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Mathematics, Physics, and Information

(An Editorial)

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1. INFORMATION IN SCIENCE

The theory of information raises a number of fundamental questions about the first postulates of science and the limits of their validity. It is hoped that among its many other functions, this new journal will offer a possibility for free discussion of these problems and for a fruitful comparison of points of view. As an example of such problems, let us examine the opinions of a pure mathematician and of a physicist about the foundations of geometry. The mathematician starts with dimensionless points, infinitely thin curves and surfaces, and continuous space time. Atomic science denies any real meaning to these definitions.

Consider a very thin tinfoil, and look at it with X-rays: you discover an atomic lattice, with isolated atoms separated by large empty intervals. The foil has a finite thickness and is not continuous. Even a monomolecular layer exhibits similar properties.

2. EXPERIMENTAL ERRORS

The mathematician very carefully defines irrational numbers. The physicist never meets any such numbers. Whatever he measures is represented by a finite number, with so many figures, and a certain amount of uncertainty. The mathematician shudders at uncertainty and tries to ignore experimental errors.

Open a book of pure mathematics and consider a theorem. It is always built on a typical scheme: given certain conditions A, B, C, which are assumed to be exactly fulfilled, it can be proven rigorously that conclusion Q must be true. Here the physicist starts wondering: how can we know that A, B, C, are exactly fulfilled? No observation can tell us that much. The only thing we may know is that A, B, C, are approximately satisfied within certain limits of error. And then what does the theorem prove? Very small errors on A, B, C, may result in a very small error on the final statement Q, or they may destroy it completely. The discussion is not complete until the problem of the stability of the theorem has been investigated, and this is another story!

3. INFORMATION IS FINITE

In a recent book,[1] I discussed at great length the problem of experimental errors. The theory of information gives us a possibility for stating correctly these questions and getting consistent answers. We are now able to define the amount of information obtained from a certain experiment, and to measure it in a precise way. We only need to know the field of uncertainty before and after the observation. The logarithm of the ratio of these two uncertainties yields the amount of information. If the final uncertainty is very small (very accurate measurement) the information obtained is very large.

The mathematician dreams of measurements of infinite accuracy, defining for instance the position of a point without any possible error. This would mean an experiment yielding an infinite amount of information and this is physically impossible. One of the most important results of the theory is known as the "negentropy principle of information." It states that any *information* obtained from an experiment must be paid for in negentropy. As D. Gabor states it: "you cannot get something for nothing, not even an observation." If an experiment yields an information ΔI, there must have been increase of entropy

$$\Delta S \geqq \Delta I \tag{1}$$

in the apparatus or in the laboratory where the experiment has been performed. An increase ΔS in entropy means a decrease

$$\Delta N = -\Delta S \tag{2}$$

in the total "negentropy." The information ΔI is paid for by a larger amount ΔN of negentropy.

$$\Delta N + \Delta I \leqq 0$$

A very large amount of information shall cost a very high price, and an infinite amount of information is unattainable. No wealth could pay for it. Another important point refers more specifically to the definition

[1] L. Brillouin, "Science and Information Theory." Academic Press, New York, 1956.

of a continuum in space and time (*loc. cit.*, Ch. XVI). A very direct discussion leads to the result that the measurement of a very small length Δx (and by "very small," a length smaller than 10^{-13} cm is meant) requires the use of a total energy ΔE such that

$$\Delta E \Delta x \geqq \tfrac{1}{2} hc \tag{3}$$

This represents a new limitation, different from the uncertainty principle and completely independent from the conditions specified in (1). Extremely small distances cannot be measured, unless a source of very high energy is used. This energy may not be completely dissipated, but it is needed for the experiment. Only a fraction ΔQ of this total energy is dissipated, according to Eq. (1)

$$\Delta S = \Delta Q / T \geqq \Delta I \tag{4}$$

if the experiment is performed under most economical conditions.

Both results lead to similar conclusions: an infinite amount of information can never be obtained. An infinitely small distance cannot be measured. Geometrical and mathematical definitions are only dreams, but the physicist cannot trust them and we should especially emphasize the impossibility of physically defining a continuum in space and time.

4. THE VIEWPOINT OF M. BORN

It is not necessary to repeat here many comments or explanations which were given in my book. Let us only recall that the negentropy principle of information gives a precise meaning to an old remark of J. von Neumann saying that an observation is an irreversible process. Condition (1) specifies the amount of irreversibility.

It may be interesting to quote another author, Max Born, and to discuss a paper he recently published in the jubilee-book[2] presented to Niels Bohr for his seventieth birthday. Under the title "Continuity, determinism and reality," Max Born presents some general remarks very close to those of the present paper, although his line of reasoning is different:

"I maintain that the mathematical concept of a point in a continuum has no direct physical significance. It has no meaning to say the value of a coordinate $x \cdots$ has a value $x = \sqrt{2}$ inch or $x = \pi$ cm." (This disposes of irrational numbers!)

"Modern physics has achieved its greatest successes by applying the

[2] *Kgl. Danske Videnskab. Selskab, Mat. fys. Medd.* (1955) **30**(2).

methodological principle that concepts which refer to distinctions beyond possible experience have no physical meaning and ought to be eliminated.... The most glaringly successful cases are Einstein's foundation of relativity based on the rejection of the concept of aether... and Heisenberg's foundation of quantum mechanics.... *I think that this principle should be applied also to the idea of physical continuity.*"

And M. Born explains that experimental errors should be taken into account right from the beginning, in the field of classical physics where they have been too often ignored.

He does not want to reject the mathematical concept of a real number, but specifies that "the situation demands a description of haziness." The probability for the value of a physical quantity to be in a given interval should be specified, instead of pretending that the value of the quantity can be known exactly.

Many examples discussed by Born ought to be given here, but we hope the reader will refer to the original paper.

Let us remember, that the general principle invoked by Born is often quoted as Bridgman's *operational* point of view.

5. THE PROBLEM OF DETERMINISM

This is again a problem which, in our opinion, belongs to the domain of metaphysics but not of physics. The famous Laplace's statement is well known: an infinitely clever fellow measures with infinite accuracy all the positions of the atoms in the whole world, and computes the behavior of this world for any distant future. This is a dream and corresponds to no physical problem. Bridgman would dismiss it as contrary to the operational method. Born discusses a very simple example which proves the fallacy of the statement: he considers one single atom, moving back and forth along a straight line, between two reflecting walls located at $x = 0$ and $x = l$. Here the prediction looks easy if the initial position x and velocity v are given. But v is measured only within an error Δv and this means an error $t\Delta v$ in x after an interval of time t. Very soon $t\Delta v$ will become larger than the length l of the strip and nobody can predict where to find the particle. A similar example is discussed in E. Borel's lectures on statistical mechanics (written down by F. Perrin[3]). You may believe in determinism, or you may deny it, or simply doubt it. Classical science can give you no answer: it is an act of

[3] E. Borel, "Mecanique statistique classique," p. 22. Gauthier-Villars, Paris, 1925.

faith. And when we come to quantum mechanics and atomic structures then we know that determinism does not apply at all.

6. PHYSICAL THEORIES

The difficulty comes when you consider that mathematics is used as a tool for building physical theories or discussing technical problems. Here we are back on the earth. We cannot measure any quantity with infinite accuracy and we correct the stiffness of mathematical statements by using statistical methods and the calculus of probability. This is practically what Born recommends (see section 4).

The methods of quantum mechanics have been extremely useful in many problems, but they definitely failed on others. The most typical example is the impossibility of computing a variety of quantities, which appear as divergent integrals. Some of these troubles have been partially corrected by the use of "renormalization" methods, but this again is only a remedy. Every physicist knows where the trouble comes from. The integrals extend from zero to infinity, where zero means the immediate neighborhood of a particle. And it is impossible to measure anything too close to the particle. The "exact" position of the center of the particle cannot be measured, and the geometry of a continuum is to blame, but we have no other method for the time being, and we are waiting for a bright idea, telling us how to rebuild mathematics on a practical basis.

The information theory may help in discovering the actual weak points of present methods. Information theory and quantum theory are in full agreement and emphasize the need for a strict obedience to operational definitions.

A practical theory seems to be shaping up progressively with the use of new discontinuous operators, rather similar to the spin operator. All these methods ignore the usual space-time continuum, and they may represent the beginning of the new theory we need.