Combining F-K filter with Minimum Entropy Stolt migration algorithm for subsurface object imaging and background permittivity estimation

Huilin ZHOU, Xing WAN*, Wei LI, Yuling JIANG

Department of Electronic Information Engineering, Nanchang University, Nanchang 330031, China

Abstract

Determination of background permittivity and removing of the direct wave is one of the main problems in data processing on Ground Penetrating Radar. The interference of direct wave will greatly affect the accuracy and the resolution of GPR imaging. We propose an F-k filtering approach for the solution, which use a difference of an apparent velocity between a direct wave and a reflection from a target. Besides, a Stolt migration algorithm combined F-K filter with minimum entropy technique is proposed to improve the focus effect of the radar image and realize the precise inversion of background relative permittivity. Synthetic data generated by FDTD have been used to test the improved migration algorithm, and the results show that the presented algorithm can achieve comparative results produced by windowed filter based Stolt migration algorithm.

Keywords: GPR; F-K filter; Stolt Migration; Minimum Entropy.

1. Introduction

Ground Penetrating Radar (GPR) is a popular electromagnetic technology in detecting and localizing subsurface object due to its advantages of high resolution and fast profiling, so it is widely used in aspects of highway construction, survey of shallow subsurface geology and geological exploration [1, 2]. However, the factors of radar wave recording mode and wave propagation characteristics make the radar profile different from geological space, mainly including the reflection events moving along the declinational direction and energy dispersions of scattering points. To eliminate the influence of factors mentioned above, migration of radar profile should be implemented for the purpose of subsurface objects location and imaging. Currently, migration methods in GPR data processing technology widely applied are Wave Equation Migration Algorithms, which includes: Kirchhoff migration [3], finite difference migration [4] and frequency-wavenumber migration [5, 6] (includes: Stolt migration, phase-shift migration and so on). Charles P. Oden [5] improved the resolution of underground radar image of lossy medium by F-K migration method and Colin Gilmore [6] did a more detailed elaboration and research for the scalar wave equation on FK phase shift migration algorithm, and showed the availability and validity of the algorithm.

* Corresponding author.
E-mail address: wanxingne@163.com.
Although the migration algorithm can achieve subsurface objects location, the imaging resolution is seriously affected by the direct wave. In order to suppress the direct wave and enhance the reflected wave from the target, several techniques, such as background subtraction and windowed filter, have been proposed, but when the target is close to the ground, the direct wave and the target echo almost reach the receiver simultaneously so that the method is invalid. So we propose to apply a frequency–spatial frequency (F-K) filtering approach. Generally, an F-K filter, which is a 2-D process in an F-K domain, utilizes the differences of apparent velocity between significant wave and interference wave to implement the separation of reflection waves that arrive from different directions and positions [7]. In this paper, we present F-K filter for direct wave suppression and SNR improvement of GPR image, and Stolt migration algorithm for subsurface object detection and underground dielectric and EM propagation velocity estimation based on minimum entropy evaluation technique.

2. F-K filter Theory

Frequency-wavenumber filter is used to suppress interference waves based on the differences between significant wave and interference wave in the frequency-wavenumber spectrum. \( E(x, z = 0, t) \) is two-dimensional radar record on the ground, the frequency spectrum of the record is:

\[
\hat{E}(k_z, \omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x, z = 0, t) \exp[-i(\omega t + k_z x)] dx dt
\]

Then, use an F-K filter as:

\[
\hat{G}(k_z, \omega) = \hat{E}(k_z, \omega) \cdot H(k_z, \omega)
\]

Here, \( H(\omega, k_z) \) is a filtering function to be presented in the following, and \( G(\omega, k_z) \) is a filtered spectrum. By applying 2-D inverse Fourier transform to \( G(\omega, k_z) \), we can obtain a time-spatial radar profile without the interference wave component as:

\[
g(x, t) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G(k_z, \omega) \exp[i(k_z x + \omega t)] dk_z d\omega
\]

Two-dimensional filter make use of differences of apparent velocity and frequency between significant wave and interference wave to achieve filtering, which can be expressed in the f-k plane, as shown in fig.1

![Fig.1 two-dimensional spectrum plane](image)

Here, the apparent velocity can be expressed as \( v = f / k_z \), the zone I is the interference zone of high velocity and the zone II is the interference zone of low velocity, the dip angle of the event in time-space domain is greater, the line in frequency-wavenumber domain is closer to the wavenumber axis. The spectrum component with zero dip is on the frequency axis, zero dip is equivalent to zero wavenumber. It is confirmed that the direct wave arrived at the same time, apparently, and its apparent velocity becomes infinitely large. As a result, a spectrum component of the direct wave in the f–k domain is distributed along the frequency axis, spatial frequency \( k_z = 0 \). In this paper, the main noise of simulation data is the direct wave, so the filter to be applied in the F–K domain is given by

\[
H(\omega, k_z) = \begin{cases} 0(\omega, 0) \\ 1(\omega, k_z \neq 0) \end{cases}
\]
3. Stolt Migration and Minimum Entropy Technique

3.1. Stolt Migration

The algorithms based on wave equation migration all make use of the wave equation to back-propagate the record wave field in order to develop an image of the subsurface geological structure. Stolt algorithm takes on the advantages of both Kirchhoff Integral Method and Finite Difference Method. Furthermore, efficiency is obtained via Fourier-based implementations employing the fast Fourier transform (FFT).

Let \( \hat{E}(k, z=0, \omega) \) be two-dimensional Fourier transform of overlaying records on the ground and underground part is composed of single layer with homogeneous medium. Stolt algorithm can be described as following:

\[
E(x, z, 0) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left( 1 + \frac{k_x^2}{k_z^2} \right)^{-1/2} \hat{E} \left( k_x, \frac{vk_x}{2} \sqrt{1 + \frac{k_x^2}{k_z^2}} \right) \exp[i(k_x x + k_z z)] dk_x dk_z
\]

\[
= \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} B(k_x, k_z) \exp[i(k_x x + k_z z)] dk_x dk_z
\]

\[
\hat{E}(k_x, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} E(x, z = 0, t) \exp[-i(\omega t + k_x x)] dx dt
\]

\[
B(k_x, k_z) = \frac{v}{2} \left( 1 + \frac{k_x^2}{k_z^2} \right)^{-1/2} \hat{E} \left( k_x, \frac{vk_x}{2} \sqrt{1 + \frac{k_x^2}{k_z^2}} \right)
\]

Where \( x \) is the horizontal coordinate, \( t \) is the time along the depth direction, \( z \) is the depth coordinate, \( v \) is the velocity of migration, and \( k_x, k_z \) are the wavenumber vectors in the horizontal and vertical component, respectively. However, the key to achieve Stolt migration is to transform the space \( \hat{E}(k_x, \omega) \) into \( (k_x, k_z) \) by interpolation.

3.2. Principle of Minimum Entropy Technique

Supposed two-dimensional image of GPR can be described by matrix \( X \) as follows

\[
X = \begin{bmatrix}
    x_{11} & x_{12} & \cdots & x_{1N} \\
    x_{21} & x_{22} & \cdots & x_{2N} \\
    \vdots & \vdots & \ddots & \vdots \\
    x_{M1} & x_{M2} & \cdots & x_{MN}
\end{bmatrix} = \begin{bmatrix}
    X_1 & X_2 & \cdots & X_N
\end{bmatrix}
\]

(8)

Where \( x_{ij} \) indicates \( i \) sampling point of the \( j \) trace data and \( X_j \) indicates the \( j \) trace data set. Entropy of image [8] is defined as follows:

\[
E(X) = \sum_{j=1}^{N} \left( \frac{1}{N} \sum_{i=1}^{N} x_{ij}^4 \left( \frac{1}{N} \sum_{i=1}^{N} x_{ij}^2 \right)^{-2} \right)
\]

(9)

According to the definition, the maximum value of entropy is 1 for the single trace data set when the data contains only peak pulse with single-unit amplitude, as to the N trace sets, the value is N. In terms of an image, the greater its entropy is, the more confusing the image target point is. Vice versa, minimizing the entropy of image after migration processing can optimize the focus effect. So the effect of migration processing can be evaluated by minimum entropy technique in order to make the focus effect optimal.

4. Experimental Results and Analysis of Data

4.1. Simulation Scene and Data Profile of Forward Simulation

The geoelectric model to simulation is shown in fig.2. (a), the relative electric property parameters of it are shown in table 1. In this paper, according to the FDTD principle with CPML absorbing boundary conditions [9, 10], we do the simulation for the model, and the forward profile is shown in fig.2. (b).
Table 1 Relative Electric Property Parameters of Each Medium

<table>
<thead>
<tr>
<th>Medium</th>
<th>Relative permittivity</th>
<th>Relative conductivity</th>
<th>Relative permeability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air layer</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Soil layer</td>
<td>9</td>
<td>0.002</td>
<td>1</td>
</tr>
<tr>
<td>Square target</td>
<td>25</td>
<td>0.005</td>
<td>1</td>
</tr>
</tbody>
</table>

4.2. Migration Imaging and Comparative Analysis of Experimental Data

In the forward profile, the direct wave reflection interferes seriously in the focus of the target migration. This paper applies windowing filter and F-K filter to process the radar profile respectively, then, according to the principle of Stolt migration we implement Stolt migration processing for filtered result by a velocity sequence, and calculate the entropy of images after migration. We extract the migrated velocity using minimum point of minimum entropy curve, and then implement migration imaging once again by the extracted velocity, and the results are as follow:
Fig. 4. (a) F-K filtered image; (b) f-k spectrum of the forward profile; (c) Stolt Migration Image of Minimum Entropy (F-K filter); (d) Relationship between Migration Velocity and Entropy

Table 2 Experimental Results after filter

<table>
<thead>
<tr>
<th></th>
<th>Minimum entropy</th>
<th>The extracted velocity</th>
<th>The background permittivity</th>
<th>The true background velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Windowing filter</td>
<td>1.624</td>
<td>9.387 cm/ns</td>
<td>10.2</td>
<td></td>
</tr>
<tr>
<td>F-K filter</td>
<td>1.774</td>
<td>9.778 cm/ns</td>
<td>9.41</td>
<td>10 cm/ns</td>
</tr>
</tbody>
</table>

In fig.3, the direct wave is effectively removed, but when the target is close to the ground, the direct wave and the target echoes almost reach the receiver simultaneously so that the method is invalid. From the fig.4 (b), we can see that the direct wave spectrum and the target reflection waves in F-K domain are separated. In fig.4(a),(c), although the F-K filtered image is a little confusing, because the part information of vertices on the target echoes the same with direct wave spectrum is also filtered, it does not obviously affect the overall estimation of radar image. According to the above results and table 2, we know that the F-K filtered image is also suitable for migration processing and the extracted velocity is even closer to the true background propagation velocity. So the F-K filter is more effective and automatic than the windowed filter.

5. Conclusion

In this paper, we present the F-K filter for direct wave suppression and SNR improvement of GPR image, then use Stolt migration algorithm combining with minimum entropy techniques for subsurface object imaging and background dielectric estimation. The experimental results based on simulated data show that the presented algorithm can achieve comparative results produced by windowed filter based Stolt migration algorithm.

Acknowledgement

This work is supported by the NSFC (No.6106202009) and Key Technology R&D Program of Jiangxi (No. 2009BGB02200).

References

[6] Gilmore C, Jeffrey I, LoVetri J. Derivation and comparison of SAR and frequency-wavenumber migration within a common


