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No-go theorems for *R* symmetries in four-dimensional GUTs

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ABSTRACT

We prove that it is impossible to construct a grand unified model, based on a simple gauge group, in four dimensions that leads to the exact MSSM, nor to a singlet extension, and possesses an unbroken R symmetry. This implies that no MSSM model with either a $\mathbb{Z}^R_{M\geqslant 3}$ or $\mathrm{U}(1)_R$ symmetry can be completed by a four-dimensional GUT in the ultraviolet. However, our no-go theorem does not apply to GUT models with extra dimensions. We also show that it is impossible to construct a 4D GUT that leads to the MSSM plus an additional anomaly-free symmetry that forbids the μ term.

1. Introduction

The scheme of supersymmetric grand unification provides an attractive framework for physics beyond the standard model (SM) of particle physics. Apart from the observation that gauge couplings seem to unify at a scale of a few times 10^{16} GeV [1] in the minimal supersymmetric standard model (MSSM), the structure of matter hints at unification. SM matter comes in three copies of $\mathbf{10}\oplus\overline{\mathbf{5}}$ representations under

$$\mathrm{SU}(5)\supset\mathrm{SU}(3)_\mathrm{C}\times\mathrm{SU}(2)_\mathrm{L}\times\mathrm{U}(1)_\mathrm{Y}=G_\mathrm{SM},\tag{1.1}$$

or, after introducing the right-handed neutrino, in form of three **16**-plets of SO(10). Arguably, the most compelling explanation of the smallness of neutrino masses is due to the see-saw mechanism [2], which also appears to require the see-saw scale to be close to M_GUT. However, despite all these hints the scheme of grand unified theories (GUTs) does not yet provide us with a clear picture. For instance, typical obstacles encountered when constructing GUTs in four dimensions include the so-called doublet–triplet splitting problem, i.e. the question why Higgs fields appear in split multiplets, and, associated to it, the prediction of too fast proton decay.

While, arguably, all known proposals for doublet–triplet splitting in four-dimensional (4D) GUTs have some weak points, up to now there exists no argument for why this is necessarily the case. One purpose of this Letter is to give such an argument.

Suppose there is indeed a doublet–triplet splitting mechanism which can be completely understood in terms of 4D physics. Then one should be able to understand in the effective theory why the μ term essentially vanishes. If the smallness of the μ term is to be 'natural' (in 't Hooft's sense [3]), there has to be a symmetry that forbids it. On the other hand, it has been shown that, if one demands consistency with grand unification and anomaly freedom, then only R symmetries may forbid the μ term [4] (cf. also the somewhat similar discussion in [5]). It therefore appears that, if one is to solve the μ problem in 'a natural way', R symmetries are instrumental.

However, we shall prove that for a spontaneously broken GUT symmetry (based on a simple Lie group) in four dimensions one cannot get the exact MSSM with residual R symmetries. This allows us to conclude that a 'natural' solution to the doublet–triplet problem is not available in four dimensions. Our proof applies to singlet extensions of the MSSM as well and, in what follows, we will use the abbreviation MSSM also for these singlet extensions.

This Letter is organized as follows. We will start with the special case of a $\mathrm{SU}(5)\times\mathbb{Z}^R_M$ in Section 2 and extend the result obtained there to more general cases in Section 2.2. Implications of our no-go theorem for model building are discussed in Section 3. Section 4 is devoted to the question of circumventing our no-go theorem in extra dimensions while Section 5 contains our summary.

2. No *R* symmetries from 4D GUTs

This section is devoted to the proof that it is impossible to construct a GUT (based on a simple gauge group) in four dimensions with a finite number of multiplets that leads to the MSSM (or any of its singlet extensions) with a residual R symmetry. While our discussion is based on Abelian discrete R symmetries, denoted by $\mathbb{Z}^R_M$ in what follows, it also applies to continuous, i.e. $\mathrm{U}(1)_R$, symmetries because in this case one can always resort to a discrete

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subgroup $\mathbb{Z}_M^R \subset \mathrm{U}(1)_R$. In our analysis, we focus on discrete $\mathbb{Z}_M^R$ symmetries with $M \geqslant 3$.[1] Our conventions are such that the superpotential carries R charge 2. We start by discussing 4D SU(5) GUTs in Section 2.1, and then consider generalizations in Section 2.2.

2.1. Massless exotics vs. unbroken $\mathbb{Z}_M^R$ in 4D SU(5) GUTs

Consider the MSSM with an additional $\mathbb{Z}_M^R$ symmetry, i.e. the symmetry group of the model is $G_{\mathrm{SM}} \times \mathbb{Z}_M^R$ (possibly amended by further symmetries). We can then ask whether this symmetry group can emerge from an SU(5) GUT by spontaneous breaking; the extension to larger GUT groups is deferred to Section 2.2.2. Since there is a residual $\mathbb{Z}_M^R$ symmetry, the symmetries at the GUT level have to contain $\mathbb{Z}_M^R$ as a subgroup. Without loss of generality, we can base our discussion on a GUT with $\mathrm{SU}(5) \times \mathbb{Z}_M^R$ symmetry, although the actual (R and/or non-R) symmetry before spontaneous breaking might be larger (and lead to stronger conditions than we need). In other words, in case there is actually a larger symmetry group above the GUT scale, the charges we will refer to will always be the ones of the $\mathbb{Z}_M^R$ subgroup.

We proceed by classifying the GUT multiplets $\mathbf{R}$ according to their $\mathbb{Z}_M^R$ charges. Most mass terms between such multiplets are prohibited by $\mathbb{Z}_M^R$. For our purposes it will be sufficient to focus on the subsector of fields with charges 0 and 2. Particles of the latter two types can only have mass terms of the form $\mathcal{M}(m, \langle H_0\rangle, \Lambda)\Psi_0\Phi_2$, where the subscripts denote the R charges and $\mathcal{M}$ is an arbitrary scalar function of SU(5) invariant mass parameters m and SU(5) breaking VEVs $\langle H_0\rangle$ (and a 'cut-off' scale Λ).

In what follows, we will show that it is impossible to:

1. spontaneously break $\mathrm{SU}(5) \to G_{\mathrm{SM}}$ by assigning a VEV to a suitable representation,
2. keep the R symmetry unbroken and to
3. avoid extra massless G_{SM} charged representations

at the same time. We will present our analysis in two steps. First, we focus on the simplest possibility of spontaneously breaking $\mathrm{SU}(5) \times \mathbb{Z}_M^R \to G_{\mathrm{SM}} \times \mathbb{Z}_M^R$ by giving a VEV to a **24**-plet and allowing only for further **24**-plets as mass partners in the model. This setting already illustrates the crucial point of our proof, namely the obstruction to decouple unwanted exotics. In the second step, we will discuss the general case where the symmetry is broken by an arbitrary reducible representation and where we allow for arbitrary further representations to render all exotics massive.

2.1.1. Breaking the GUT symmetry using only 24-plets

Since we wish to leave $\mathbb{Z}_M^R$ unbroken, the **24**-plet that is supposed to break the GUT symmetry to the SM group has to carry $\mathbb{Z}_M^R$ charge 0. The branching rule for **24** is

$$\mathbf{24} = (\mathbf{8},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{3})_0 \oplus (\mathbf{1},\mathbf{1})_0 \oplus (\mathbf{3},\mathbf{2})_{-5/6} \oplus (\bar{\mathbf{3}},\mathbf{2})_{5/6}. \tag{2.1}$$

In the course of spontaneous symmetry breakdown the last two SM representations $(\mathbf{3},\mathbf{2})_{-5/6} \oplus (\bar{\mathbf{3}},\mathbf{2})_{5/6}$ get absorbed in the longitudinal components of the extra gauge bosons. However, we are now left with chiral superfields transforming as $(\mathbf{8},\mathbf{1})_0$ and $(\mathbf{1},\mathbf{3})_0$ and carrying R charge 0. The crucial observation here is that the mass term $m\mathbf{24}\mathbf{24}$ for the adjoint is forbidden: although the **24**-plet is a real SU(5) representation the mass term is prohibited by the $\mathbb{Z}_M^R$ symmetry because $0 + 0 \not\equiv 2 \pmod{M}$. Therefore, in order to give masses to the extra $(\mathbf{8},\mathbf{1})_0$ and $(\mathbf{1},\mathbf{3})_0$ fields, we would have to introduce further fields furnishing the same representations and carrying R charge 2. Yet we cannot simply introduce these desired SM representations, rather we have to add complete SU(5) multiplets. That is, we have to introduce one or more multiplets that contain $(\mathbf{8},\mathbf{1})_0$ and $(\mathbf{1},\mathbf{3})_0$ and carry R charge 2. Here, in the first step, we consider the possibility to add a **24**-plet with R charge 2, the **24** being the smallest multiplet containing $(\mathbf{8},\mathbf{1})_0$ and/or $(\mathbf{1},\mathbf{3})_0$. While this, in principle, allows us to write mass terms for $(\mathbf{8},\mathbf{1})_0$ and $(\mathbf{1},\mathbf{3})_0$, we are now left with extra chiral fields transforming as $(\mathbf{3},\mathbf{2})_{-5/6}$ and $(\bar{\mathbf{3}},\mathbf{2})_{5/6}$ and carrying R charge 2. Now of course, we may add another **24**-plet with R charge 0, but this will lead us just back to the problem we started with: extra massless $(\mathbf{8},\mathbf{1})_0$ and $(\mathbf{1},\mathbf{3})_0$ representations. So we conclude that adding an arbitrary but finite number of **24**-plets with R charges 0 or 2 cannot solve the problem; we will always obtain massless exotics.

2.1.2. General case

Could one rectify the situation by introducing representations different from **24** as the GUT breaking Higgs and as mass partners? In what follows, we will show that this is not the case.

Instead of using just a **24** to break SU(5) to the SM, we will use an arbitrary, finite, possibly reducible representation $\mathbf{H}_0$, such as one or several **75**-plets. This representation $\mathbf{H}_0$ has to fulfill two requirements:

(i) it has $\mathbb{Z}_M^R$ charge 0 (as suggested by the subscript) in order to leave this symmetry unbroken and
(ii) it lies within the congruence class [9,10] of the **24**.

The second property must hold because only SM singlet components of SU(5) representations may attain VEVs. They can only originate from the adjoint congruence class because of the following reasoning. The decomposition of an SU(5) representation into SM representations can be accomplished by using an invertible projection matrix P [10, Section 6],

$$P = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ -1/3 & 1/6 & -1/6 & 1/3 \end{pmatrix}, \tag{2.2}$$

which maps SU(5) weights (in the Dynkin basis) to the corresponding SM weights. SU(5) has five congruence classes of mutually disjoint weight lattices. Using the projection P, one can map the weight lattice of each congruence class onto an equivalence class of SM weights. (Of course, there are further SM representations that do not fit into complete SU(5) multiplets.) These equivalence classes are disjoint. The SM singlet lies in the class that originates from the SU(5) congruence class of the adjoint, and also the representation $(\mathbf{3},\mathbf{2})_{-5/6} \oplus (\bar{\mathbf{3}},\mathbf{2})_{5/6}$, which is needed to make the extra SU(5) gauge bosons massive, lies in the same class. Hence we can choose $\mathbf{H}_0$ from the adjoint SU(5) congruence class. Of course, there may be additional fields with R charge 0 that do not obtain a VEV and therefore do not have to be in this class, but they do not interfere with the following arguments.

In order to arrive at the precise SM spectrum, we allow for an additional finite, possibly reducible representation $\mathbf{R}_2$ with R charge 2. All non-trivial SM representations contained in $\mathbf{H}_0$ except for $(\mathbf{3},\mathbf{2})_{-5/6} \oplus (\bar{\mathbf{3}},\mathbf{2})_{5/6}$ have to obtain masses by pairing up with representations in $\mathbf{R}_2$ in order to avoid massless exotics. However, in the following we will show that this is impossible.

[1] Discrete R symmetries of order two are no 'true' R symmetries since any global supersymmetric theory possesses a symmetry under which the superspace coordinates transform as $\theta \to -\theta$ and all spin-1/2 fermions get multiplied by -1. In particular, using this 'automatic' symmetry one can easily convince oneself that the so-called R parity of the MSSM [6] is equivalent to matter parity [7] (cf. also the discussion in [8]).

Without loss of generality, we can restrict $\mathbf{R}_2$ to belong to the SU(5) congruence class of the adjoint. This is because the complex conjugates of representations from the adjoint congruence class lie in the same class. Therefore no SM representation coming from a different congruence class can pair up with representations coming from $\mathbf{H}_0$. Clearly, one can also remove those representations from $\mathbf{H}_0$ and $\mathbf{R}_2$ for which one can write down $\mathrm{SU}(5)\times\mathbb{Z}_M^R$ invariant mass terms. For notational simplicity we will call the remaining representations again $\mathbf{H}_0$ and $\mathbf{R}_2$, respectively.

Now we take the highest weights Λ_0 from $\mathbf{H}_0$ and Λ_2 from $\bar{\mathbf{R}}_2$, respectively.[2] They cannot be equal because otherwise the corresponding representations could pair up and, therefore, would have been removed in the previous step. Thus we arrive at two cases: (i) $\Lambda_0 > \Lambda_2$ and (ii) $\Lambda_2 > \Lambda_0$.

Case 1: $\boldsymbol{\Lambda_0 > \Lambda_2}$. Using the projection matrix P, the SU(5) representation with highest weight $\Lambda_0 = (a_1, a_2, a_3, a_4)$ introduces an SM representation $\mathbf{r} = \mathbf{r}(P(\Lambda_0))$ with highest weight $P(\Lambda_0) = P\cdot(a_1, a_2, a_3, a_4)^{\mathrm{T}}$. In what follows, we will show that (i) $\mathbf{r}$ is neither any of the desired representations $(\mathbf{3},\mathbf{2})_{-5/6}$ or $(\bar{\mathbf{3}},\mathbf{2})_{5/6}$ nor (ii) SM matter nor (iii) an SM singlet and that (iv) it cannot pair up with any partner from $\mathbf{R}_2$.

(i) Since P establishes a one–to–one correspondence between SU(5) and SM weights, we can use its inverse to calculate the inverse image of the highest weight of $(\mathbf{3},\mathbf{2})_{-5/6} = ((1,0),(1),-5/6)$,

$$P^{-1}\cdot(1,0,1,-5/6)^{\mathrm{T}} = (1,0,1,-1)^{\mathrm{T}}, \tag{2.3}$$

which is not a highest weight of SU(5). Hence it cannot be equal to Λ_0. The same holds for the highest weight of $(\bar{\mathbf{3}},\mathbf{2})_{5/6}$. Therefore $\mathbf{r}\neq(\mathbf{3},\mathbf{2})_{-5/6}$ and $\mathbf{r}\neq(\bar{\mathbf{3}},\mathbf{2})_{5/6}$.

(ii) $\mathbf{r}$ can also not be part of the SM matter content because $\mathbf{H}_0$ was chosen to be in the congruence class of the adjoint and matter originates from different classes.

(iii) Furthermore, we can exclude the case that $\mathbf{r}$ is an SM singlet because otherwise the highest weight would be $\Lambda_0 = (0,0,0,0)$ and we could not give masses to the extra SU(5) gauge bosons.

(iv) In addition to that, there is no chance that $\mathbf{r}$ can pair up because the necessary partner $\bar{\mathbf{r}}$ is not contained in $\mathbf{R}_2$. This is true because, by assumption, the highest weight Λ_2 of $\bar{\mathbf{R}}_2$ is smaller than Λ_0.

Therefore $\mathbf{r}$ is a massless, SM charged exotic.

Case 2: $\boldsymbol{\Lambda_2 > \Lambda_0}$. For analogous reasons as in case 1, $\mathbf{r}' = \overline{\mathbf{r}(P(\Lambda_2))}$ can neither be SM matter nor a singlet nor can it pair up with a partner from $\mathbf{H}_0$ to obtain a mass. As it originates from $\mathbf{R}_2$, its R charge is 2 and therefore it can also not be used to give masses to the extra SU(5) gauge bosons. Again we are left with at least one massless, SM charged exotic in representation $\mathbf{r}'$.

Altogether we have seen that, if one wants to break SU(5) to the SM with a finite number of multiplets while leaving a $\mathbb{Z}_M^R$ unbroken, one will necessarily obtain massless, SM charged exotics.

Let us illustrate the main point with an easy example, based on $\mathbf{H}_0 = \mathbf{24}$ and $\mathbf{R}_2 = \mathbf{75}$ (such that $\bar{\mathbf{R}}_2 = \mathbf{R}_2$). The highest weights of the two sets are $\Lambda_0 = (1,0,0,1)$ and $\Lambda_2 = (0,1,1,0)$, respectively. Out of the two highest weights, Λ_2 is the higher one and we are left with a massless, SM charged field $\mathbf{r}(P(0,1,1,0)) = (\mathbf{8},\mathbf{3})_0$ with R charge 2. There are, of course, further massless exotic states.

At this point, a remark is in order. The restriction to a finite number of multiplets is crucial for our proof. Our analysis is, in this sense, very similar to the one by Goodman and Witten [11], where obstructions for building 4D GUT models with a finite number of multiplets have been identified. As discussed in [11] and as we shall see explicitly in Section 4, in theories with compact extra dimensions, which from a 4D perspective appear to have infinitely many states, our no-go theorem does not apply.

2.2. Further no-go theorems

It is straightforward to extend the no-go theorem to the case of singlet extensions of the MSSM as well as to GUTs with gauge groups containing SU(5) as a subgroup (such as SO(10)).

2.2.1. No-go for singlet extensions of the MSSM

As already mentioned in the introduction, our arguments also apply to the case of singlet extensions of the MSSM. This is because the presence of additional singlets cannot lead to a decoupling of the charged states. Therefore we will still be left with charged light states beyond the MSSM spectrum.

2.2.2. No-go for GUTs with simple gauge group $G\supset\mathrm{SU}(5)$

In the case of a GUT with simple gauge group G containing SU(5) as a subgroup, the multiplets will become larger and the constraints derived in Section 2 get tighter. To see this, one can decompose all representations of G into irreducible representations with respect to the SU(5) subgroup. Adding representations of G can therefore not circumvent our no-go theorem. One may now wonder whether the extra gauge bosons from $G/\mathrm{SU}(5)$ may provide mass partners for the unwanted exotics discussed in Section 2.1.2. However, these gauge bosons come in SU(5) congruence classes which are different from the one containing the adjoint. This is because the difference between weights of extra gauge bosons and a weight of a representation in the SU(5) adjoint congruence class is not an SU(5) root. (For instance, in $\mathrm{SO}(10)/\mathrm{SU}(5)$ one has extra gauge bosons transforming as $\mathbf{10}\oplus\overline{\mathbf{10}}$ while in the case of SU(6) one gets extra $\mathbf{5}\oplus\bar{\mathbf{5}}$ states.) Therefore the extra gauge bosons from $G/\mathrm{SU}(5)$ cannot pair up with the unwanted exotics discussed in Section 2.1.2 and thus cannot interfere with our proof. Hence our no-go theorem from Section 2 applies to the case of a GUT based on a simple group $G\supset\mathrm{SU}(5)$ as well. In particular, it also holds in the case of $G = \mathrm{SO}(10)\supset\mathrm{SU}(5)\times\mathrm{U}(1)_\chi$ and therefore excludes R symmetries in another important class of 4D GUT models.

3. Implications for model building

As already stated in the introduction, assuming that (in the effective MSSM theory) matter is contained in GUT multiplets and that the theory is anomaly-free, only R symmetries can control the μ term [12, Section 2.1]. (The role of anomaly-free discrete R symmetries in controlling the μ parameter has also been discussed earlier in [13]). Yet, as we have shown, such symmetries are not available if the MSSM is to be completed by a 4D GUT. Therefore it is not possible to obtain a 'natural' (in 't Hooft's sense), i.e. symmetry-based, solution to the doublet–triplet splitting problem in four dimensions.

This applies in particular to the five $\mathbb{Z}_M^R$ symmetries recently discussed in [12]. These are the only family-independent, anomaly-free symmetries for the MSSM which (i) commute with SU(5) in the matter sector, (ii) forbid the μ term at tree level, (iii) allow for the usual Yukawa couplings and the dimension five neutrino

[2] The term highest weight can be defined using the following ordering: $\lambda > \mu$ if and only if the first non-zero coefficient n_i in the expansion $\lambda - \mu = \sum_i n_i\alpha_i$, where $n_i\in\mathbb{N}_0$ and α_i are the simple roots, is greater than zero.

mass operator and (iv) suppress proton decay. Our no-go theorems tell us that these symmetries, providing simple and simultaneous solutions to the μ and proton decay problems, are not available in 4D GUT model building.

4. $\mathbb{Z}_4^R$ MSSM from GUTs in extra dimensions

As mentioned above, our no-go theorems do not apply in the presence of extra dimensions, where new ways of GUT symmetry breaking arise [14,15]. Let us discuss the case of breaking by a discrete Wilson line. This Wilson line breaks the GUT symmetry in the same way as an adjoint VEV would do, i.e. $\mathrm{SU}(5) \to G_{\mathrm{SM}}$ in the phenomenologically interesting case. However, a $\mathbb{Z}_2$ (or more generally a $\mathbb{Z}_N$) Wilson line is quantized. Hence there are no continuous deformations (i.e. $(\mathbf{8},\mathbf{1})_0$ or $(\mathbf{1},\mathbf{3})_0$ fields). From the 4D point of view, the symmetry breaking is not spontaneous. Or, adopting the point of view suggested in [11], there are infinitely many representations such that each of the unwanted states can find a mass partner to pair up. Therefore this mechanism evades our no-go theorems.

Wilson line breaking of the GUT symmetry has been implemented in the context of MSSM Calabi–Yau compactifications [16,17]. More recently, it has also been realized in heterotic orbifold compactifications [18]. At this point it is worthwhile to point out that there is a slightly confusing terminology. What is traditionally called a "discrete Wilson line on an orbifold" [19] is in fact a discrete Wilson line on the underlying torus and a difference between "local shifts" on the orbifold (see [20] for an explanation of local shifts). An appealing feature of the orbifold models is that there the discrete R symmetries are not imposed by hand, rather they originate from the Lorentz symmetry of compact dimensions [21], and their appearance can be related to the fact that orbifolds are highly symmetric compactifications. More importantly for phenomenology, it has been demonstrated explicitly that the remnant $\mathbb{Z}_M^R$ symmetries can be of the type discussed above. Specifically, in a global $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold model vacua with the precise MSSM spectrum and a residual $\mathbb{Z}_4^R$ symmetry have been identified. This $\mathbb{Z}_4^R$ is the unique $\mathbb{Z}_M^R$ symmetry for which the discrete matter charges commute with SO(10) [4]. It forbids the μ term and dimension five proton decay operators at tree level, and contains matter parity as a subgroup. In summary, we see that grand unified theories in extra dimensions allow us to circumvent our no-go theorems (and, arguably, provide us with the most compelling way of doublet–triplet splitting). In the context of heterotic orbifolds it is rather straightforward to realize the phenomenologically attractive $\mathbb{Z}_4^R$ in MSSM vacua, and, moreover one obtains a simple geometric intuition for how this discrete R symmetry emerges.

5. Summary

We have shown that 4D GUTs cannot provide an ultraviolet completion of the MSSM with a residual $\mathbb{Z}_{M\geqslant 3}^R$ symmetry, nor with a continuous R symmetry. These theories fail because, as we demonstrated, one will necessarily have additional SM charged states at low energies.

Given that, assuming (i) matter charges that commute with SU(5) and (ii) anomaly freedom, only R symmetries can forbid the μ term in the MSSM, we have argued that it is not possible to obtain a 'natural', i.e. symmetry-based, solution to the doublet–triplet problem in four dimensions. In particular, none of the five generation-independent, anomaly-free discrete R symmetries, which forbid the μ term and suppress proton decay in the MSSM, can be implemented in a 4D GUT (based on a simple gauge group).

On the other hand, as we have discussed, higher-dimensional models of grand unification (with an explicit string completion) can give us precisely the MSSM with a residual $\mathbb{Z}_4^R$ symmetry. In such models, the doublet–triplet splitting has a very simple solution and the μ parameter is related to the gravitino mass. In these constructions, the discrete R symmetries are not imposed by hand, rather they originate from the Lorentz symmetry of compact dimensions.

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