1]^T & [\sigma_1 b_1 \sigma_1]^T \end{bmatrix}. \tag{4.23}$$

The last ingredient for constructing the fermionic contribution to the dilaton is the explicit expression for the element of $SU(4)/SO(5)$ in the gauge (4.3), (4.6):

$$g_S = \frac{1}{\Delta_S} \begin{bmatrix} 1 & b & ab & a \\ -b & 1 & a & -ab \\ ab & -a & 1 & -b \\ -a & -ab & b & 1 \end{bmatrix} \begin{bmatrix} 1 - iX_3 & X_1 & -iX_5 & 0 \\ -X_1 & 1 + iX_3 & 0 & iX_5 \\ -iX_5 & 0 & 1 + iX_3 & X_1 \\ 0 & iX_5 & -X_1 & 1 - iX_3 \end{bmatrix} \tag{4.24}$$

$$\Delta_S = \sqrt{1 + a^2} \sqrt{1 + b^2} \sqrt{1 + (X_1)^2 + (X_3)^2 + (X_5)^2}.$$

The element of $SU(2, 2)/SO(4, 1)$ is obtained by making the analytic continuation (4.13) in the last expression. Notice that the symmetry

$$X_1 \leftrightarrow X_3, \quad a \leftrightarrow b, \tag{4.25}$$

which was obvious in the $SO(6)$ parameterization (4.3), (4.6), is less explicit in (4.24).

Evaluation of the fermionic contribution to the dilaton involves a straightforward but tedious calculation of the determinant

$$\det[(Ad_f - 1 - (\lambda^{-2} - 1)P_\lambda)|_{\hat{f}_1 \oplus \hat{f}_3}], \tag{4.26}$$

and the results are rather complicated. We collect them and discuss some of their features in the next two subsections.

4.3. Dilaton for the gauged WZW model

Geometry with $\lambda = 0$ describes the gauged WZW model, and the solution in this case is given by the frames (4.12), along with their AdS₅ counterpart and the dilaton

$$e^{-2\Phi} = 2^{20} \frac{a^2 b^2 (a^2 - b^2)^2 X_1^2 X_3^2}{(1 + a^2)^3 (1 + b^2)^3 X^2} \frac{\tilde{a}^2 \tilde{b}^2 (\tilde{a}^2 - \tilde{b}^2)^2 \tilde{X}_1^2 \tilde{X}_3^2}{(1 + \tilde{a}^2)^3 (1 + \tilde{b}^2)^3 \tilde{X}^2} \left[\frac{X^2 \tilde{X}^2}{X^2 + \tilde{X}^2 - X^2 \tilde{X}^2} \right]^8 \tag{4.27}$$

$$X^2 = 1 + X_1^2 + X_3^2 + X_5^2, \quad \tilde{X}^2 = 1 - \tilde{X}_1^2 - \tilde{X}_3^2 + \tilde{X}_5^2.$$

The bosonic contribution to the dilaton is obtained by dropping the expression in the brackets, and the bosonic coset does not require any Ramond–Ramond fluxes. The situation for the supercoset is different, as we have already seen in the AdS₃ × S³ case: the Ramond–Ramond fluxes are turned on even at $\lambda = 0$. In the present case we were not able to construct the fluxes explicitly, but we verified that the solution (4.12)–(4.27) can be supported by F_5 .

Recall that the stress–energy tensor for the self-dual five-form,

$$T_{mn} = \frac{1}{96} F_{mabcd} F_n{}^{abcd} \tag{4.28}$$

satisfies the Rainich conditions [30]¹⁵:

$$T_m^m \equiv \text{Tr } T = 0, \quad \text{Tr } T^3 = 0, \quad \text{Tr } T^5 = 0, \quad \text{Tr } T^7 = 0, \quad \text{Tr } T^9 = 0, \quad (4.29)$$

and for geometry supported only by the dilaton and the metric the T_{mn} can be expressed as¹⁶

$$T_{mn} = R_{mn} + 2\nabla_m \nabla_n \Phi. \quad (4.30)$$

The right-hand side vanishes for the “bosonic” dilaton, while for the full solution (4.27) it gives a nontrivial result which satisfies the constraints (4.29). It would be very interesting to find the corresponding flux F_5 .

4.4. Special cases for $\lambda \neq 0$

Although the dilaton for arbitrary values of λ can be computed by evaluating the appropriate determinants, unfortunately the results are not very illuminating. In this subsection we will collect the answers for some special cases which give manageable expressions. Since the general expression for the bosonic dilaton is already given by (4.8), we will focus only on the fermionic contribution to (2.16):

$$e^{2\Phi_F} = \det[(Ad_f - 1 - (\lambda^{-2} - 1)P_\lambda)|_{\hat{f}_1 \oplus \hat{f}_3}]. \quad (4.31)$$

First we observe that at $\lambda = 0$ the expression for $e^{2\Phi_F}$ depends only on X_k and \tilde{X}_k . While this property does not hold for general values of λ , setting $a = \tilde{a} = b = \tilde{b}$ we still find an interesting result:

$$e^{2\Phi_F} \Big|_{a=\tilde{a}=b=\tilde{b}=0} = \left[\frac{(1 - \mu X \tilde{X})^2 - (1 - X^2)(1 - \tilde{X}^2)}{X^2 \tilde{X}^2} \right]^8, \quad \mu \equiv \frac{2\lambda}{\lambda^2 + 1}. \quad (4.32)$$

In the opposite case, where all X are switched off, the expression is much more complicated, for example at $\lambda = 1$ it has the form

$$e^{2\Phi_F} \Big|_{X_m=\tilde{X}_m=0, \lambda=1} = F^{12} \left[8F - 2 + (FP_{-1} - 2)^2 - 2(P_1 - 1)^2 + 2P_2 \right]^2, \quad (4.33)$$

$$F \equiv (a^2 + 1)(b^2 + 1)(\tilde{a}^2 + 1)(\tilde{b}^2 + 1),$$

$$P_k \equiv (a^2 + 1)^k + (b^2 + 1)^k + (\tilde{a}^2 + 1)^k + (\tilde{b}^2 + 1)^k.$$

In particular, we observe that the last expression is fully symmetric under interchanging the elements of the list $(a, b, \tilde{a}, \tilde{b})$. This property persists for all values of λ , as long as $X_m = \tilde{X}_m = 0$, but the general expression is not very illuminating, so we will not write it here.

The last two interesting cases correspond to looking only at the sphere or only at the AdS space:

¹⁵ For a recent discussion of the original Rainich conditions for electromagnetism and their generalizations to higher dimensions see, for example, [31,10].

¹⁶ In this paper we are working in the string frame.

$$\begin{aligned}
 e^{2\Phi_F}\Big|_S &= \left[\frac{(AB - \mu X)^2 + \mu^2[(AbX_3)^2 + (aBX_1)^2 - a^2b^2X^2]}{A^2B^2X^2} \right]^8, \\
 e^{2\Phi_F}\Big|_{AdS} &= \left[\frac{(\tilde{A}\tilde{B} - \mu\tilde{X})^2 - \mu^2[(\tilde{A}\tilde{b}\tilde{X}_3)^2 + (\tilde{a}\tilde{B}\tilde{X}_1)^2 + \tilde{a}^2\tilde{b}^2\tilde{X}^2]}{\tilde{A}^2\tilde{B}^2\tilde{X}^2} \right]^8, \\
 A &= \sqrt{1+a^2}, \quad B = \sqrt{1+b^2}, \quad \tilde{A} = \sqrt{1+\tilde{a}^2}, \quad \tilde{B} = \sqrt{1+\tilde{b}^2}.
 \end{aligned}
 \tag{4.34}$$

The complexity of our results beyond $\lambda = 0$ suggests that the full solution for the λ -deformation of $AdS_5 \times S^5$ cannot be constructed unless one finds better coordinates, and we leave this problem for future investigation.

5. Discussion

In this article we have constructed the supergravity background describing the λ -deformation of $AdS_3 \times S^3$ supercoset and reported some progress towards the analogous result for $AdS_5 \times S^5$. Our main result is summarized by equations (3.15), (3.9), (3.28), (3.38). In the $AdS_5 \times S^5$ case we have constructed the metric and the dilaton describing the supercoset, and while the results presented in section 4 are rather complicated, there are striking similarities with lower-dimensional cases. For example, at the WZW point, where the expression (4.27) for the ten-dimensional dilaton is rather simple, one finds a very close analogy with the six-dimensional case (3.58), and we hope that a further exploration of such analogies will lead to construction of full gravity solution for the deformed $AdS_5 \times S^5$.

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Appendix A. λ -deformation for $AdS_2 \times S^2$

For comparison with the results obtained in this article, we review the geometry of λ -deformed $AdS_2 \times S^2$ constructed in [22]. We also extend the solution of [22] by one free parameter which makes the fluxes symmetric between the sphere and AdS space. Applying the procedure reviewed in section 2 to a coset $SU(2)/U(1)$, the authors of [22] constructed the metric and the supercoset version of the dilaton (2.19)¹⁷:

$$\begin{aligned}
 ds^2 &= \frac{-dx^2 + dy^2}{1 - \kappa x^2 + \kappa^{-1}y^2} + \frac{dp^2 + dq^2}{1 - \kappa p^2 - \kappa^{-1}q^2}, \\
 e^\Phi &= \frac{\kappa - x^2 + y^2 - p^2 - q^2 + 2\sqrt{1 - \kappa^2}xp}{\sqrt{-(1 - \kappa x^2 + \kappa^{-1}y^2)}\sqrt{1 - \kappa p^2 - \kappa^{-1}q^2}},
 \end{aligned}
 \tag{A.1}$$

where

$$\kappa = \frac{1 - \lambda^2}{1 + \lambda^2}.
 \tag{A.2}$$

¹⁷ The solution corresponding to the bosonic dilaton (2.18) had been constructed earlier in [17].

This background is supported by the Ramond–Ramond flux

$$\begin{aligned}
 A &= \frac{c_1}{M} [ydx - (x - \sqrt{1 - \kappa^2}p)dy] + \frac{c_2}{M} [qdp - (p - \sqrt{1 - \kappa^2}x)dq], \\
 M &= \frac{\kappa - x^2 + y^2 - p^2 - q^2 + 2\sqrt{1 - \kappa^2}xp}{\sqrt{-(1 - \kappa x^2 + \kappa^{-1}y^2)(1 - \kappa p^2 - \kappa^{-1}q^2)}}, \quad c_1^2 + c_2^2 = 4\kappa^{-1},
 \end{aligned}
 \tag{A.3}$$

which solves the supergravity equations

$$\begin{aligned}
 R_{mn} + 2\nabla_m \nabla_n \Phi &= \frac{e^{2\Phi}}{2} (F_{mp} F_n{}^p - \frac{1}{4} g_{mn} F_{kl} F^{kl}), \\
 \partial_n (\sqrt{-g} F^{mn}) &= 0, \quad \nabla^2 e^{-2\Phi} = 0,
 \end{aligned}
 \tag{A.4}$$

and article [22] presented the answer (A.3) for $c_2 = 0$.

It is interesting that the flux (A.3) has a free parameter which interpolates between the components on the sphere and on AdS, while the $\text{AdS}_3 \times S^3$ solution (3.38) has no freedom. This difference can already be seen for the undeformed $\text{AdS}_p \times S^p$, and it can be traced to the different structure of “electric–magnetic” duality groups in four and six dimensions ($U(1)$ in 4d vs Z_2 in 6d).

Since in this article we use parameterization of cosets in terms of X, A coordinates introduced in (4.3), we will conclude this appendix by writing the relations between coordinate systems used in [17,22] and a three-dimensional version of (4.3)–(4.6) describing $SO(3)/SO(2)$:

$$\begin{aligned}
 g_{so} &= \begin{bmatrix} 1 & & 0 \\ 0 & (1+A)(1-A)^{-1} & \end{bmatrix} \begin{bmatrix} b-1 & bX & 0 \\ -bX & 1-bX^2 & -bX \\ 0 & -bX & 1 \end{bmatrix}, \\
 A &= \begin{bmatrix} 0 & a \\ -a & 0 \end{bmatrix}, \quad b = \frac{2}{1+X^2}.
 \end{aligned}
 \tag{A.5}$$

To compare this with the parameterization in terms of the Euler’s angles used in [17,20],

$$g^{trig} = \exp[i(\phi_1 - \phi_2)\sigma_3/2] \exp(i\omega\sigma_2) \exp[i(\phi_1 + \phi_2)\sigma_3/2],
 \tag{A.6}$$

we follow the procedure outlined in section 3.5. Specifically, computing the matrix D (3.51) and comparing the result with a general parameterization (4.3) applied to $SO(3)$, we find

$$\begin{aligned}
 X_1 &= -\frac{4(\cos^2 \omega \sin 2\phi_1 + \sin^2 \omega \sin 2\phi_2)}{4 + \cos[2(\omega - \phi_1)] + \cos[2(\omega + \phi_1)] + 2 \cos 2\phi_1 + 4 \cos 2\phi_2 \sin^2 \omega}, \\
 X_2 &= -\frac{4 \sin 2\omega \sin(\phi_1 - \phi_2)}{4 + \cos[2(\omega - \phi_1)] + \cos[2(\omega + \phi_1)] + 2 \cos 2\phi_1 + 4 \cos 2\phi_2 \sin^2 \omega}, \\
 a &= \frac{\cos \phi_2 \tan \omega}{\cos \phi_1}.
 \end{aligned}
 \tag{A.7}$$

A $U(1)$ gauge transformation relates this to (A.5) with

$$\begin{aligned}
 X &= -\frac{4\sqrt{\sin^2(2\phi_1) + \sin^2(\phi_1 - \phi_2) \sin^2(2\omega)}}{4 + \cos[2(\omega - \phi_1)] + \cos[2(\omega + \phi_1)] + 2 \cos 2\phi_1 + 4 \cos 2\phi_2 \sin^2 \omega}, \\
 a &= \frac{\cos \phi_2 \tan \omega}{\cos \phi_1}.
 \end{aligned}
 \tag{A.8}$$

The authors of [17] fixed the gauge by setting $\phi_2 = 0$, while the authors of [22] chose $\phi_2 = \phi_1$ and changed coordinates as

$$\omega = \arccos \sqrt{\kappa p^2 + \kappa^{-1} q^2}, \quad \phi_1 = \arccos \frac{\sqrt{\kappa} p}{\sqrt{\kappa p^2 + \kappa^{-1} q^2}} \tag{A.9}$$

to arrive at (A.1).

Appendix B. Parametrization of $\mathfrak{psu}(1, 1|2)$ and $\mathfrak{psu}(2, 2|4)$

In this appendix we briefly summarize the parameterization of $\mathfrak{psu}(1, 1|2)$, $\mathfrak{psu}(2, 2|4)$, and their cosets used in sections 3 and 4. We will mostly follow the notation of [23,24], although our parameterization of fermions differs from the one in [23], and we will comment on the difference.

The Lie superalgebras $\mathfrak{psu}(n, n|2n)$ can be defined in terms of $(4n) \times (4n)$ supermatrices

$$\mathcal{M} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}, \tag{B.1}$$

with even $(2n) \times (2n)$ blocks A, D and odd $(2n) \times (2n)$ blocks B, C . The graded Lie bracket is defined as

$$[\mathcal{M}, \mathcal{M}'] = \begin{bmatrix} AA' + BC' - A'A + B'C & AB' + BD' - A'B - B'D \\ CA' + DC' - C'A - D'C & CB' + DD' + C'B - D'D \end{bmatrix}. \tag{B.2}$$

Matrix \mathcal{M} is subject to the hermiticity condition

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \Sigma A^\dagger \Sigma^{-1} & -i \Sigma C^\dagger \\ -i B^\dagger \Sigma^{-1} & D^\dagger \end{bmatrix}, \tag{B.3}$$

where Σ is a hermitian matrix of signature (n, n) . Convention for $\mathfrak{su}(n, n)$ represented by A fixes the matrix Σ and the parameterization of fermions B, C .

For $\mathfrak{psu}(1, 1|2)$ we choose $\Sigma = \text{diag}(1, -1)$. This leads to the relation

$$C = -i B^\dagger \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \tag{B.4}$$

or more explicitly

$$B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}, \quad C = \begin{bmatrix} -i b_{11}^\dagger & i b_{21}^\dagger \\ -i b_{12}^\dagger & i b_{22}^\dagger \end{bmatrix}. \tag{B.5}$$

To construct the algebra for the coset

$$\frac{PSU(1, 1|2)_l \times PSU(1, 1|2)_r}{SU(1, 1)_{diag} \times SU(2)_{diag}}, \tag{B.6}$$

we take two copies of $\mathfrak{psu}(1, 1|2)$,

$$\mathcal{M}' = \begin{bmatrix} \mathcal{M}_1 & 0 \\ 0 & \mathcal{M}_2 \end{bmatrix}, \tag{B.7}$$

and project to the subgroup H by imposing the relation (3.21)

$$\mathcal{P}^{-1} \mathcal{M}' \mathcal{P} = \mathcal{M}', \tag{B.8}$$

as discussed in section 3.2. Notice that AdS_3 and S_3 blocks are mixed in the matrix (B.7), and to make the separation more explicit we rearrange the components of the matrix \mathcal{M}' using the parameterization (B.1) for \mathcal{M}_1 and \mathcal{M}_2 . Specifically we define

$$\mathcal{M} = \left[\begin{array}{cc|cc} A_1 & 0 & B_1 & 0 \\ 0 & A_2 & 0 & B_2 \\ \hline C_1 & 0 & D_1 & 0 \\ 0 & C_2 & 0 & D_2 \end{array} \right]. \tag{B.9}$$

The top left block of this matrix describes AdS space, the bottom right block describes the sphere, and the matrix \mathcal{P} corresponding to this supercoset is given by (3.26):

$$\mathcal{P} = \left[\begin{array}{cccc} 0 & 1_{2 \times 2} & 0 & 0 \\ 1_{2 \times 2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1_{2 \times 2} \\ 0 & 0 & 1_{2 \times 2} & 0 \end{array} \right]. \tag{B.10}$$

In particular, this matrix does not mix the B_i and C_i components, so in section 3.2 we computed the fermionic contribution to the dilaton by treating the holomorphic and anti-holomorphic components (b_{ij} and b_{ij}^\dagger) as independent variables.

Let us now discuss the $\mathfrak{psu}(2, 2|4)$ superalgebra, which emerges in the description of strings on $AdS_5 \times S^5$ [28]. In this case equation (B.3) involves 4×4 blocks, and we choose the matrix Σ involved in the hermiticity condition (B.3) to be

$$\Sigma = \begin{bmatrix} 0 & \sigma_3 \\ \sigma_3 & 0 \end{bmatrix} \tag{B.11}$$

This choice leads to a relation between 2×2 blocks of B and C in (B.1):

$$B \equiv \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix}, \quad C \equiv \begin{bmatrix} c_1 & c_2 \\ c_3 & c_4 \end{bmatrix} = -i \begin{bmatrix} b_3^\dagger \sigma_3 & b_1^\dagger \sigma_3 \\ b_4^\dagger \sigma_3 & b_2^\dagger \sigma_3 \end{bmatrix}. \tag{B.12}$$

A choice of holomorphic and anti-holomorphic fermions is no longer convenient since the coset projection mixes them. As discussed in section 4.2, for the $\mathfrak{psu}(2, 2|4)$ supercoset, the condition (3.21) is replaced by (4.20)

$$\mathcal{P}^{-1} \mathcal{M} \mathcal{P} = \mathcal{M}^T, \tag{B.13}$$

with \mathcal{P} given by (4.21). An explicit calculation shows that projection (B.13) chooses the elements which satisfy

$$B = \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix}, \quad C = \begin{bmatrix} -[\sigma_1 b_4 \sigma_1]^T & [\sigma_1 b_2 \sigma_1]^T \\ [\sigma_1 b_3 \sigma_1]^T & [\sigma_1 b_1 \sigma_1]^T \end{bmatrix} \tag{B.14}$$

in addition to (B.12). The coset corresponds to the generators included in (B.12), but not in (B.14). In other words, generators satisfying both (B.14) and (B.12) survive under projection P_3 , and P_1 is defined as $P_1 = 1 - P_3$.

We conclude this appendix by relating our conventions with notation used in [23]. We chose a different embedding of the coset into $SU(4) \times SU(2, 2)$, and this led to a following relation between our generators and the ones used by Arutyunov and Frolov (AF) [23]:

$$T_{su(4)} = RT_{su(4)}^{AF} R^{-1}, \quad R = \begin{bmatrix} i & 0 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \tag{B.15}$$

$$T_{su(2,2)} = \tilde{R}T_{su(2,2)}^{AF} \tilde{R}^{-1}, \quad \tilde{R} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}. \tag{B.16}$$

While our generators are convenient for evaluating the λ -deformation, the generators of Arutyunov and Frolov are better suited for imposing kappa symmetry. Specifically, elimination of this freedom in the notation of [23] gives

$$B^{AF} = \begin{bmatrix} 0 & b_2 \\ b_3 & 0 \end{bmatrix}, \quad C^{AF} = \begin{bmatrix} 0 & c_2 \\ c_3 & 0 \end{bmatrix} \tag{B.17}$$

while in our notation

$$B = \begin{bmatrix} b_1 & b_2 \\ \sigma_3 b_1 \sigma_3 & -\sigma_3 b_2 \sigma_3 \end{bmatrix}, \quad C = \begin{bmatrix} i\sigma_3 b_1^\dagger & i b_1^\dagger \sigma_3 \\ -i\sigma_3 b_2^\dagger & i b_2^\dagger \sigma_3 \end{bmatrix}. \tag{B.18}$$

The expressions for kappa symmetry are not used in this paper.

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