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A new look at anomaly cancellation in heterotic M-theory

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Abstract

This Letter considers anomaly cancellation for eleven-dimensional supergravity on a manifold with boundary and theories related to heterotic M-theory. The Green–Schwartz mechanism is implemented without introducing distributions. The importance of the supersymmetry anomaly in constructing the low energy action is discussed and it is argued that a recently proposed action for low-energy heterotic M-theory gives a supersymmetric theory to all orders in the gravitational coupling κ.

1. Introduction

Horava and Witten have argued that the strong coupling limit of the ten-dimensional heterotic string is eleven-dimensional supergravity with gauge multiplets confined to two ten-dimensional hypersurfaces forming the boundary of the eleven-dimensional spacetime manifold [1,2]. This theory is a very promising starting point for phenomenological models based on compactifications to four dimensions (see, for example, [3–7]). In applications such as these, it is important to know the action in as much detail as possible.

The form of the low-energy action originally put forward was found by relying partly on anomaly cancellation and supersymmetry. Gauge and gravitational anomalies in the theory cancel via a novel modification of the Green–Schwartz mechanism involving the supergravity 3-form. The cancellation, which involves some remarkable algebraic coincidences, requires that the matter action contains a factor of order $\kappa^{2/3}$ compared to the supergravity action, where κ is the eleven-dimensional gravitational coupling strength.

Imposing local supersymmetry on the action fixes all of the terms at order $\kappa^{2/3}$. However, when the same procedure is applied to order $\kappa^{4/3}$, singular terms depending on the square of the delta-function start to arise. This problem has recently been overcome by modifying the boundary conditions on the gravitino and the supergravity 3-form, so that now an action can be constructed which is non-singular and supersymmetric to higher orders [8]. The theory reduces to the correct Yang–Mills supergravity theory in 10 dimensions for the low-energy heterotic string by dimensional reduction, with the boundary conditions playing a role in obtaining the correct 10-dimensional gaugino terms [9].

The main issue to be addressed in this Letter is the effect of the new boundary conditions on anomaly cancellation. As we extend the theory to higher orders in the gravitational coupling, we have to take account of the supersymmetry anomaly which appears at order κ^2 (i.e. κ^2 times the gravitational action). The most important point is that the existence of a supersymmetry anomaly implies that the classical action should not be supersymmetric at this order. However, it is reasonable to suppose that the supersymmetry anomaly, like the gauge anomaly, is cancelled by the Green–Schwartz mechanism, and the action should therefore be supersymmetric up to the variation of the Green–Schwartz terms [10,11]. This was not appreciated in [9], where it was shown that the supersymmetric variation of the new action for heterotic M-theory reduced to just the variation of the Green–Schwartz terms. Now, taking into account the supersymmetry anomaly, this theory is supersymmetric to all orders in κ, at least when truncated to terms up to first order in the Riemann tensor.

A heuristic argument can be made for the cancellation of the supersymmetry anomaly by the Green–Schwartz terms. The Wess–Zumino consistency conditions relate the supersymme-

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try anomaly to the gauge anomaly [10,11]. When the Green–Schwartz terms are added to the effective action, the total has vanishing gauge variation. If the consistency conditions have a unique solution, then the effective action with the Green–Schwartz terms should have vanishing supersymmetry variation as well. Below, we shall see how this works out explicitly.

We shall consider the gauge and supersymmetry anomaly cancellation now in more detail. We work throughout on the 'downstairs picture' of a manifold with boundary, rather than lifting to the covering space $R^{10}\times S_1$. For the present, we truncate the action to first order in the Riemann tensor. The gauge anomaly from the chiral fermion on one of the boundary components can be described by a formal 12-form $I_{12}(F)$ [12]. To generate the anomaly, we introduce the notation T, such that locally $dT\omega=\omega$ for a closed form ω. The anomalous variation of the chiral fermion effective action under gauge transformations δ_α is given by integrating a 10-form I_{10}^A, defined by

$$I_{10}^A = T\delta_\alpha T I_{12}. \tag{1}$$

The anomalous variation of the chiral fermion effective action under supersymmetry variations has been discussed by Itoyama et al. [10,11]. It is given by the sum of two different 10-forms, $I_{10}^S + I_{10}^{S'}$, where

$$I_{10}^S = l_\eta T I_{12} \tag{2}$$

and $I_{10}^{S'}$ is a gauge invariant expression which we leave for later. The operator l_η is an anti-derivative defined by $l_\eta A=0$ and $l_\eta F=\delta_\eta A$.

In the case of the gauge group E_8,

$$I_{12} = \frac{1}{12(4\pi)^5}\left(\operatorname{tr} F^2\right)^3 \tag{3}$$

and we have

$$I_{10}^A = \frac{1}{12(4\pi)^5}\operatorname{tr}(\alpha F)\left(\operatorname{tr} F^2\right)^2 \tag{4}$$

$$I_{10}^S = \frac{1}{12(4\pi)^5}\operatorname{tr}(\delta_\eta A A)\left(\operatorname{tr} F^2\right)^2 + \frac{1}{3(4\pi)^5}\operatorname{tr}(\delta_\eta A F)T\left(\operatorname{tr} F^2\right)^2. \tag{5}$$

Now the key observation of Horava and Witten was that I_{10}^A can be cancelled by a variation of the CGG term in the supergravity action [13]. This can be done by requiring $G\sim \operatorname{tr} F^2$ on the boundary and $\delta_\alpha C\sim\delta(x^{11})\operatorname{tr}(\alpha F)$. If we follow this route further, we are eventually lead to the theory with $\delta(x^{11})^2$ terms [2].

An alternative way to arrange the Green–Schwartz cancellation was first described in [14]. If we let $\delta_\alpha C\sim da$, where a is any 2-form which satisfies $a=\operatorname{tr}(\alpha F)$ on the boundary, and require that $G\sim\operatorname{tr} F^2$ on the boundary, then the variation of the Green–Schwartz term is a total derivative,

$$\delta CGG \sim d(aGG). \tag{6}$$

This integrates to give a boundary term which can cancel the anomaly (4). Similarly, if we add an extra supersymmetry variation $\delta_\eta C\sim df$ to the 3-form, where $f=\operatorname{tr}(\delta_\eta AA)$ on the boundary, then part of the supersymmetry anomaly I_{10}^S is cancelled [9]. (The rest of the anomaly is cancelled by the usual variation of C, as we shall see shortly.)

The gauge and supersymmetry variations of C are precisely those which are required to maintain the gauge and supersymmetry invariance of the 3-form boundary condition given in [9], namely

$$C = \frac{\sqrt{2}}{12}\epsilon(\omega_Y+\omega_\chi) \tag{7}$$

on the boundary, where ω_Y is the Chern–Simons form $T\operatorname{tr} F^2$ and

$$\omega_\chi = \frac{1}{4}\bar\chi^a\Gamma_{ABC}\chi^a. \tag{8}$$

The constant ϵ in Eq. (7) is related to the gauge coupling constant λ by $\epsilon=\kappa^2/2\lambda^2$. This boundary condition is responsible for obtaining the correct combination of two-form and gaugino fields in the action when the theory is reduced to 10 dimensions [9].

Since the Chern–Simons form has a gauge transformation $\delta_\alpha\omega_Y=d(\operatorname{tr}\alpha F)$, the boundary condition remains valid if the variation of C is given by

$$\delta_\alpha C = \frac{\sqrt{2}}{12}\epsilon\, da \tag{9}$$

where $a=\operatorname{tr}(\alpha F)$ on the boundary. (Some details of the use of p-form boundary conditions in quantum field theory can be found in [15,16]. A more careful treatment would consider the Abelian BRST variations of the boundary condition, but these are similar in form to the Abelian gauge variations with the gauge parameter replaced by the ghost field, and the argument remains essentially unchanged.)

The fermion term (8) in the boundary condition is required to make the boundary condition supersymmetric. It also plays an important role in obtaining the correct ten-dimensional reduction. Unfortunately, the gaugino enters into the variation of the CGG term through the value of $G=6dC$ on the boundary,

$$G = \frac{\epsilon}{\sqrt{2}}\left(\operatorname{tr} F^2 + d\omega_\chi\right). \tag{10}$$

The gauge anomaly has no fermion terms, therefore in order to avoid spoiling the anomaly cancellation, we have to add boundary corrections to the Green–Schwartz terms. The CGG term is taken from the usual supergravity action (with gravitational coupling $\kappa^2/2$ [17]),

$$S_C = -\frac{2\sqrt{2}}{\kappa^2}\int_{\mathcal{M}} CGG. \tag{11}$$

The boundary terms can only involve ω_Y, ω_χ and F and they must vanish when $\omega_\chi=0$. The unique combination which has the desired effect is

$$S_3 = -\frac{\epsilon^3}{6\kappa^2}\int_{\partial\mathcal{M}} \omega_Y\omega_\chi\left(2\operatorname{tr} F^2 + d\omega_\chi\right). \tag{12}$$

The variation of S_C and S_3 using (9) is then

$$\delta_\alpha S_C + \delta_\alpha S_3 = -\frac{\epsilon^3}{6\kappa^2}\int_{\partial\mathcal{M}} \operatorname{tr}(\alpha F)\big(\operatorname{tr} F^2\big)^2. \tag{13}$$

This has the desired form to cancel the gauge anomaly (4), and fixes the value of ϵ,

$$\epsilon = \frac{1}{4\pi}\left(\frac{\kappa}{4\sqrt{2}\pi}\right)^{2/3}. \tag{14}$$

This agrees with [17], which corrected a factor of 2 in [2]. The result differs by a factor of 3 from the one obtained on the covering space in [18]. The difference is possibly due to the way in which the theory is lifted to the covering space.

If our assumptions are correct, then the supersymmetric variation of the Green–Schwartz terms should now cancel the supersymmetry anomaly, i.e.

$$\delta_\eta S_C + \delta_\eta S_3 + \int \big(I^S_{10} + I^{S'}_{10}\big) = 0. \tag{15}$$

A supersymmetry variation of S_C and S_3 allows us to read off the non-gauge-invariant part of the anomaly

$$I^S_{10} = \frac{\epsilon^3}{6\kappa^2}\operatorname{tr}(\delta_\eta A A)\big(\operatorname{tr} F^2\big)^2 + \frac{2\epsilon^3}{3\kappa^2}\operatorname{tr}(\delta_\eta A F)\omega_Y \operatorname{tr} F^2. \tag{16}$$

This is in complete agreement with (5), proving that this part of the supersymmetry anomaly does indeed cancel. We also generate the gauge invariant part of the supersymmetry anomaly,

$$I^{S'}_{10} = \frac{\epsilon^3}{\kappa^2}\operatorname{tr}(\delta_\eta A F)\omega_\chi\big(2\operatorname{tr} F^2 + d\omega_\chi\big) + \frac{\epsilon^3}{6\kappa^2}(\delta_\eta\omega_\chi)\omega_\chi\big(3\operatorname{tr} F^2 + 2d\omega_\chi\big). \tag{17}$$

In these expressions, we have included local supersymmetry transformations

$$\delta_\eta A = \frac{3}{2}\bar\eta\Gamma_A\chi, \tag{18}$$

$$\delta_\eta\omega_\chi = \frac{1}{8}\bar\eta\Gamma_{ABC}\Gamma^{DE}\chi^a\hat F^a{}_{DE} + \frac{3}{8}\bar\eta\Gamma_D\psi_{[A}\bar\chi^a\Gamma^D{}_{BC]}\chi^a, \tag{19}$$

where $\hat F_{AB} = F_{AB} - \bar\psi_{[A}\Gamma_{B]}\chi$. The supersymmetry anomaly in ten dimensions has been calculated previously up to four Fermi terms [19,20]. Our result has a similar form, although a direct comparison is not possible because our result contains contributions from the eleventh dimension (indicated by the presence of the gravitino field ψ_A).

We can make use of the anomaly (16) in connection with the action of heterotic M-theory. The action S proposed in [9] consisted of usual supergravity action with boundary terms S_0 and a boundary matter action

$$S_1 = -\frac{2\epsilon}{\kappa^2}\int_{\partial\mathcal{M}} dv\left(\frac{1}{4}F^a{}_{AB}F^{aAB} + \frac{1}{2}\bar\chi^a\Gamma^A D_A\big(\Omega^{**}\big)\chi^a + \frac{1}{4}\bar\psi_A\Gamma^{BC}\Gamma^A F^{a*}_{BC}\chi^a\right), \tag{20}$$

where $F^* = (F+\hat F)/2$, Ω is the supergravity spin connection and $\Omega^{**}{}_{ABC} = \Omega_{ABC} + \frac{1}{24}\psi^D\Gamma_{ABCDE}\psi^E$. We have now discovered a new result that we must also add the term S_3 for the anomaly cancellation to work properly. In [9], it was shown that the supersymmetry variation of the action was

$$\delta_\eta S = \frac{2\sqrt{2}}{\kappa^2}\int_{\partial\mathcal{M}} \delta_\eta CCG, \tag{21}$$

up to one possible four Fermi term and all orders in κ. We now recognize this as the variation of the Green–Schwartz term, and therefore it cancells with the supersymmetry anomaly. The extra four Fermi terms in (17) explain also why there was a four-Fermi term left in the variation. Up to the limitations of truncating out the higher order curvature terms, the action $S = S_0 + S_1 + S_3$ describes a theory which is supersymmetric to all orders in κ.

It is interesting to see how the Green–Schwartz terms behave in the weakly coupled string limit when the 11-dimensional theory is reduced to 10 dimensions. The reduction ansatz for the C field which is consistent with the boundary condition (7) is [9]

$$C_{ABC} = -\frac{\sqrt{2}}{12}\epsilon y(\omega_{2Y} + \omega_{2\chi}) + \frac{\sqrt{2}}{12}\epsilon(1-y)(\omega_{1Y} + \omega_{1\chi}), \tag{22}$$

$$C_{11AB} = \frac{1}{6}B_{AB}, \tag{23}$$

where the subscripts 1 and 2 refer to the gauge multiplets on the hypersurfaces at $y=0$ and $y=1$. The CGG term reduces to the usual Green–Schwartz term '$B(\operatorname{tr} F^2)^2$' for the low-energy limit of the heterotic superstring (for the gauge group $E_8\times E_8$) plus some terms depending on fermion fields [21]. The new boundary term S_3 reduces to an $O(\alpha'^3)$ fermion term in the ten-dimensional action. These fermion terms must be consistent with the supersymmetry of the action which has not been broken in the reduction, but they could also be checked directly against the six point heterotic string loop amplitude.

The treatment of gauge and gravitational anomalies in the original Horava–Witten model included terms which are higher order in the Riemann tensor [2]. The 12-form which generates the anomalies was obtained from the gauginos and boundary effects on the gravitino [1],

$$I_{12} = \frac{1}{12(2\pi)^5}\big(I_4^3 - 4I_4X_8\big), \tag{24}$$

where $I_4 = \operatorname{tr} F^2 - \frac{1}{2}\operatorname{tr} R^2$ and $X_8 = -\frac{1}{8}\operatorname{tr} R^4 + \frac{1}{32}(\operatorname{tr} R^2)^2$. The gauge, gravitational and supersymmetry anomalies due to the first term in I_{12} can be removed by the CGG term as described above, provided that the boundary condition on C is modified,

$$C = \frac{\sqrt{2}}{12}\epsilon\left(\omega_Y - \frac{1}{2}\omega_L + \omega_\chi - \frac{1}{2}\omega_\psi\right), \tag{25}$$

where $\omega_L = T\operatorname{tr} R^2$ is the Lorentz Chern–Simons term and ω_ψ, according to dimensional analysis, is bilinear in the derivative of the gravitino. The construction of a fully supersymmetric theory with this boundary condition has not yet been done, but it seems inevitable that R^2 terms will also appear in the action [22,23]. These terms would be needed to ascertain the precise form of ω_ψ.

Similarly, the X_8 terms in the gauge anomaly should be cancelled by a Green–Schwartz term CX_8 in eleven-dimensions [2,22]. This can be done with $\delta_\alpha C \sim da$ as above, but the cancellation is not exact, because the eleven-dimensional curvature appears in the Green–Schwartz term but, so far, only the ten-dimensional curvature appears in the anomaly. This problem could be removed by adding additional boundary terms to the action, but the most likely possibility is that there are new contributions to the anomaly depending on the extrinsic curvature of the boundary.

In conclusion, it is possible to cancel the gauge anomalies in eleven-dimensional supergravity with boundaries without introducing singular gauge transformations. The CGG term in the supergravity action acts as a Green–Schwartz term, but since there are fermions present in the boundary conditions it is necessary to introduce an extra boundary term in the action depending on the gaugino field. It is interesting that the boundary conditions and action appear to be well-determined from gauge and supersymmetry invariance. This agrees with recent work by van Nieuwenhuizen and Vassilesvich [24], who have found that supersymmetry severely restricts the boundary conditions for pure supergravity. Given also that eleven-dimensional supergravity with more than two boundaries can now be consistently formulated (at least as $\kappa \to 0$) [25], it looks increasingly likely that the manifold with boundary picture is the more fundamental way of formulating heterotic M-theory.

We have seen the supersymmetry anomaly has to be taken into account when constructing the action and, at least in the limit of small curvature, the action for supergravity with E_8 gauge multiplets on the boundaries gives a fully supersymmetric quantum field theory. It has been show elsewhere that this theory reduces to the same 10-dimensional Yang–Mills supergravity theory as the low energy heterotic string [9]. However, dimensional reduction from eleven to four dimensions involves curvature terms in the internal dimensions which are not small, but comparable in size to the gauge field strength [3–7]. It would be very desirable to find a supersymmetric action which includes the R^2 terms suggested by the gauge and gravity anomalies, and then we would have confidence in using this version of heterotic M-theory as a basis for particle physics phenomenology.

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