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Convective Heat Transfer Flow along a Sinusoidal Wavy Surface in a Porous Medium with Variable Permeability

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Abstract

A numerical study on convective heat transfer flow along a sinusoidal wavy surface in a fluid saturated sparsely packed porous medium is undertaken by considering variable porosity, variable permeability and variable thermal conductivity. The Darcy model is used to describe the flow in porous medium. A coordinate transformation is used to incorporate the waviness in governing equations, and the resultant boundary layer equations are solved by employing Local Non-similarity method. The obtained numerical results for fluid flow characteristics and Nusselt number are presented graphically for different values of ratio of solid thermal conductivity and fluid conductivity, porosity at the edge of the boundary layer and amplitude of the wavy surface for two cases uniform permeability (UP) and variable permeability (VP).

Keywords: Variable Permeability; Wavy Surface; Darcy Porous Medium;

1. Introduction

The study of the transport process of convective heat transfer from irregular (sinusoidal wavy) surface in a porous medium is of great importance in engineering and industrial applications such as grain storage containers, heat transfer devices, condensers in refrigerators, solar collectors, electrical and nuclear cooling components, design of building energy systems, chemical catalytic reactors, compact heat exchangers etc. The concept of convective boundary layer flow from vertical wavy surface in a porous medium was first investigated by Rees and Pop [1]. Kumari et.al [2] investigated heat transfer flow of a non-Newtonian convective fluid over a vertical wavy surface. Many researchers have been investigated heat and mass transfer flow of convective micropolar fluid (see Wang and Chen [3]), power law fluid (see Cheng [4]) and Nanofluid (see Mahdy and Ahmed [5]) past a semi-infinite vertical wavy surface in a porous medium. Recently, Srinivasacharya et al. [6] studied cross diffusion effects on mixed convective flow along a wavy surface in a porous medium with variable properties.

In all the above studies uniform porosity has been considered and investigated. The uniform porosity is always not applicable in industrial applications, such as packed bed exchangers, drying and fixed bed catalytic reactors where its...
value is not unique at the wall and away from the wall. In this problem, therefore we considered non-uniform porosity to investigate convective heat transfer along a vertical wavy surface. Chandrasekhara et.al [7] and Chandrasekhara [7] investigated on natural and mixed convective heat transfer flow in a porous medium adjacent to a horizontal surface. Recently, Mohammadein and El-Shaer [9] investigated influence of variable permeability on mixed convection flow over a vertical surface in a porous medium. Pal and Shivakumara [10] investigated free and forced convection from a heated vertical plate in a sparsely packed porous medium.

In view of the above applications, the present paper is presented to investigate convective boundary layer flow past a sinusoidal wavy surface embedded in a porous medium with variable permeability. This type of study finds applications in the fields of manufacturing, ceramics, petroleum geology, aerodynamics, pharmaceutics etc.

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2. Mathematical Formulation

Consider the two dimensional, incompressible, steady, laminar Newtonian fluids adjacent to a vertical wavy surface embedded in a sparsely packed fluid saturated porous medium with variable properties. We define the wavy surface with

$$\bar{y} = \hat{\sigma}(\bar{x}) = \bar{a} \sin \left( \frac{\pi \bar{x}}{l} \right)$$  \hspace{1cm} (1)

where $\bar{a}$ is the amplitude of the wavy plate surface and $l$ is the characteristics of wavy length. The plate is maintained with uniform temperature $T_w$, which is higher than the ambient fluid (free stream) temperature $T_\infty$. The Darcy law is used to describe the porous medium. All the properties of the fluid and the porous region are assumed to be
constant except thermal diffusivity, permeability and porosity. In view of the above assumptions and Boussinesq approximation, the governing equations for the conservation of mass, momentum and energy are (see Vafai [11], Chandrashekara [7]):

\[
\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0
\]  
(2)

\[
\frac{\partial}{\partial \bar{y}} \left( \frac{\mu_{ef} \varepsilon(\bar{y}) K(\bar{y})}{1} \bar{u} \right) = \frac{\partial}{\partial \bar{x}} \left( \frac{\mu_{ef} \varepsilon(\bar{y})}{K(\bar{y})} \bar{v} \right) + \rho g \left( \beta_1 \frac{\partial T}{\partial \bar{y}} + \beta_e \frac{\partial C}{\partial \bar{y}} \right)
\]  
(3)

\[
\bar{u} \frac{\partial T}{\partial \bar{x}} + \bar{v} \frac{\partial T}{\partial \bar{y}} = \frac{\partial}{\partial \bar{x}} \left( \alpha(\bar{y}) \frac{\partial T}{\partial \bar{x}} \right) + \frac{\partial}{\partial \bar{y}} \left( \alpha(\bar{y}) \frac{\partial T}{\partial \bar{y}} \right)
\]  
(4)

The associated boundary conditions are

\[
\bar{u} = 0, \quad \bar{v} = 0, \quad T = T_w, \quad \text{at} \quad \bar{y} = \bar{\sigma}(\bar{x}) = \bar{a} \sin \left( \frac{\bar{x}}{l_1} \right)
\]

\[
\bar{u} \to 0, \quad T \to T_\infty, \quad \text{as} \quad \bar{y} \to \infty
\]  
(5)

where \(\bar{u}\) and \(\bar{v}\) are velocity components in \(\bar{x}\) and \(\bar{y}\) directions respectively. \(\mu_{ef}\) is the effective viscosity, \(K(\bar{y})\) is the variable permeability of the porous medium, \(\varepsilon(\bar{y})\) is the variable porosity of the porous medium, \(\rho\) is the density of the fluid, \(\beta_1\) is the thermal expansion coefficient, \(g\) is the acceleration due to gravity, \(\alpha(\bar{y})\) is the variable effective thermal diffusivity respectively. The porous region and fluid properties, namely permeability \(K(\bar{y})\), porosity \(\varepsilon(\bar{y})\) and thermal conductivity \(\alpha(\bar{y})\) are assumed to be vary exponentially from the wall and is given by (see Chandrasekhar et.al (1984))

\[
K(\bar{y}) = k_0 \left( 1 + de \right), \quad \varepsilon(\bar{y}) = \varepsilon_0 \left( d^e \right), \\
\alpha(\bar{y}) = \alpha_0 \left[ \varepsilon_0 \left( d^e \right) + b \left( 1 - \varepsilon_0 \left( d^e \right) \right) \right]
\]  
(6)

Introducing the stream function \(\bar{\psi}\) through \(\bar{u} = \frac{\partial \bar{\psi}}{\partial \bar{y}}, \text{and} \bar{v} = -\frac{\partial \bar{\psi}}{\partial \bar{x}}\) and the following dimensional variables

\[
x = \bar{x}/l, \quad y = \bar{y}/l, \quad a = \bar{a}/l, \quad \sigma = \bar{\sigma}/l, \quad \psi^* = \frac{\bar{\psi}}{\alpha_0}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}
\]  
(7)

and the following transformations

\[
x = \xi, \quad \eta = \frac{y - a \sin(x)}{(1 + \alpha^2 \cos^2(\xi)) \alpha_0^{1/2}}, \quad \psi^* = Ra^{1/2} \xi^{1/2} f(\eta), \quad \theta = \theta(\eta)
\]  
(8)

into eqns. (2)-(5) and letting \(Ra \to \infty\), they reduces into boundary layer equations as follows:

\[
\left( 1 + \sigma^2(\xi) \right) \left( 1 + d^e e^{-\eta} \right) \frac{\partial f}{\partial \eta} + \left( 1 + \sigma^2(\xi) \right) \frac{\partial^2 f}{\partial \eta^2} = \frac{\partial \theta}{\partial \eta}
\]  
(9)

\[
\left( 1 + \sigma^2(\xi) \right) \left[ \varepsilon_0 + b (1 - \varepsilon_0) + \varepsilon_0 d^e e^{-\eta} (b - 1) \right] \frac{\partial^2 \theta}{\partial \eta^2} + \frac{1}{2} \frac{\partial \theta}{\partial \eta} + \frac{\partial f}{\partial \eta} \frac{\partial \theta}{\partial \eta} + \frac{\partial f}{\partial \eta} \frac{\partial \theta}{\partial \xi} = \frac{\partial \psi}{\partial \eta}
\]  
(10)

where \(b\) is the ratio of thermal conductivity of the solid to thermal conductivity of the fluid, \(\varepsilon_0\) is the porosity at the edge of boundary layer, \(Ra = \frac{gk_b b(T_w - T_\infty)}{\varepsilon_0 \nu_0^{1/2}}\) is the modified - Rayleigh number and \(\nu = \frac{\mu_{ef}}{\rho}\) is the effective kinematic viscosity of the fluid in porous medium. The associated boundary conditions are given by

\[
f(0) = 0, \quad \theta(0) = 1, \quad f'(\infty) = 0, \quad \theta(\infty) = 0
\]  
(11)

The engineering design quantities of physical interest include Nusselt number which is defined as

\[
Nu_{\xi} = -\frac{\theta'(0)Ra^{1/2}_{\xi}}{(1 + \alpha^2 \cos^2(\xi))^{1/2}}
\]  
(12)
3. Numerical method

The set of equations (9) and (10) is solved by using local non-similarity solution method which has been used by many researchers (see Sparrow et.al [12] and Narayana et.al [13]). The resulting boundary value problems from this method are solved by employing shooting technique. For the first level of truncation, the terms involving $\xi \frac{\partial}{\partial \xi}$ are assumed to be negligible, and this is valid for $\xi \ll 1$. Therefore we get the following set of equations by neglecting the terms with $\xi \frac{\partial}{\partial \xi}$:

\[
\left(1 + \sigma^2(\xi)\right) \left(1 + d^2 e^{-\eta} \right) \left(1 + d^2 e^{-(\eta)} \right) \frac{\partial f}{\partial \eta} + \left(1 + \sigma^2(\xi)\right) \left(1 + d^2 e^{-(\eta)} \right) \frac{\partial^2 f}{\partial \eta^2} = \frac{\partial \theta}{\partial \eta}
\]

\[
\left(1 + \sigma^2(\xi)\right) \left[\epsilon_0 + b(1 - \epsilon_0) + \epsilon_0 d^* e^{-\eta}(b - 1)\right] \frac{\partial^2 \theta}{\partial \eta^2} + \frac{1}{2} f \frac{\partial \theta}{\partial \eta} + \epsilon_0 d^* e^{-\eta}(b - 1) \left(1 + \sigma^2(\xi)\right) \frac{\partial \theta}{\partial \eta} = 0
\]

The associated boundary conditions are given by:

\[
f(0) = 0, \quad \theta(0) = 1, \quad f'(\infty) = 0, \quad \theta(\infty) = 0
\]

For the second level truncations, we introduce the variable $g = \frac{\partial f}{\partial x}$ and $h = \frac{\partial \theta}{\partial x}$ and recover the neglected terms at the first level of truncation. Thus, the governing equations at this level are:

\[
\left(1 + \sigma^2(\xi)\right) \left(1 + d^* e^{-\eta} \right) \left(1 + d^* e^{-(\eta)} \right) \frac{\partial f}{\partial \eta} + \left(1 + \sigma^2(\xi)\right) \left(1 + d^* e^{-(\eta)} \right) \frac{\partial^2 f}{\partial \eta^2} = \frac{\partial \theta}{\partial \eta}
\]

\[
\left(1 + \sigma^2(\xi)\right) \left[\epsilon_0 + b(1 - \epsilon_0) + \epsilon_0 d^* e^{-\eta}(b - 1)\right] \frac{\partial^2 \theta}{\partial \eta^2} + \frac{1}{2} f \frac{\partial \theta}{\partial \eta} + \epsilon_0 d^* e^{-\eta}(b - 1) \left(1 + \sigma^2(\xi)\right) \frac{\partial \theta}{\partial \eta} = \xi \left[f' h - g\theta \right]
\]

The associated boundary conditions are given by:

\[
f(0) = 0, \quad \theta(0) = 1, \quad f'(\infty) = 0, \quad \theta(\infty) = 0
\]

At third level, we differentiate eqns. (16) and (17) with respect to $\xi$ and neglect the terms $\frac{\partial g}{\partial \xi}$ and $\frac{\partial h}{\partial \xi}$ to get the following equations:

\[
\frac{1 + d^* e^{-\eta}}{(1 + d^2 e^{-\eta})^2} \left[\left(1 + \sigma^2(\xi)\right) g' + 2\sigma\xi(\xi)\sigma\xi(\xi) f' \right] + \frac{1 + d^* e^{-\eta}}{(1 + d^2 e^{-\eta})} \left[\left(1 + \sigma^2(\xi)\right) g' + 2\sigma\xi(\xi)\sigma\xi(\xi) f'' \right] = g'
\]

\[
\left(\epsilon_0 + b(1 - \epsilon_0) + \epsilon_0 d^* e^{-\eta}(b - 1)\right) \left[\left(1 + \sigma^2(\xi)\right) h'' + 2\sigma\xi(\xi)\sigma\xi(\xi) \theta'' \right] + \frac{1}{2} \left[g\theta' + f h' \right] + \epsilon_0 d^* e^{-\eta}(b - 1) \left[\left(1 + \sigma^2(\xi)\right) h' + 2\sigma\xi(\xi)\sigma\xi(\xi) \theta' \right] = \left(f' h - g\theta \right) + \xi \left(g' h - g\theta' \right)
\]

The corresponding boundary conditions are:

\[
g(0) = 0, \quad h(0) = 0, \quad g'(\infty) = 0, \quad h(\infty) = 0
\]

The set of equations (16) (17) and (19) (20) together with boundary conditions (18) and (21) are solved by employing shooting technique that uses Runge-kutta fourth method and Newton-Raphson method (see Mallikarjuna et al. [14]).
4. Results and Discussion

Numerical solutions have been carried out and presented in figs. 1-4. The numerical problem comprises two dependent fluid characteristics \((f, \theta)\) and five thermo-physical parameters, namely \(d, d^*, b, a\) and \(\epsilon_0\). The following default parameter values i.e., \(a = 0.5, \epsilon_0 = 0.5, \xi = 1, b = 0.5\) and \(d = 3.0, d^* = 1.5\) for variable permeability (VP) and \(d = 0, d^* = 0\) for uniform permeability (UP) are considered (unless otherwise stated). Fig. 1 represent the variation of the flow velocity and temperature distribution for different values of \(b\), ratio of thermal conductivity of the solid and fluid for different cases of UP and VP. Increasing \(b\) which implies an increase in thermal conductivity of porous matrix (a decrease in fluid thermal conductivity) is observed to accelerate the flow i.e. increases velocity as shown in fig. 1. Decreasing of thermal conductivity of the fluid with increase in \(b\) values diminishes the thermal diffusivity of the fluid regime and enhances thermal energy in the boundary layer. The temperature distribution is therefore enhanced, as observed in fig.1. Thus both momentum and thermal boundary layer thicknesses are enhanced with increasing \(b\) values. Figs. 2 depicts the velocity and temperature distributions for various values of the porosity \((\epsilon_0)\). Increasing \(\epsilon_0\) implies to increase porosity at the edge of the boundary layer leads to increase the flow i.e. accelerates the flow as well as increase the temperature profile with rise in \(\epsilon_0\). It may also observe from figs. 1 and 2 that fluid characteristics are more influenced for the case of UP compare to VP results. Figs. 3 illustrate the variation of Nusselt number for different values of \(b\) and \(\epsilon_0\). Increasing \(b\) is observed to strongly enhance the Nusselt number for VP and reduce for UP as shown in fig. 3. Increasing \(\epsilon_0\) is evidently strongly retards the Nusselt number as illustrated in fig. 3. Fig. 4 represents the variation of rate of heat transfer (Nusselt number) for different values of the amplitude of the wavy surface \((a)\). It is worth to mention that wavy surface becomes flat surface for \(a = 0\). Increasing \(a\) from 0 through 0.3 to maximum value 0.5, results in a distinct deceleration in the Nusselt number for both cases of VP and UP.

5. Conclusion

A mathematical model of steady, laminar, viscous, incompressible heat transfer flow of a natural convective boundary layer fluid over a vertical wavy surface in a porous medium with variable permeability has been analyzed. The governing equations are non-dimensionalised using specified transformations and then transformed into coupled, boundary layer equations by incorporating the waviness and another set of transformations. The resultant equations are solved numerically and presented the results graphically on fluid flow characteristics and Nusselt number for different values of ratio of thermal conductivity of the solid to fluid \((b)\), porosity at the edge of the boundary layer \((\epsilon_0)\) and amplitude of the wavy surface for two cases UP and VP. Increasing \(b\) results to enhance flow velocity, temperature distributions for both cases of UP and VP. But Nusselt number increases for the case of VP and reduces for UP case. An increase in \(\epsilon_0\), leads to accelerates the flow velocity and enhance the temperature distribution for both cases of UP and VP. Conversely Nusselt number is decreased with rising \(\epsilon_0\). Amplitude of the Nusselt number increases for larger values of amplitude of the wavy surface. This type of study finds applications in the fields of manufacturing, ceramics, petroleum geology, aerodynamics, pharmaceutics etc.
Fig. 2. Velocity and temperature profile for different values of $\epsilon_0$.

Fig. 3. Nusselt number for different values of $b$ and $\epsilon_0$

Fig. 4. Nusselt number for different values of amplitude ($a$).

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