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# Dynamics of quintessence with thermal interactions

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**Abstract**

The cosmological dynamics of minimally coupled scalar field that couple to the background matter with thermal interactions is investigated in exponential potential. The conditions for the existence and stability of various critical points as well as their cosmological implications are obtained. Although we show that the effects of thermal interaction such as depressing the equation-of-state parameter of quintessence, is only important at the early time, the evolution of equation-of-state parameter of quintessence is manifested. The upper bound is required on the coupling between quintessence and relativistic relic particles such as photons and neutrinos.


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Astronomical measurements from Supernovae Ia [1], galaxies clustering, for example, Sloan Digital Sky Survey (SDSS) [2] and anisotropies of Cosmic Microwave Background Radiation (CMBR) [3] independently suggest that a large fraction of the energy density of our Universe is so-called dark energy, which has negative pressure, or equation of state with $w = p/\rho < 0$, and accelerates the expansion of the Universe. The origin of the dark energy remains elusive from the point of view of general relativity and standard particle physics. One candidate source of this missing energy component is a slowly evolving and spatially homogeneous scalar field, referred to as "quintessence" with $w > -1$ [4–6] and "phantom" with $w < -1$ [7–10], respectively. Since current observational constraint on the equation of state of the dark energy lies in the range $-1.38 < w < -0.82$ [11], it is still too early to rule out any of the above candidates. To study the global property of the cosmological system containing dark energy, phase space analysis is proved to be a powerful tool.

Because dark energy redshifts more slowly than ordinary matter or radiation, there exists an important problem that various proposals of the dark energy should explain, that is, why the energy density

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of matter and dark energy should be comparable at the present epoch. A form of quintessence called "tracker fields", whose evolution is largely insensitive to initial conditions and at late times begin to dominate the universe with a negative equation of state, was introduced to avoids the above problem [12]. Another approach to solve the puzzle is introducing an interaction term in the equations of motion, which describes the energy flow between the dark energy and the rest matter (mainly the dark matter) in the universe. It is found that, with the help of a suitable coupling, it is possible to reproduce any scaling solutions.

The effects of thermal coupling between the quintessence field and the ordinary matter particles has been investigated [13]. Hsu and Murray set the quintessence field to be a static external source for a Euclidean path integral depicting the thermal degree of freedom and let the time-like boundary conditions of the path integral have a period which is decided by the inverse of temperature. They show that if in the early universe matter particles are in thermal equilibrium, quantum gravity [14] will induce an effective thermal mass term for quintessence field $\phi$, which takes the form

$$\left(\frac{\beta}{M_P}\right)^2\phi^2T^4, \tag{1}$$

where $T = \sqrt[4]{\rho_{r0}}/a$ is the temperature of the universe, $M_P$ the Planck mass scale, $\beta$ a dimensionless constant, $a$ the scale factor (with current value $a_0 = 1$) and $\rho_{r0}$ the energy density of radiation at the present epoch, and find that even Planck-suppressed interactions between matter and the quintessence field can alter its evolution qualitatively. For convenience, Hsu and Murray [13] use an approximation that the back reaction on matter of quintessence is neglectable. However, this back-reaction effect, in fact, is significant both at the early time and late time. In this Letter, we investigate the dynamics of the cosmology with quintessence using the phase space analysis, which has the above thermal coupling to the matter in a complete manner. We shall seek the conditions for the existence and stability of various critical points as well as their cosmological implications. As will be shown in this Letter, although the effects of thermal interaction, such as depressing the equation-of-state parameter of quintessence, is only important at the early time, the evolution of equation-of-state parameter of quintessence is manifested. The upper bound is required on the coupling between quintessence and relativistic relic particles such as photons and neutrinos.

We first study the phase space of quintessence with a thermal coupling to ordinary matter particles in spatially flat FRW cosmological background

$$ds^2 = dt^2 - a^2(t)\,d\mathbf{x}^2. \tag{2}$$

For the spatially homogeneous scalar field minimally coupled to gravity with thermal interaction (1), the evolution is governed by the Klein–Gordon equation

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 2\left(\frac{\beta}{M_P}\right)^2\phi T^4, \tag{3}$$

where the overdots denote the derivative with respect to cosmic time and the prime denotes the derivative with respect to $\phi$. Here the Hubble parameter $H \equiv \dot{a}/a$ is determined by the Friedmann equation

$$H^2 = \frac{\kappa^2}{3}\left[\rho_m + \frac{1}{2}\dot{\phi}^2 + V(\phi)\right] \tag{4}$$

and

$$\dot{H} = -\frac{\kappa^2}{2}\left(\rho_m + p_m + \dot{\phi}^2\right), \tag{5}$$

where $\kappa^2 \equiv 8\pi/M_p^2$, $\rho_m$ and $p_m$ are the energy density and pressure of the barotropic matter. According to Eqs. (3)–(5) and the conservation of energy, $\rho_m$ satisfies the following continuous equation

$$\dot{\rho}_m + 3H(\rho_m + p_m) = -2\left(\frac{\beta}{M_P}\right)^2\phi\dot{\phi}T^4, \tag{6}$$

and $p_m = (\gamma - 1)\rho_m$, where $\gamma$ is a constant, $0 \leqslant \gamma \leqslant 2$, such as radiation ($\gamma = 4/3$) or dust ($\gamma = 1$). It is clear that when the thermal coupling parameter $\beta$ becomes zero, Eqs. (3)–(6) will return to those of the standard one scalar field quintessence scenario. In current situation, both quintessence and barotropic matter are not conserved, but the overall energy is conserved. The energy density ratio of quintessence and barotropic matter satisfies the following conditions

$$\dot{r} \equiv \left(\frac{\dot{\rho}_m}{\rho_\phi}\right) = r\left(\frac{Q}{\rho_m} + \frac{Q}{\rho_\phi} - 3H\gamma + 3H\gamma_\phi\right), \tag{7}$$

where the parameter $Q = 2(\beta/M_p)^2\phi\dot{\phi}\rho_{\gamma,0}/a^4$, $\gamma_\phi = 1 + w_\phi$, in which $w_\phi$ is the so-called parameter of state of quintessence which is defined by $w_\phi = p_\phi/\rho_\phi$, and

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi), \tag{8}$$

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi). \tag{9}$$

Rewriting Eqs. (3) and (6), we have

$$\dot{\rho}_m + 3H\left(w_m^{\rm eff} + 1\right)\rho_m = 0, \tag{10}$$

$$\dot{\rho}_\phi + 3H\left(w_\phi^{\rm eff} + 1\right)\rho_\phi = 0, \tag{11}$$

where $w_m^{\rm eff} = \gamma - 1 + \frac{2\beta^2\phi\dot{\phi}\rho_{\gamma,0}}{3M_p^2 Ha^4\rho_m}$ and $w_\phi^{\rm eff} = w_\phi - \frac{2\beta^2\phi\dot{\phi}\rho_{\gamma,0}}{3M_p^2 Ha^4\rho_m}$, respectively. Now, introducing the following variables:

$$x = \frac{\kappa\dot{\phi}}{\sqrt{6}H}, \tag{12}$$

$$y = \frac{\kappa\sqrt{V(\phi)}}{\sqrt{3}H}, \tag{13}$$

$$z = \frac{\kappa}{M_p}\frac{\phi}{\sqrt{3}H}\frac{\sqrt{\rho_{r0}}}{a^2}, \tag{14}$$

$$\xi = \frac{\sqrt{6}}{\kappa\phi}, \tag{15}$$

$$\lambda = -\frac{\sqrt{6}V'(\phi)}{\kappa V(\phi)}, \tag{16}$$

$$\Gamma = \frac{V(\phi)V''(\phi)}{V'^2(\phi)}, \tag{17}$$

$$N = \ln a, \tag{18}$$

the equation system (3)–(6) becomes the following system:

$$\begin{aligned}
\frac{dx}{dN} &= \frac{3}{2}x\left[\gamma\left(1-x^2-y^2\right)+2x^2\right] \\
&\quad - \left(3x + \beta^2 z^2\xi - \frac{1}{2}\lambda y^2\right), \\
\frac{dy}{dN} &= \frac{3}{2}y\left[\gamma\left(1-x^2-y^2\right)+2x^2\right] - \frac{1}{2}\lambda xy, \\
\frac{dz}{dN} &= \frac{3}{2}z\left[\gamma\left(1-x^2-y^2\right)+2x^2\right] - 2z + xz\xi, \\
\frac{d\xi}{dN} &= -x\xi^2, \\
\frac{d\lambda}{dN} &= -x\lambda^2(\Gamma - 1).
\end{aligned} \tag{19}$$

The energy density parameter and the equation-of-state parameter of quintessence, $\Omega_\phi$ and $w_\phi$, satisfy the constraint equation

$$\Omega_\phi \equiv \frac{\kappa^2\rho_\phi}{3H^2} = 1 - \frac{\kappa^2\rho_m}{3H^2} = x^2 + y^2, \tag{20}$$

and $w_\phi = (x^2 - y^2)/(x^2 + y^2)$, respectively. Since the energy density of the barotropic fluid $\rho_m$ is semi-positive-definite, any cosmological model can be represented as a trajectory in the phase space that is bounded within the unit disc, i.e., $0 \leqslant x^2 + y^2 \leqslant 1$. From the definitions of these new variables, It is not difficult to reduce the effective parameter of state of quintessence to

$$w_\phi^{\rm eff} = w_\phi - \frac{2}{3}\frac{\beta^2 xz^2\xi}{1 - x^2 - y^2}. \tag{21}$$

To be concrete, we consider the quintessence with an exponential potential energy density, i.e.,

$$V(\phi) = V_0\exp(-\lambda_0\kappa\phi), \tag{22}$$

where the parameter $V_0$ and $\lambda_0$ are two constants. Exponential potentials have been studied extensively in various situations, and these are of interest for two main reasons. Firstly, they can be derived from a good candidate of fundamental theory for such being string/M-theory; secondly, the motion equations can be written as an autonomous system in the situation. The author of an earlier work [15] considered a scalar field with a single exponential potential in a homogeneous and isotropic universe. Subsequently, it was extended to include barotropic matter [16,17] and multiple scalar fields [18], and generalized for anisotropic universe [19]. According to the definitions, the parameters $\lambda$ and $\Gamma$ both become constants and are equal to $\lambda_0$ and 1, respectively. Under the circumstance, the equations (19) constitute an autonomous system as follows,

$$\begin{aligned}
\frac{dx}{dN} &= \frac{3}{2}x\left[\gamma\left(1-x^2-y^2\right)+2x^2\right] \\
&\quad - \left(3x + \beta^2 z^2\xi - \frac{1}{2}\lambda_0 y^2\right), \\
\frac{dy}{dN} &= \frac{3}{2}y\left[\gamma\left(1-x^2-y^2\right)+2x^2\right] - \frac{1}{2}\lambda_0 xy,
\end{aligned}$$

$$\frac{dz}{dN} = \frac{3}{2}z\left[\gamma\left(1-x^2-y^2\right)+2x^2\right]-2z+xz\xi,$$
$$\frac{d\xi}{dN} = -x\xi^2. \tag{23}$$

From the equation of state of matter (6) and Eqs. (12)–(15), we obtain that

$$\frac{d\rho_m}{dN} + 3\gamma\rho_m + 2V_0\beta^2\frac{xz^2\xi}{y^2}e^{-\sqrt{6}\lambda_0/\xi} = 0, \tag{24}$$

where we have chosen the exponential potential (22). $\rho_m$ can be directly expressed as

$$\rho_m(N) = e^{-3\gamma N}\left(\rho_{m,0} + 2V_0\beta^2\int_0^N \frac{xz^2\xi}{y^2}e^{3\gamma N-\sqrt{6}\lambda_0/\xi}\,dN\right), \tag{25}$$

where the parameter $\rho_{M,0}$ is the present value of energy density of matter. It is easy to find that in the case of $\beta = 0$, the energy density of matter will evolve in the manner of $\rho_m \sim a^{-3}$ for $\gamma = 1$ as usual.

When barotropic matter is under consideration and/or the equations of motion are too difficult to solve analytically, phase space methods become particularly useful, because numerical solutions with random initial conditions usually do not expose all the interesting properties. In Table 1, we list the critical points, conditions for their existence and the cosmological parameters there.

To investigate the stability of these critical points, we can write the variables near these points $(x_c, y_c, z_c, \xi_c)$ in the form $x = x_c + \eta_1$, $y = y_c + \eta_2$, $z = z_c + \eta_3$ and $\xi = \xi_c + \eta_4$ with $\eta_i$, $i = 1, 2, 3, 4$, the perturbations of the variables about the critical points to the first order. This leads to the equation of motion,

$$\mathbf{U}' = \mathbf{A}\cdot\mathbf{U}, \tag{26}$$

where the 4-column vector $\mathbf{U} = (\eta_i)^T$, $i = 1, 2, 3, 4$ represents the perturbations of the variables and $\mathbf{A}$ is a constant $4\times 4$ matrix. For stability we require the all 4 eigenvalues of $\mathbf{A}$ to be negative. For the critical points listed in Table 1, we find the eigenvalues of the linear perturbation matrix for different stable critical points, see Table 2.

Critical points of type A correspond to the scalar field dominated solutions ($\Omega_\phi = 1$), which exist for sufficiently flat potentials, $\lambda^2 < 24$. Moreover, for the

Table 1
The critical points and their physical properties for models with exponential potential

| Type | Critical points $(x_c, y_c, z_c, \xi_c)$ | $\lambda$ | $\Omega_\phi$ | $w_\phi$ | Stability |
|---|---|---|---|---|---|
| A | $\frac{\lambda}{6}, \pm\sqrt{1-\frac{\lambda^2}{36}}, 0, 0$ | any | 1 | $-1+\frac{\lambda^2}{18}$ | stable for $\lambda^2 < 24$ and $\gamma > \frac{\lambda^2}{18}$ |
| B | $3\frac{\gamma}{\lambda}, \pm 3\sqrt{\frac{\gamma(2-\gamma)}{\lambda^2}}, 0, 0$ | $\lambda \neq 0$ | $\frac{18\gamma}{\lambda^2}$ | $\gamma - 1$ | stable for $\gamma < \frac{4}{3}$ and $\lambda^2 > 18\gamma$ |
| C | 0, 0, 0, any | any | 0 | undefined | unstable |
| D | $\pm 1, 0, 0, 0$ | any | 1 | 1 | unstable |
| E | $0, \pm 1, 0, 0$ | $\lambda = 0$ | 1 | $-1$ | stable |

Table 2
The eigenvalues of the critical points for the exponential potential

| Type | Critical points $(x_c, y_c, z_c, \xi_c)$ | Eigenvalues |
|---|---|---|
| A | $\frac{\lambda}{6}, \pm\sqrt{1-\frac{\lambda^2}{36}}, 0, 0$ | $0, -3+\frac{\lambda^2}{12}, -3\gamma+\frac{\lambda^2}{6}, -2+\frac{\lambda^2}{12}$ |
| B | $3\frac{\gamma}{\lambda}, \pm 3\sqrt{\frac{\gamma(2-\gamma)}{\lambda^2}}, 0, 0$ | $0, \frac{3}{4}\left[(\gamma-2)-\sqrt{(2-\gamma)\left(2-9\gamma+\frac{144\gamma^2}{\lambda^2}\right)}\right]$ |
| | | $-2+\frac{3\gamma}{2}, \frac{3}{4}\left[(\gamma-2)+\sqrt{(2-\gamma)\left(2-9\gamma+\frac{144\gamma^2}{\lambda^2}\right)}\right]$ |
| C | 0, 0, 0, any | $0, -3+\frac{3\gamma}{2}, \frac{3\gamma}{2}, -2+\frac{3\gamma}{2}$ |
| D | $\pm 1, 0, 0, 0$ | $0, 6-3\gamma, 1, 3\mp\frac{\lambda}{2}$ |
| E | $0, \pm 1, 0, 0$ | $0, -3, -2, -3\gamma$ |

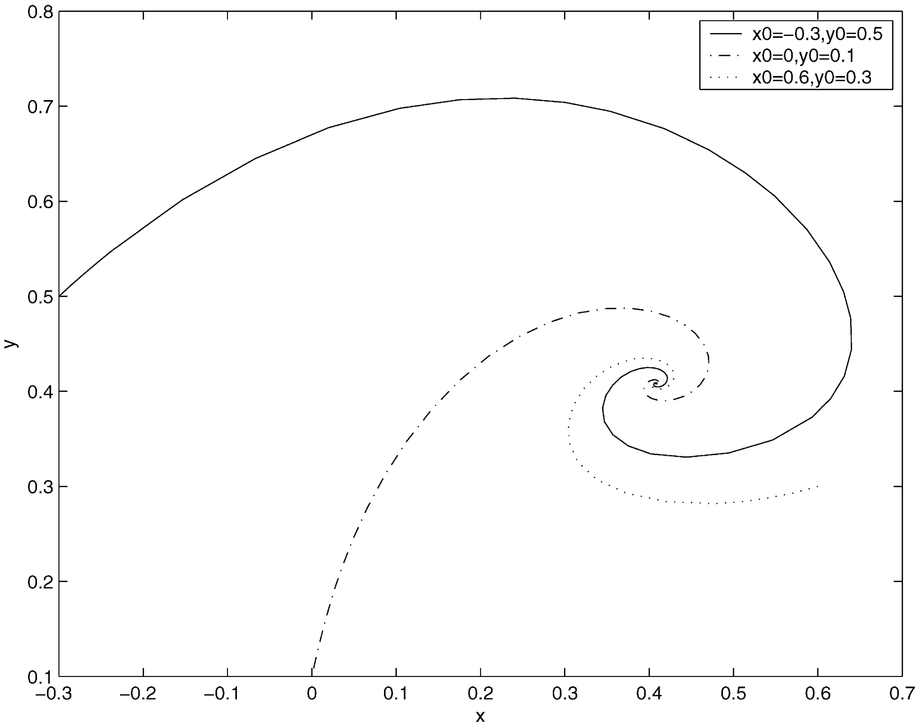


Fig. 1. The portrait of scaling behavior of the quintessence with thermal interaction for exponential models $V(\phi) = V_0 \exp(-\lambda_0 \kappa \phi)$, where $\lambda_0 = 25$ and the parameter $\gamma = 1$.

barotropic index $\gamma > \lambda^2/18$, they are stable to the extent that this kind of solutions represent late-time attractors in the presence of a barotropic fluid. Note that type E is a special case of type A, i.e., $\lambda = 0$.

Critical points of type B correspond to another kind of late-time attractor where neither the quintessence field nor the barotropic fluid completely dominates the evolution of the universe. They are known as scaling solutions where the energy density of quintessence is proportionated to that of barotropic fluid at late time with $\Omega_\phi = 18\gamma/\lambda^2$. It is remarkable that the barotropic index $\gamma$ is required to be no more than $4/3$ for the sake of stability.

Critical points of type C denote the barotropic fluid dominated solutions where $\Omega_\phi = 0$. They are unstable for all reasonable values of $\gamma$ and $\lambda$ (i.e., however steep the potential). The ones of type D are the so-called kinetic-dominated solutions where the late-time evolution of the universe is dominated by the kinetic energy of quintessence with a stiff equation of state, i.e., $w_\phi = 1$. These solutions are unstable as one expects.

As shown in Table 1, all the critical points exist when and only when $z = 0$, that is to say that $\beta$ does not appear in the final expressions that determine the critical points. This means that with the evolution of the universe the thermal coupling (1) between the quintessence and matter always goes asymptotically to vanish away. Fig. 1 shows a scaling solution that the quintessence goes towards an attractor.

In the above, we have studied the phase space of scalar field with thermal interactions to cosmic matter in a flat FRW cosmological background. The critical points indicate that these stable attractor phases corresponding to the vanishing of the thermal interaction between the quintessence and the barotropic matter. However, what will be the effects of the thermal interactions in the circumstances? In fact, the introduction of the interacting term will change the evolutionary tracks of the dynamical systems, which was not well manifested by the above qualitative analysis. Therefore, we study their dynamical evolution numerically and compare the results with the cases of no coupling quintessence scenario. In Fig. 2, the evolu-

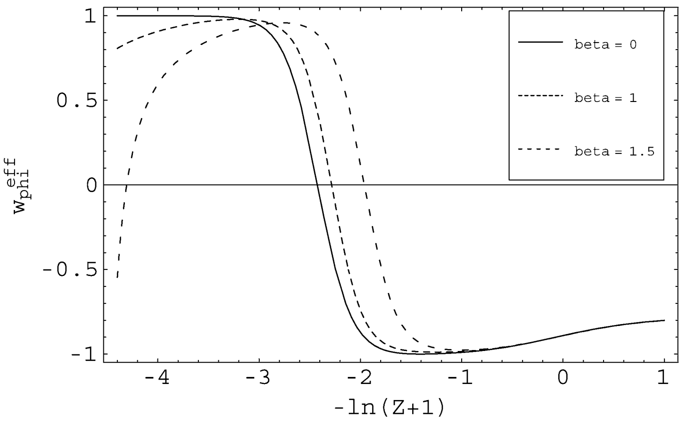


Fig. 2. Evolution of the effective parameter of state $w_\phi^{\mathrm{eff}}$ for $\beta = 1.5$, $\beta = 1$ and $\beta = 0$, respectively, where we let $\gamma = 1$, $\lambda_0 = 2$ and $z$ in the label of horizontal axis denote red shift.

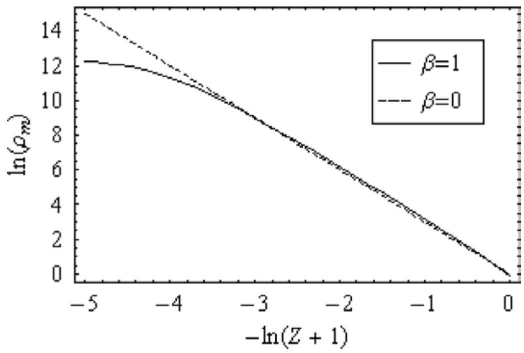


Fig. 3. Evolution of the energy density of barotropic matter $\rho_m$ for $\beta = 1$ and $\beta = 0$, respectively, where we let $\rho_{m,0} = 1$, $\gamma = 1$, $\lambda_0 = 2$ and $z$ in the label of horizontal axis denote red shift.

tion of equation-of-state parameter of quintessence is manifested. Obviously, the effective equation-of-state parameter of quintessence $w_\phi^{\mathrm{eff}}$ that incorporate the effect of the thermal coupling between quintessence and matter is smaller than that of quintessence without coupling to the background matter.

In conclusion, we have investigated the cosmological dynamics of scalar field, quintessence, that couple to the background matter with thermal interactions in exponential potential. The conditions for the existence and stability of various critical points are obtained by phase space analysis. We find that incorporating the effect of thermal coupling between the quintessence and background matter does not qualitatively alter the late-time evolution of the components in the universe. Just as the ordinary situation that there is no direct coupling between quintessence and background barotropic matter [16], for the parameters $\lambda^2 < 18\gamma$, the universe is dominated by quintessence at late time; and for $\lambda^2 > 18\gamma$, quintessence does not entirely dominate the universe but remains a fixed fraction of the total matter at late time. However, we also find that the effects of thermal interaction make the equation-of-state parameter of quintessence become lower than the one in the standard quintessence scenario of dark energy at the early time and depress the energy density of matter simultaneously, see Fig. 3. In other words, large coupling to relic particles such as neutrinos can be ruled out as they lead to a problematic equation-of-state parameter, which is consistent with Ref. [13]. The fact that $\phi$ is probably close to the Planck energy suggests that more interactions should figure in to the behavior of quintessence. However, this will possibly lead to a relic density problem at nucleosynthesis. Finally, it is worth emphasizing that this thermal interaction may be helpful to understand the coincidence of $\Omega_m \sim \Omega_\phi$.

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## References

[1] A.G. Riess, et al., Astron. J. 116 (1998) 1009;
S. Perlmutter, et al., Astrophys. J. 517 (1999) 565;
J.L. Tonry, et al., Astrophys. J. 594 (2003) 1.
[2] M. Tegmark, et al., Astrophys. J. 606 (2004) 702.
[3] D.N. Spergel, et al., Astrophys. J. Suppl. 148 (2003) 175.
[4] P.J.E. Peebles, B. Ratra, Rev. Mod. Phys. 75 (2003) 599;
T. Padmanabhan, Phys. Rep. 380 (2003) 235.
[5] R.R. Caldwell, R. Dave, P.J. Steinhardt, Phys. Rev. Lett. 80 (1998) 1582.
[6] X.-Z. Li, J.G. Hao, D.-J. Liu, Class. Quantum Grav. 19 (2002) 6049.
[7] R.R. Caldwell, Phys. Lett. B 545 (2002) 23;
R.R. Caldwell, M. Kamionkowski, N.N. Weinberg, Phys. Rev. Lett. 91 (2003) 071301.
[8] S.M. Carroll, M. Hoffman, M. Trodden, Phys. Rev. D 68 (2003) 023509.
[9] J.G. Hao, X.-Z. Li, Phys. Rev. D 67 (2003) 107303;
J.G. Hao, X.-Z. Li, Phys. Rev. D 68 (2003) 043501;
J.G. Hao, X.-Z. Li, Phys. Rev. D 70 (2004) 043509;
D.-J. Liu, X.-Z. Li, Phys. Rev. D 68 (2003) 067301.
[10] P. Singh, M. Sami, N. Dadhich, Phys. Rev. D 68 (2003) 023522;
S. Nojiri, S.D. Odintsov, Phys. Lett. B 562 (2003) 147;
S. Nojiri, S.D. Odintsov, Phys. Lett. B 571 (2003) 1;
P.F. Gonzales-Diaz, Phys. Rev. D 68 (2003) 021303;
L.P. Chimento, R. Lazkoz, Phys. Rev. Lett. 91 (2003) 211301;
H. Stefancic, Phys. Lett. B 580 (2004) 5.
[11] A. Melchiorri, L. Mersini, C.J. Odman, M. Trodden, Phys. Rev. D 68 (2003) 043509.
[12] I. Zlatev, L. Wang, P.J. Steinhardt, Phys. Rev. Lett. 82 (1999) 896;
P.J. Steinhardt, L. Wang, I. Zlatev, Phys. Rev. D 59 (1999) 123504.
[13] S. Hsu, B. Murray, Phys. Lett. B 595 (2004) 16.
[14] M. Kamionkowski, J. March-Russell, Phys. Lett. B 282 (1992) 137;
R. Holman, S. Hsu, T.W. Kephart, E.W. Kolb, R. Watkins, J. March-Rusell, Phys. Lett. B 282 (1992) 132;
S.M. Carroll, Phys. Rev. Lett. 81 (1998) 3067.
[15] J.J. Halliwell, Phys. Lett. B 185 (1987) 341.
[16] E.J. Copeland, A.R. Liddle, D. Wands, Phys. Rev. D 57 (1998) 4686.
[17] R.J. van den Hoogen, A.A. Coley, D. Wands, Class. Quantum Grav. 16 (1999) 1843;
I.P.C. Heard, D. Wands, Class. Quantum Grav. 19 (2002) 5435.
[18] X.-Z. Li, J.G. Hao, Phys. Rev. D 69 (2004) 107303;
J.G. Hao, X.-Z. Li, Class. Quantum Grav. 21 (2004) 4771;
J.G. Hao, X.-Z. Li, Phys. Lett. B 606 (2005) 7.
[19] A.P. Billyard, A.A. Coley, R.J. van de Hoogen, Phys. Rev. D 58 (1998) 123501.