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On the stiffness and damping coefficients of constant flow valve compensated conical hydrostatic journal bearing with micropolar lubricant

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Abstract

A theoretical analysis for performance characteristics of a conical multirecess hydrostatic journal bearing compensated with constant flow valve has been carried out considering micropolar lubricant. The numerical solution of the modified Reynolds equation for the conical bearing has been done using finite element method with necessary boundary conditions. The performance characteristics have been presented for various values of the restrictor design parameter, load, micropolar parameters, semi-cone angle and aspect ratio (L/D ratio) of the bearing at zero speed. It has been observed that the bearing develops large stiffness and damping coefficients with increase in restrictor design parameter, micropolar effect and semi-cone angle.

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Keywords: constant flow valve restrictor; conical bearing; micropolar lubricant; finite element method.

Nomenclature

a	Axial bearing land width 'mm', $\bar{a}_b = a/L$
A_p	Pocket area, mm^2

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A_b	Bearing area, mm^2
C	Radial clearance, mm
C_{ij}	Fluid film damping coefficients (N/mm^2), ($i,j=x,z$), $\bar{C}_{ij}=C_{ij}\left(C^3/\mu r_j^4\right)$
D_m	Mean journal diameter of conical shaft, mm
F	Fluid film reaction, N
F_x, F_z	Fluid film reaction component 'N', $\bar{F}_x$,$\bar{F}_z$= (F_x, F_z /$p_s\, r_j^2$)
h	Fluid film thickness 'mm', $\bar{h}_{\min}=h/C$
l	Bearing characteristic length 'mm', $l_m=C/l$
L	Bearing length, mm
n_l^e	Number of nodes per element
N	Coupling number
N_i	Shape function
p	Pressure, Pa
p_s	Supply pressure 'Pa', $\bar{p}_{\max}=p/p_s$
$\bar{Q}_c$	Constant flow valve restrictor design parameter
R	Radius of journal at radius R of cone, mm
r_j	Journal mean radius, mm
S_{ij}	Fluid film thickness coefficients (N/mm), ($i,j=x,z$), $\bar{S}_{ij}=S_{ij}\left(C/p_s r_j^2\right)$
t	Time 's', $\bar{t}= t\left(C^2 p_s/r_j^2\mu\right)$
ω_j	Journal speed (rev/s), $\Omega=\omega_j\left(\mu_r r_j^2/C^2 p_s\right)$
X_j, Z_j	Journal center coordinate, $\bar{X}_j, \bar{Z}_j$= (X_j.Y_j) $/r_j$
x,y,z	Cartesian coordinates

Greeks

γ	Semi cone angle, deg
α	Circumferential coordinate
β	Axial coordinate
∂	Material coefficients for micropolar lubricant, N-s
κ	Spin viscosity coefficients (Pa-s)
μ	Dynamic viscosity (Pa-s)
λ	Aspect ratio, L/D_m
θ	Inter-recess angle, deg
φ	Attitude angle, deg
ε	Eccentricity ratio, e/C
ϕ	Micropolar function

Matrices and vectors

$\left[\bar{F}_{ij}\right]$	Assembled fluidity matrix
$\{\bar{p}\}$	Nodal pressure vector
$\{\bar{Q}\}^e$	Nodal flow vector
$\{\bar{R}_{H_i}\}^e$	Column vector due to hydrodynamic terms
$\left\{\bar{R}_{x_{ji}}\right\}^e$,$\{\bar{R}_{z_{ji}}\}^e$	Nodal vectors due to journal centre velocities

1. Introduction

In order to improve the performance of bearings various design geometries are being exploited by several researchers. Among them, conical bearing geometry is the one which has been adopted by several researchers to enhance the bearing performances. Conical bearing is considered as the modification of cylindrical, multi-lobe, lemon, wave and tapered bearings. This bearing has significant advantages which supports both radial and axial loads and are generally used in high speed turbines, precision machine tools etc. The literature depicts that conical geometry when implemented over hydrostatic / hybrid journal bearings with the use of various restrictors improves the bearing performances and stability. The characteristics of multirecess conical hydrostatic thrust bearings were studied theoretically taking into account the effect of rotational lubricant inertia by Prabhu and Ganesan [1]. The steady state performance of externally pressurized gas lubricant conical bearings under rotation has been presented by Srinivasan and Prabhu [2]. They also investigated a theoretical study of the behavior of capillary compensated annular recess conical hydrostatic thrust bearings under conditions of eccentricity and rotation. Kalita et al. [3, 4] analyzed the theoretical and experimental aspects of the characteristics of externally pressurized central recess conical bearings with non-constant film thickness under the assumption of isothermal laminar flow of a viscous incompressible fluid. Khalil et al. [5] has presented theoretical investigation on the effect of laminar and turbulent flow in conical thrust bearing. An experimental and theoretical studies dealing with the effect of solid additives in lubricating oils on the steady state performance of conical journal bearings using mass transfer theory has been studied by Yousif and Nacy [6].

Sinha et al. [7] reported an externally pressurized non-constant gap conical bearing rotating with a uniform angular velocity considering incompressible lubricant and rotational inertia. Yang and Jeng [8] studied the effect of lubrication of a conventional finite diameter conical cylindrical bearing using a power law, non-Newtonian lubricants. Hannaon and Braun [9] have presented numerical solution of a fully thermally coupled generalized universal Reynolds equation and its application to conical bearings. Hong et al. [10] have presented a theoretical and experimental method to recognize the dynamic performance of an externally pressurized deep/shallow pockets hybrid conical bearings compensated by flat capillary restrictors. Verma et al. [11] have analyzed the multirecess hydrostatic journal bearing operating with micropolar lubricant. Sharma et al. [12] have studied theoretically the performance of a 4 pocket capillary compensated hydrostatic conical journal bearing system for various cone angles and have shown the effect of wear on its performance. Sharma and Rajput [13- 15] have demonstrated the influence of micropolar lubricant on a 4 pocket conical hybrid journal bearing system compensated with capillary and membrane restrictor. Rana et. al [17] have presented static characteristics of constant flow valve compensated conical hydrostatic journal bearing under micropolar fluid lubrication.

The objective of the present work is to study the performance characteristics of a four-pocket conical hydrostatic journal bearing compensated with constant flow valve in terms of stiffness and damping coefficients, for various values of the restrictor design parameter, load, micropolar parameters, semi-cone angle and aspect ratio of the bearing at zero speed.

2. Mathematical model

The four pocket hydrostatic conical journal bearing is shown in Fig.1. The journal is assumed to rotate with uniform angular velocity about its equilibrium position. For hydrostatic bearing design it must be possible to support the full operating load at zero speed.

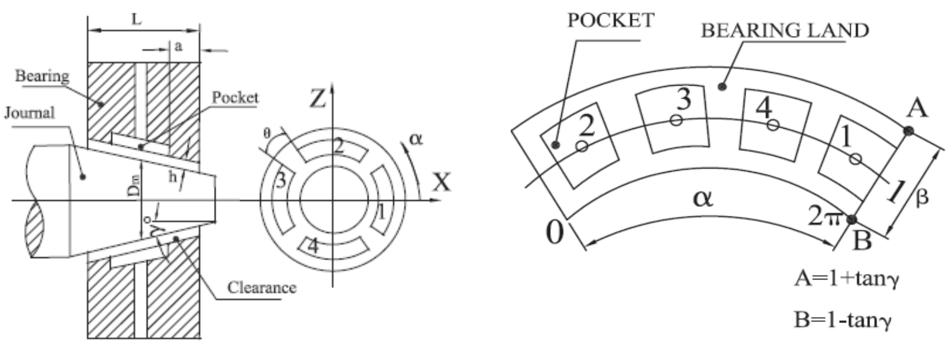

Fig.1: Four pocket hydrostatic conical journal bearing

2.1. Fluid film thickness

The film thickness expression in non-dimensional form for conical journal bearing:

$$\bar{h} = \left(1 - \bar{X}_j cos\alpha - \bar{Z}_j sin\alpha\right) cos\gamma \quad (1)$$

2.2. Restrictor flow

The constant flow valve restrictor should be able to supply a fixed quantity of lubricant flow through it; hence the flow $\bar{\boldsymbol{Q}}_{\boldsymbol{R}}$ of lubricant flow through it is expressed as:

$$\bar{Q}_R = constant = \bar{Q}_c \quad (2)$$

Here, $\bar{\boldsymbol{Q}}_{\boldsymbol{R}}$ and $\bar{\boldsymbol{Q}}_{\boldsymbol{c}}$ represent the restrictor flow and pocket flow, respectively.

2.3. Reynolds equation

The Reynolds equation for conical bearing is as follows [14]:

$$\frac{1}{\beta^2}\frac{\partial}{\partial\alpha}\left[\frac{\bar{\phi}(N,l_m,\bar{h})}{12}\frac{\partial\bar{p}}{\partial\alpha}\right] + \frac{sin^2\gamma}{\beta}\frac{\partial}{\partial\beta}\left[\frac{\beta\bar{\phi}(N,l_m,\bar{h})}{12}\frac{\partial\bar{p}}{\partial\beta}\right] = \frac{\Omega}{2}\frac{\partial\bar{h}}{\partial\alpha} + \frac{\partial\bar{h}}{\partial\bar{t}} \quad (3)$$

where, $\beta = \frac{Rsin\gamma}{r_j}$, $\bar{p} = \frac{p}{p_s}$, $\bar{h} = \frac{h}{C}$, $\bar{t} = t\left(\frac{C^2 p_s}{r_j^2\mu}\right)$, $\bar{\phi}(N, l_m, \bar{h}) = 1 + \frac{12}{\bar{h}^2 l_m^2} - \frac{6N}{\bar{h} l_m} coth\left(\frac{N\bar{h}l_m}{2}\right)$

$N = \left(\frac{k}{2\mu+k}\right)^{1/2}$, $\quad l = \left(\frac{\partial}{4\mu}\right)^{1/2}$, $\quad l_m = C/l$

2.4. Finite element formulation

The solution of the Reynolds equation has been carried out using finite element method. The lubricant flow in bearing is discretized using simplest four noded quadrilateral isoparametric elements. The pressure variation is assumed to vary linearly over an element and is expressed as follows:

$$\bar{p} = \sum_{i=1}^{n_l^e} \bar{p}_i N_i$$

where N_i is the elemental shape function and n_l^e is the number of nodes per element of 2D flow field discretized solution domain.

With the use of Galerkin's orthogonality conditions following usual assembly procedure, global system equation is obtained as follows:

$$\left[\bar{F}_{ij}\right]\{\bar{p}\}=\{\bar{Q}_i\}^e + \Omega\left\{\bar{R}_{H_i}\right\}^e + \dot{\bar{X}}_J\left\{\bar{R}_{x_{ji}}\right\}^e + \dot{\bar{Z}}_J\left\{\bar{R}_{z_{ji}}\right\}^e \quad (4)$$

where,

$$[\bar{F}_{ij}] = \iint_{A^e} \frac{\bar{\phi}(N,\bar{l},\bar{h})}{12}\left[\frac{1}{sin\gamma}\frac{\partial N_i}{\partial\alpha}\frac{\partial N_j}{\partial\alpha} + \beta^2 sin\gamma\frac{\partial N_i}{\partial\beta}\frac{\partial N_j}{\partial\beta}\right] d\alpha d\beta$$

$$\{\bar{Q}_i\}^e = \int_{\Gamma^e}\frac{\bar{\phi}(N,\bar{l},\bar{h})}{12}\left[\frac{1}{sin\gamma}\frac{\partial\bar{p}}{\partial\alpha}l_1 + \beta^2 sin\gamma\frac{\partial\bar{p}}{\partial\beta}l_2\right]N_i d\Gamma^e - \frac{\Omega}{2}\int_{\Gamma^e}\frac{\beta^2}{sin\gamma}\bar{h}l_1 N_i d\Gamma^e$$

$$\bar{R}_{H_i}^e = \iint_{A^e}\frac{\bar{h}}{2}\frac{\partial N_i}{\partial\alpha}\frac{\beta^2}{sin\gamma}d\alpha d\beta, \quad \bar{R}_{X_{ji}}^e = \iint_{A^e} N_i cos\alpha cos\gamma\frac{\beta^2}{sin\gamma}d\alpha d\beta, \ \bar{R}_{Z_{ji}}^e = \iint_{A^e} N_i sin\alpha cos\gamma\frac{\beta^2}{sin\gamma}d\alpha d\beta,$$

l_1 and l_2 are direction cosines and i, j =1, 2 …..n_l^e

2.5. Stiffness and damping coefficients

Fluid film stiffness coefficients

$$\begin{bmatrix} \bar{S}_{xx} & \bar{S}_{xz} \\ \bar{S}_{zx} & \bar{S}_{zz} \end{bmatrix} = -\begin{bmatrix} \partial\bar{F}_x/\partial\bar{X}_j & \partial\bar{F}_x/\partial\bar{Z}_j \\ \partial\bar{F}_z/\partial\bar{X}_j & \partial\bar{F}_z/\partial\bar{Z}_j \end{bmatrix} \quad (5)$$

Fluid film damping coefficients

$$\begin{bmatrix} \bar{C}_{xx} & \bar{C}_{xz} \\ \bar{C}_{zx} & \bar{C}_{zz} \end{bmatrix} = -\begin{bmatrix} \partial\bar{F}_x/\partial\dot{\bar{X}}_j & \partial\bar{F}_x/\partial\dot{\bar{Z}}_j \\ \partial\bar{F}_z/\partial\dot{\bar{X}}_j & \partial\bar{F}_z/\partial\dot{\bar{Z}}_j \end{bmatrix} \quad (6)$$

3. Computational procedure

The solution of a hydrostatic conical journal bearing system requires the solution of the micropolar fluid flow equation with the restrictor flow equation as constraint and appropriate boundary conditions. The modified Reynolds equation governing the flow of the micropolar lubricant is solved along with restrictor flow equation by finite element method so as to obtain fluid film pressures. The iterative procedure is repeated until the converged solution for the fluid film pressure field is obtained as shown in solution scheme [17].

Boundary conditions:

1. Flow of lubricant through the constant flow valve is equal to the bearing input flow.
2. All the nodes situated on the recess have equal pressure.
3. Nodes situated on the external boundary of the bearing have zero pressure.
4. At the trailing edge of positive region, the Reynolds boundary condition is applied i.e.

$$\bar{p} = \frac{\partial\bar{p}}{\partial\alpha} = 0.0$$

4. Results and discussion

The mathematical model developed is used to compute the performance characteristics of a constant flow valve compensated 4 pocket conical hydrostatic journal bearing. Since, for hydrostatic bearing design it must be possible to support the full operating load at zero speed, the results of zero speed are presented here. The results are computed for the following dimensionless values of bearing operating and geometric parameters:

$$\Omega = 0.0,\ \lambda = 1.0,\ \bar{a}_b = 0.14,\ \overline{W}_r = 0.1 - 1.0,\ \bar{Q}_c = 0.1 - 0.3,\ \theta = 18^0,$$
$$A_p/A_b = 0.333,\ N^2 = 0.5,\ l_m = 20,\ \gamma = 0^0 - 40^0$$

The validation of the code has been shown in Table 1 & 2 by the present author in paper [17]. The results were in good agreement with the available literature. The variations of direct stiffness coefficients ($\bar{S}_{xx}$, $\bar{S}_{zz}$) with constant flow valve restrictor parameter ($\bar{Q}_c$) for different semi-cone angles (γ) are shown in Fig.2. The direct stiffness coefficients are found to be increased with increase in $\bar{Q}_c$ and γ. And the variations of direct damping coefficients ($\bar{C}_{xx}$, $\bar{C}_{zz}$) with constant flow valve restrictor parameter ($\bar{Q}_c$) for different semi-cone (γ) are shown in Fig.3. These damping coefficients are found to be decreased with increase in $\bar{Q}_c$ but increases with increase in γ.

The $\bar{S}_{xx}$ & $\bar{S}_{zz}$ are found to be increased by 261.97%, 227.48% and 204.86% for $\gamma = 0^0, 10^0$ & 40^0 respectively when the restrictor design parameter $\bar{Q}_c$ varies from 0.1 to 0.3. Whereas, $\bar{C}_{xx}$ & $\bar{C}_{zz}$ decrease by 8.5%, 16.8% and 30.8% for $\gamma = 0^0, 10^0$ & 40^0 respectively when the restrictor design parameter $\bar{Q}_c$ varies from 0.1 to 0.3.

The variations of direct stiffness coefficients ($\bar{S}_{xx}$, $\bar{S}_{zz}$) with radial load ($\overline{W}_r$) for different semi-cone angles (γ) are shown in Fig.4. The stiffness coefficients are found to be almost constant with increase in $\overline{W}_r$ and these coefficients increase with increase in γ. The variations of direct damping coefficients ($\bar{C}_{xx}$, $\bar{C}_{zz}$) with radial load

($\bar{W}_r$) for different semi-cone angle are shown in Fig.5. These damping coefficients are also found to be constant with increase in $\bar{W}_r$ however, it increases with increase in γ. As the value of semi-cone angle increases the value of fluid-film reaction components (radial and axial) changes, hence the values of fluid film stiffness coefficients are expected to increase.

The variations of direct stiffness coefficients ($\bar{S}_{xx}$, $\bar{S}_{zz}$) and direct damping coefficients ($\bar{C}_{xx}$, $\bar{C}_{zz}$) with characteristic length (l_m) for different semi-cone angles (γ) are shown in Figs.6 & 7. These coefficients decrease with increase in l_m and increase with increase in γ. The variations of stiffness coefficients against bearing design parameter (λ) have been shown in Fig.8. The stiffness coefficients decrease with an increase in λ. A decrease of 20.63% and 11.59% have been observed for $\gamma = 10^0$ and $\gamma = 40^0$ when aspect ratio changes from 0.9 to 1.3 for $\bar{S}_{xx}$ and $\bar{S}_{zz}$. The variations of direct damping coefficients ($\bar{C}_{xx}$, $\bar{C}_{zz}$) vs. λ are plotted in Fig.9. These coefficients are found to be increased with an increase in aspect ratio. An increase of 149.84 % & 115.47 % are observed for $\gamma = 10^0$ and $\gamma = 40^0$ when λ varies from 0.9 to 1.3 for $\bar{C}_{xx}$ and $\bar{C}_{zz}$.

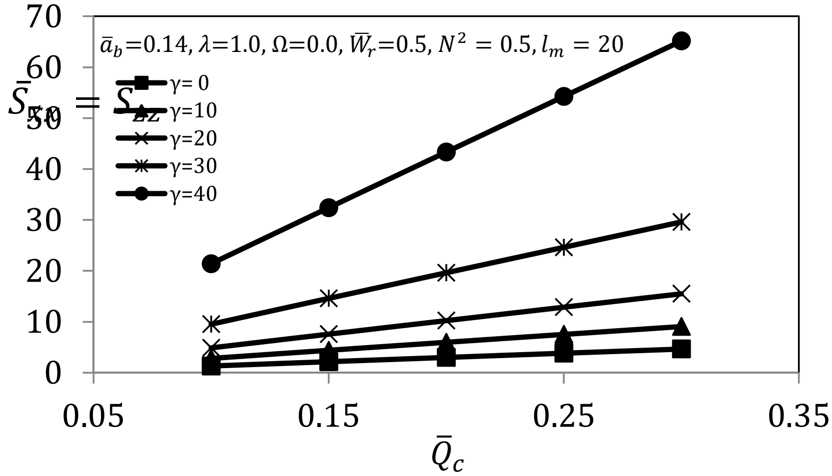

Fig. 2: Variation of direct stiffness coefficients ($\bar{S}_{xx} = \bar{S}_{zz}$) vs. constant flow valve restrictor ($\bar{Q}_c$)

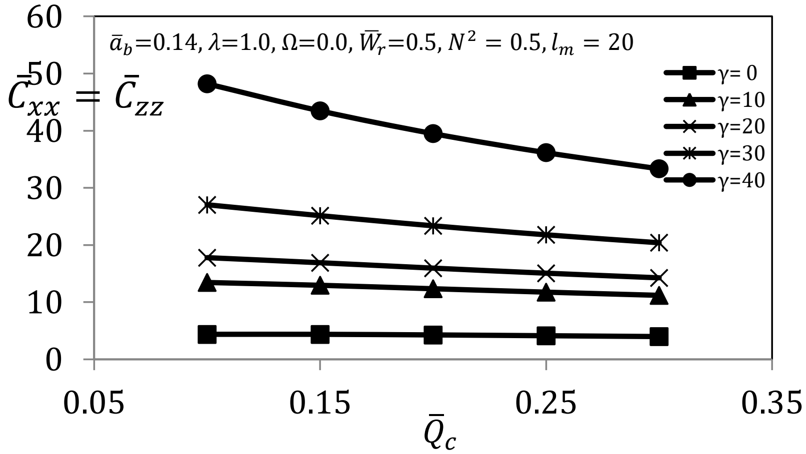

Fig. 3: Variation of direct damping coefficients ($\bar{C}_{xx} = \bar{C}_{zz}$) vs. constant flow valve restrictor ($\bar{Q}_c$)

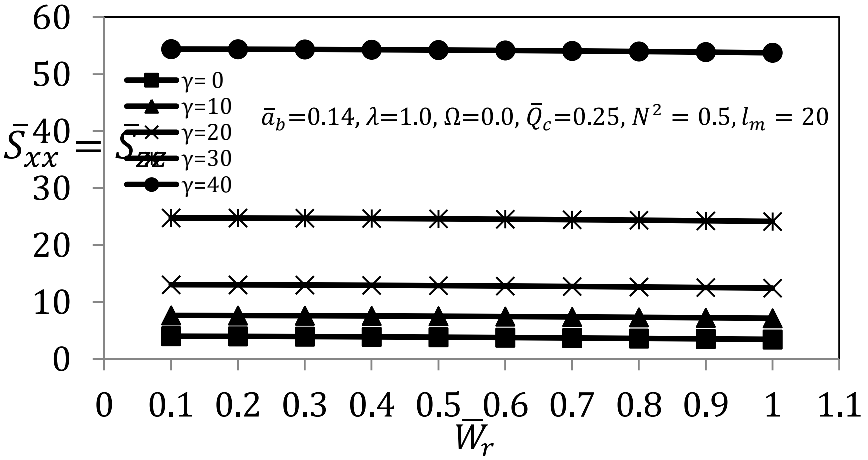

Fig. 4: Variation of direct stiffness coefficients ($\bar{S}_{xx} = \bar{S}_{zz}$) vs. radial load ($\bar{W}_r$)

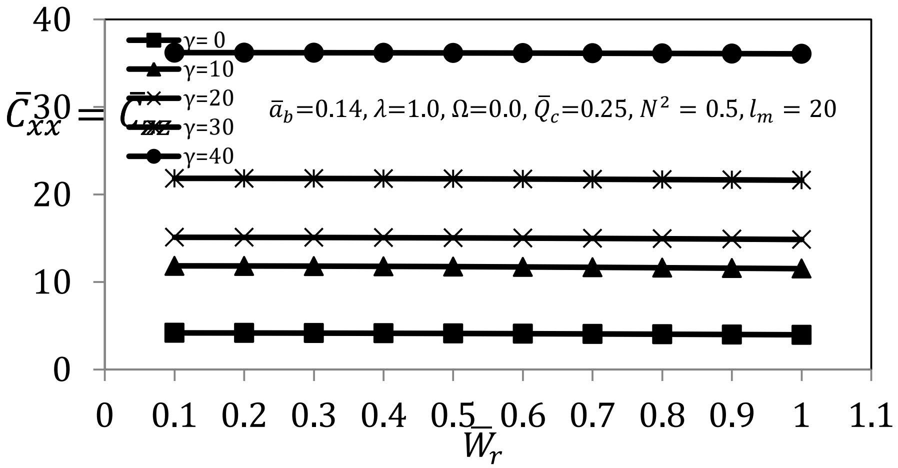

Fig. 5: Variation of direct damping coefficients ($\bar{C}_{xx} = \bar{C}_{zz}$) vs. radial load ($\bar{W}_r$)

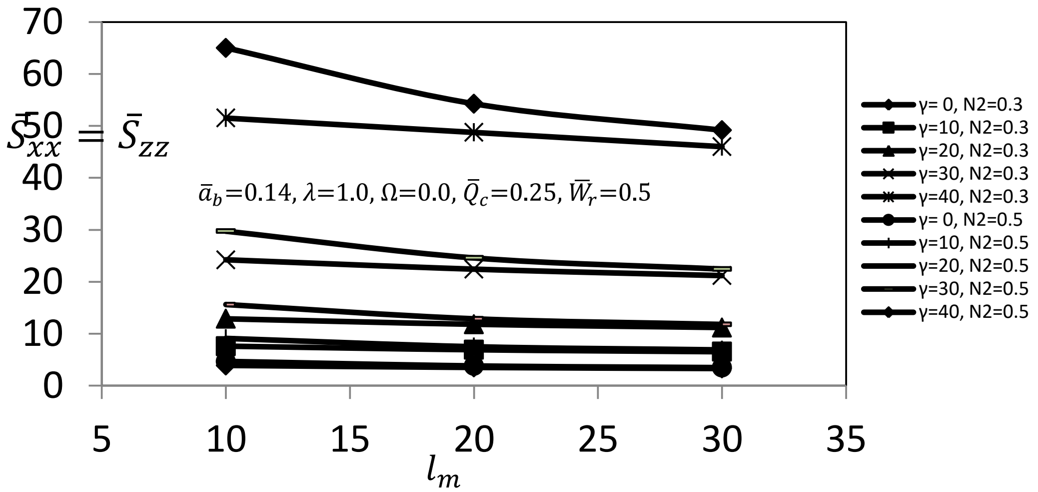

Fig. 6: Variation of direct stiffness coefficients ($\bar{S}_{xx} = \bar{S}_{zz}$) vs. characteristic length (l_m)

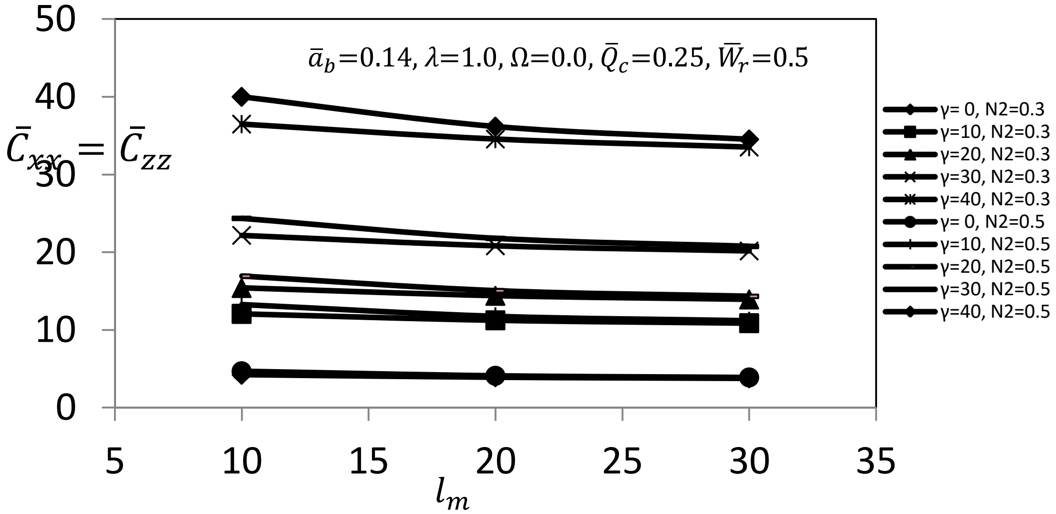

Fig. 7: Variation of direct damping coefficients ($\bar{C}_{xx} = \bar{C}_{zz}$) vs. characteristic length (l_m)

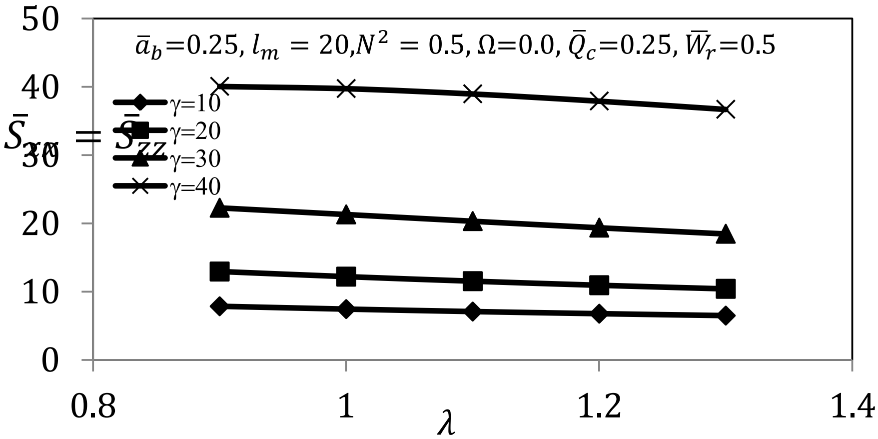

Fig. 8: Variation of direct stiffness coefficients ($\bar{S}_{xx} = \bar{S}_{zz}$)vs. aspect ratio ($\lambda$)

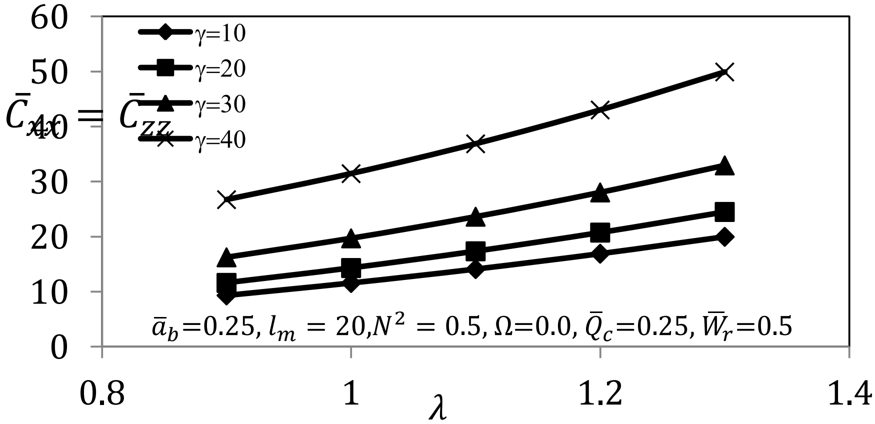

Fig. 9: Variation of direct damping coefficients ($\bar{C}_{xx} = \bar{C}_{zz}$) vs. aspect ratio ($\lambda$)

5. Concluding remarks

A four pocket constant flow valve compensated hydrostatic conical journal bearing operating with micropolar lubricant has been analyzed theoretically to determine the stiffness and damping coefficients of the bearing under zero speed condition. The following conclusions are made from this study which may be useful for bearing design.

1. It is observed that the bearing develops large stiffness and damping coefficients with increase in constant flow valve parameter, micropolar effect and semi-cone angle.
2. For a given load and aspect ratio, the value of direct damping coefficients are found to be increased with increase in semi-cone angle (γ) but decreased with increase in constant flow valve parameter$(\bar{Q}_c)$.
3. For a chosen value of semi-cone angle, the values of direct stiffness coefficients are more at lower values of aspect ratio. However, the values of direct damping coefficients are more at higher values of aspect ratio of bearing.

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