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Neutrino lumps in quintessence cosmology

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ABSTRACT

Neutrinos interacting with the quintessence field can trigger the accelerated expansion of the Universe. In such models with a growing neutrino mass the homogeneous cosmological solution is often unstable to perturbations. We present static, spherically symmetric solutions of the Einstein equations in the same models. They describe astrophysical objects composed of neutrinos, held together by gravity and the attractive force mediated by the quintessence field. We discuss their characteristics as a function of the present neutrino mass. We suggest that these objects are the likely outcome of the growth of cosmological perturbations.

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The mechanism responsible for the onset of the accelerating phase in quintessence cosmology remains undetermined. Explaining the emergence of an accelerating phase in recent cosmological times constitutes one of the most difficult challenges of quintessence models—the coincidence problem. A possible trigger for the acceleration has been proposed recently [1,2], arising through the interaction of the quintessence field with a matter component whose mass grows with time. This matter component may be identified with neutrinos [1–3]. In the proposed scenario the neutrinos remain essentially massless until recent times. When their mass eventually grows close to its present value, their interaction with the quintessence field (the cosmon) almost stops its evolution. The potential energy of the cosmon becomes the dominant contribution to the energy density of the Universe. Cosmological acceleration ensues.

For the coupled neutrino–cosmon fluid the squared sound speed c_s^2 may become negative—a signal of instability [3]. Indeed, the sign of c_s^2 oscillates in the accelerating phase for one of the proposed models [2]. A natural interpretation of this instability is that the Universe becomes inhomogeneous with the neutrinos forming denser structures. Within the linear approximation the neutrino fluctuations can be followed in these models until a redshift around one, when the neutrino overdensities become non-linear [4]. One suspects that some form of subsequent collapse of these fluctuations will result into bound neutrino lumps. In this Letter we present static, spherically symmetric solutions of the Einstein equations that describe such structures and study their characteristics. Astrophysical objects composed of neutrinos have also been studied in [5,6].

We assume that the energy density of the Universe involves a gas of weakly interacting particles (neutrinos). The mass m of the particles depends on the value of a slowly varying cosmon field ϕ [7]. For the field equation

$$\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\mu} \left(\sqrt{-g} g^{\mu\nu} \frac{\partial \phi}{\partial x^\nu} \right) = \frac{dU}{d\phi} - \frac{1}{m} \frac{dm(\phi(x))}{d\phi} T^\mu{}_\mu, \quad (1)$$

we approximate the neutrino energy–momentum tensor as $T^\mu{}_\nu = \text{diag}(-\rho, p, p, p)$. The cosmology of [1,2] also assumes the presence of another gas of particles (dark matter) whose mass is independent of ϕ .

We consider stationary, spherically symmetric configurations, with metric

$$ds^2 = -B(r) dt^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) + A(r) dr^2. \quad (2)$$

For the neutrinos we assume a Fermi–Dirac distribution, with locally varying density—the Thomas–Fermi approximation. The local chemical potential satisfies $\mu(r) = \mu_0/\sqrt{B(r)}$ [6,8]. Stable configurations are prevented from collapsing by the pressure generated through the exclusion principle. We concentrate on vanishing temperature of the neutrino gas. We do not expect qualitative changes of our solution for a non-zero temperature. For simplicity we consider one neutrino species, with the generalization to degenerate neutrino masses being straightforward.

We parametrize the particle mass by a dimensionless function \tilde{m} , defined according to $m(\phi) = \sigma \tilde{m}[(\phi - \bar{\phi})/M]$, with σ an arbitrary energy scale and $M = (16\pi G)^{-1/2} \simeq 1.72 \times 10^{18}$ GeV. Here $\bar{\phi}$ is a fixed reference value, close to the present value of the quintessence field. Hence, σ is of the order of the present neutrino mass, in the eV range or somewhat below. For concreteness, we consider a cosmon potential of the form $U(\phi) = C\sigma^4 \exp[-a(\phi - \bar{\phi})/M]$ with $a = \mathcal{O}(1)$. However, the effect of the potential on our solutions is negligible. For this reason, the pre-

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dicted astrophysical objects are largely independent of the form of the potential, and depend mainly on the interaction between dark energy and neutrinos. The present cosmological value of ϕ is given by the requirement that $U(\phi)$ constitute about 3/4 of the critical energy density $U(\phi) \simeq 10^{-11} \text{ (eV)}^4$. The cosmological value of ϕ is taken as the asymptotic value ϕ_{as} of our local solutions for large r , obeying $(\phi_{\text{as}} - \bar{\phi})/M = \bar{\phi}_{\text{as}} \simeq (1/a)[25.3 + \ln C + 4 \ln(\sigma/\text{eV})]$.

The equations of motion become more transparent if we define the dimensionless variables $\tilde{\phi} = (\phi - \bar{\phi})/M$ and $\tilde{r} = \sigma^2 r/M$. All other dimensionful quantities are multiplied with appropriate powers of σ , in order to form dimensionless quantities denoted as tilded. We use $\tilde{B} = B/\tilde{\mu}_0^2 = B\sigma^2/\mu_0^2$ and $\tilde{\mu}(\tilde{r}) = 1/\sqrt{\tilde{B}(\tilde{r})}$. We define the radius \tilde{R} of the compact object by the value of \tilde{r} at which the fermionic density becomes negligible. The physical radius is

$$R/\text{Mpc} \simeq 1.1 \times 10^{-2} (\sigma/\text{eV})^{-2} \tilde{R}.$$

The mass of the object is given by its Schwarzschild radius \tilde{R}_s . For $\tilde{r} \rightarrow \infty$ we have $B = 1/A = 1 - \tilde{R}_s/\tilde{r}$. In units of the solar mass, the mass of the neutrino lump is

$$M_{\text{tot}}/M_{\odot} \simeq 1.2 \times 10^{17} (\sigma/\text{eV})^{-2} \tilde{R}_s.$$

Another important characteristic is the total neutrino number, which we assume to be conserved. It is

$$N \simeq 5.1 \times 10^{81} (\sigma/\text{eV})^{-3} \tilde{N},$$

with

$$\tilde{N} = \int_0^{\infty} 4\pi \tilde{r}^2 \tilde{n} \sqrt{A} d\tilde{r}.$$

The field equations read [8]

$$\begin{aligned} \tilde{\phi}'' + \left(\frac{2}{\tilde{r}} - \frac{A'}{2A} + \frac{\tilde{B}'}{2\tilde{B}} \right) \tilde{\phi}' \\ = A \left[\frac{d\tilde{U}}{d\tilde{\phi}} + \frac{1}{\tilde{m}} \frac{d\tilde{m}}{d\tilde{\phi}} (\tilde{\rho} - 3\tilde{p}) \right] = -A \frac{d(\tilde{p} - \tilde{U})}{d\tilde{\phi}}, \\ \frac{1}{\tilde{r}^2} \frac{1}{A} - \frac{1}{\tilde{r}^2} - \frac{1}{\tilde{r}} \frac{A'}{A^2} = \frac{1}{2} \left[-\frac{1}{2A} \tilde{\phi}'^2 - \tilde{U}(\tilde{\phi}) - \tilde{\rho} \right], \\ \frac{1}{\tilde{r}^2} \frac{1}{A} - \frac{1}{\tilde{r}^2} + \frac{1}{\tilde{r}} \frac{\tilde{B}'}{BA} = \frac{1}{2} \left[\frac{1}{2A} \tilde{\phi}'^2 - \tilde{U}(\tilde{\phi}) + \tilde{p} \right], \end{aligned} \quad (3)$$

where a prime denotes a derivative with respect to \tilde{r} . We also have

$$\begin{aligned} \tilde{n} &= \frac{1}{3\pi^2} (\tilde{\mu}^2 - \tilde{m}^2)^{3/2}, \\ \tilde{p} &= \frac{1}{24\pi^2} \left[\tilde{\mu} \sqrt{\tilde{\mu}^2 - \tilde{m}^2} (2\tilde{\mu}^2 - 5\tilde{m}^2) \right. \\ &\quad \left. + 3\tilde{m}^4 \ln \left(\frac{\tilde{\mu} + \sqrt{\tilde{\mu}^2 - \tilde{m}^2}}{\tilde{m}} \right) \right], \\ \tilde{\rho} &= \frac{1}{8\pi^2} \left[\tilde{\mu} \sqrt{\tilde{\mu}^2 - \tilde{m}^2} (2\tilde{\mu}^2 - \tilde{m}^2) \right. \\ &\quad \left. - \tilde{m}^4 \ln \left(\frac{\tilde{\mu} + \sqrt{\tilde{\mu}^2 - \tilde{m}^2}}{\tilde{m}} \right) \right], \end{aligned} \quad (4)$$

for $\tilde{\mu} \geq \tilde{m}$, and $\tilde{n} = \tilde{\rho} = \tilde{p} = 0$ for $\tilde{\mu} < \tilde{m}$. Finally, $\tilde{U}(\tilde{\phi}) = C \exp(-a\tilde{\phi})$.

We need four initial conditions for the system of Eqs. (3). Two of them are imposed by the regularity of the solution at $\tilde{r} = 0$: $\tilde{\phi}'(0) = 0$, $A(0) = 1$. The value of $\tilde{B}(0)$ is the only free integration constant. Since $AB(r \rightarrow \infty) = 1$ one has $A\tilde{B}(\tilde{r} \rightarrow \infty) = (\mu_0/\sigma)^{-2}$. As a result, the choice of $\tilde{B}(0)$ determines the chemical potential

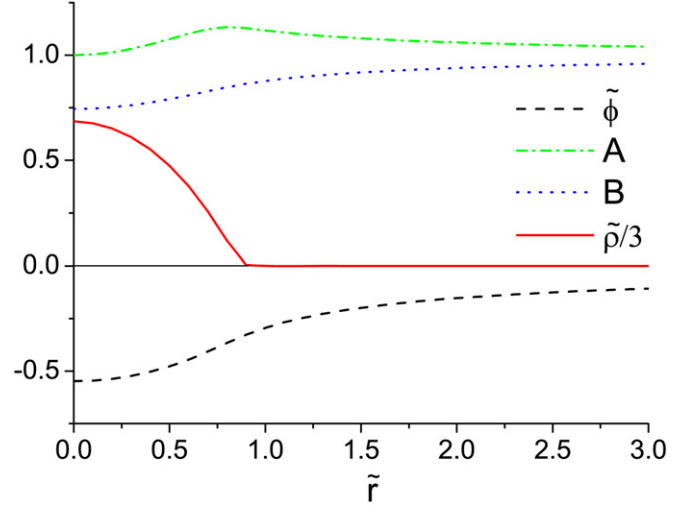


Fig. 1. Radial dependence of $\tilde{\phi}$, $\tilde{\rho}$, A and B for a neutrino lump.

and, therefore, the total number of neutrinos in the lump. Finally, $\tilde{\phi}(0)$ must be chosen so that $\tilde{\phi}(\tilde{r} \rightarrow \infty)$ reproduces correctly the present value $\tilde{\phi}_{\text{as}}$ of the cosmological solution. (We assume that the time scale of the cosmological solution is very large and neglect the time dependence of $\tilde{\phi}(\tilde{r} \rightarrow \infty)$.)

We consider two types of models, distinguished by the dependence of the particle mass on the field:

Model I assumes $\tilde{m}(\tilde{\phi}) = -1/\tilde{\phi}$ [2], with the field $\tilde{\phi}$ taking negative values.

Model II assumes $\tilde{m}(\tilde{\phi}) = \exp(-b\tilde{\phi})$ [1], with $b < 0$. (Notice that $a = \alpha/\sqrt{2}$, $b = \beta/\sqrt{2}$ in comparison to [1], where a different convention for M is used.)

In both cases we are interested in values of the field near $\tilde{\phi} = 0$. For model II we can choose $\tilde{\phi}$ such that $\tilde{\phi}_{\text{as}} = 0$, implying that $\sigma = m_\nu(t_0)$ equals the present neutrino mass. One infers for the quintessence potential $\ln C = -25.3 - 4 \ln(\sigma/\text{eV})$. The parameter a is fixed by requiring that during the early stages of the cosmological evolution the dark energy be subleading and track the radiation or the dark matter. During the radiation and matter dominated epochs, the dark energy follows a “tracker” solution with a constant density parameter $\Omega_{h,\text{early}} = n/(2a^2)$, where $n = 3(4)$ for matter (radiation) [1]. Observations require a to be large, typically $a \gtrsim 7$ [9]. We use $a = 7$ in the following. The future of our Universe is described by a different attractor, for which the dark energy dominates. Our present era coincides with the transition between the two cosmic attractors. The influence of the neutrinos on the evolution of the cosmon field is determined by the second term in the r.h.s. of Eq. (1). Demanding that today this term be equal to the first term, that arises from the potential, fixes the present neutrino fraction to the value $\Omega_\nu(t_0) = -(b/a)\Omega_h(t_0)$ [1]. For a realistic cosmology with present dark energy fraction $\Omega_h(t_0) \simeq 3/4$ one has to adjust b to the neutrino mass. For one dominant neutrino species we have $b = -a(36 \text{ eV}/m_\nu(t_0))$. For model I we need to know how close $\tilde{\phi}_{\text{as}}$ is to zero, with $\sigma = -\tilde{\phi}_{\text{as}} m_\nu(t_0)$. As compared to model II, we have now an effective $\tilde{\phi}$ -dependent $b(\tilde{\phi}) = -1/\tilde{\phi}$, which results in the condition $\tilde{\phi}_{\text{as}} = -(1/a)m_\nu(t_0)/(36 \text{ eV})$ or $\sigma = (1/a)m_\nu^2(t_0)/(36 \text{ eV})$.

In Fig. 1 we present a typical solution describing a static astrophysical object in model I. The chemical potential has the value $\tilde{\mu}_0 \simeq 2.9$. The scalar field becomes more negative near the center of the solution, so that the neutrinos become lighter there. The asymptotic value is $\tilde{\phi}_{\text{as}} = -0.02$, which corresponds to $m_\nu(t_0) \simeq 5 \text{ eV}$. The pressure and density of the fermionic gas vanish for

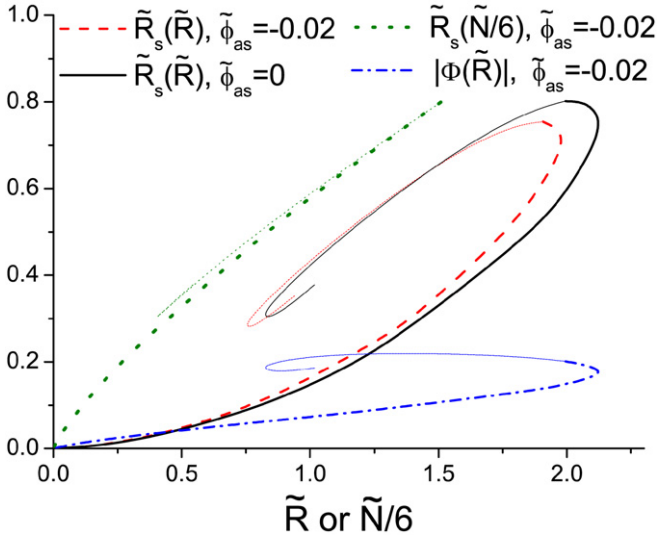


Fig. 2. Mass vs. size for neutrino lumps in model I.

$\tilde{r} \geq \tilde{R} \simeq 0.91$. The mass of the object can be deduced from the asymptotic form of A or B for $\tilde{r} \rightarrow \infty$. We find $\tilde{R}_s \simeq 0.12$. The total fermionic number is $\tilde{N} \simeq 0.88$. The form of the solutions in model II is similar to the one depicted in Fig. 1.

The variation of the chemical potential results in a whole class of solutions, depicted by the solid line in Fig. 2. We display the dimensionless Schwarzschild radius \tilde{R}_s as a function of the dimensionless radius of the object \tilde{R} . There is a maximal value for the mass, denoted by the end of the thick line. The continuation of the curve has the form of a spiral and is depicted by a thinner line. This branch is unstable to perturbations that can lead to gravitational collapse [10]. In order to demonstrate this fact, we plot in the same figure \tilde{R}_s as a function of $\tilde{N}/6$ (dotted line). This curve has two branches. The one depicted by a thinner line corresponds to the thinner line of the curve $\tilde{R}_s(\tilde{R})$. There are two possible values of \tilde{R}_s that correspond to the same value of the total neutrino number \tilde{N} . The value on the thinner line has a larger value of \tilde{R}_s and results in a larger mass. The corresponding configuration is unstable towards one with the same \tilde{N} located on the thicker line. The characteristics of the solutions depend only very mildly on the value of $\tilde{\phi}_{as}$, as demonstrated by the comparison of the solid and dashed curves. All the values of $m_\nu(t_0)$ in the range $[0, 5]$ eV correspond to $\tilde{\phi}_{as}$ in the range $[-0.02, 0]$. The respective $\tilde{R}_s(\tilde{R})$ curves lie between the solid and dashed curves of Fig. 2.

A striking feature is the existence of neutrino lumps with arbitrarily small mass. They correspond to the lower left corner of the figure, where both \tilde{R} and \tilde{R}_s vanish. For such objects the contribution from gravity is negligible and their existence is a consequence of the attractive force mediated by the scalar field. Such configurations are not generic, but depend crucially on the assumed form of $\tilde{m}(\tilde{\phi})$. A completely different form of solutions appears in model II. In Fig. 2 we also depict the gravitational potential $\Phi(\tilde{r}) = -\tilde{R}_s/(2\tilde{r})$ at a distance $\tilde{r} = \tilde{R}$ equal to the radius of the astrophysical object.

The function $\tilde{R}_s(\tilde{R})$ in model II displays a different behaviour. In Fig. 3 we plot this function for four different values of b , namely $b = -500, -50, -4, -1$. Realistic neutrino masses correspond to large, negative b . For $b = -1$ (solid line) there is a maximal value for the mass of the astrophysical objects and a branch of unstable solutions. The maximal value of \tilde{R}_s is comparable for model I and model II with $b = -1$, even though the corresponding radius is larger by an order of magnitude in the second case. For $b = -4$ (dashed line) the maximal value of \tilde{R}_s and the corresponding \tilde{R} increase by roughly two orders of magnitude.

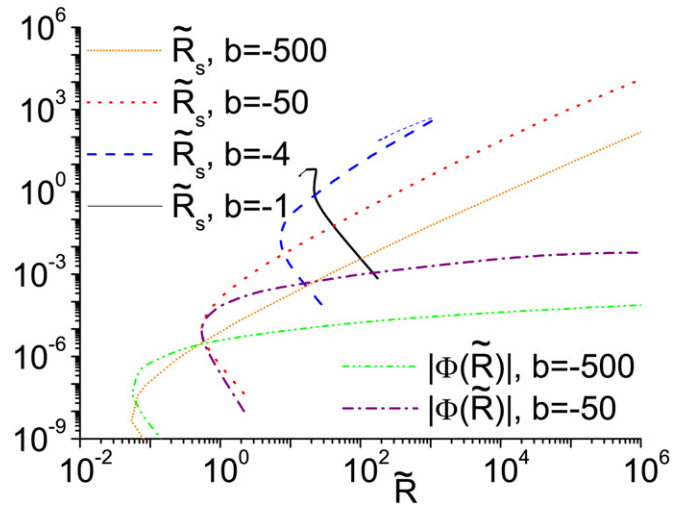


Fig. 3. Same for model II.

The crucial qualitative difference with model I concerns the form of the solutions with low values of \tilde{R}_s . In model I for $\tilde{R}_s \rightarrow 0$ we have $\tilde{R} \rightarrow 0$, while in model II we have $\tilde{R} \rightarrow \infty$. The attractive interaction mediated by the scalar field in model II is not sufficiently strong to lead to bound objects with a small fermion number. Gravity must play a role for compact objects to exist. As $|b|$ increases the dependence of \tilde{m} on $\tilde{\phi}$ becomes more pronounced. The effective neutrino mass in the interior of a compact object can become smaller without a large variation of $\tilde{\phi}$ (and a significant energy cost through the field derivative term). This has two significant effects: (a) Objects with smaller \tilde{N} and \tilde{R} can exist. As a result the bending of the curve $\tilde{R}_s(\tilde{R})$ for low \tilde{R}_s takes place for smaller \tilde{R} . (b) The configurations that are gravitationally unstable (indicated by the spiral in the upper part of the curve) are shifted toward larger values of \tilde{R}_s . The reason is that the neutrinos are essentially massless in the interior of such configurations, carrying only kinetic energy. This makes the gravitational collapse difficult.

The curves $\tilde{R}_s(\tilde{R})$ in model II with $b = -500$ and -50 are also depicted in Fig. 3. We have not managed to determine numerically a maximal value of \tilde{R}_s , as objects with huge values of \tilde{R}_s , \tilde{R} (larger by more than twenty orders of magnitude than the ones depicted) are possible. For comparison we note that in model I we have a maximal value $(\tilde{R}_s)_{\max} = 0.80$ with a corresponding radius $(\tilde{R})_{\max} = 2.0$. In Fig. 3 we observe minimal values of the radius, $(\tilde{R})_{\min} = 0.54$ for $b = -50$ and $(\tilde{R})_{\min} = 0.054$ for $b = -500$. The corresponding values of the Schwarzschild radius are $(\tilde{R}_s)_{\min} = 1.1 \times 10^{-5}$ and $(\tilde{R}_s)_{\min} = 1.1 \times 10^{-8}$, respectively. It is apparent that for small \tilde{R}_s we have the scaling behaviour $\tilde{R} \sim b^{-1}$, $\tilde{R}_s \sim b^{-3}$. This can be understood by noticing that in the limit $A', B' \rightarrow 0$, $A \rightarrow 1$, and for negligible $d\tilde{U}/d\tilde{\phi}$, the factors of b in Eq. (3) can be eliminated through the redefinitions $b\tilde{\phi} \rightarrow \tilde{\phi}$, $b\tilde{r} \rightarrow \tilde{r}$. In Fig. 3 we also depict the surface gravitational potential $\Phi(\tilde{R}) = -\tilde{R}_s/(2\tilde{R})$ as a function of \tilde{R} for the cases $b = -500$ and -50 .

In Fig. 4 we display the size R of the astrophysical objects as a function of the present neutrino mass $m_\nu \equiv m_\nu(t_0)$. Restoring physical units requires the scale σ . We use for model I $a = 7$ or $(\sigma/\text{eV})^{1/2} \simeq 0.063(m_\nu/\text{eV})$. The function $\tilde{R}_s(\tilde{R})$ has a very mild dependence on $\tilde{\phi}_{as}$ for $0 \leq \tilde{\phi}_{as} \leq 0.02$ (see Fig. 2). For given \tilde{R} the variation of σ (or equivalently m_ν) produces a class of astrophysical objects of variable physical size. They all generate the same surface gravitational potential $\Phi = -\tilde{R}_s/(2\tilde{R})$. In Fig. 4 we depict three such classes. The first two contain objects with strong gravitational potentials, while the last one contains objects that

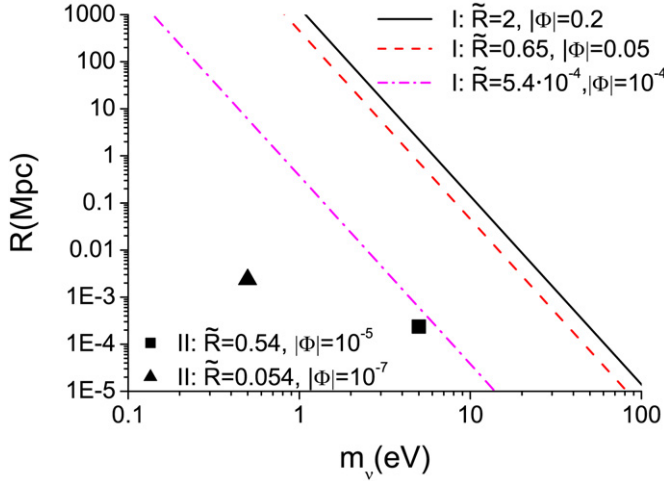


Fig. 4. Size of neutrino lumps as a function of the neutrino mass.

generate weaker fields. Solutions with $\tilde{R} \rightarrow 0$ produce curves parallel to those in Fig. 4, but located closer to the lower left corner. In the same figure we also depict two solutions of model II. In this model the neutrino mass is uniquely determined by the value of b . The two points in Fig. 4 correspond to the minimal values of \tilde{R} for $b = -50$ and -500 . These are $\tilde{R} = 0.54$ and $\tilde{R} = 0.054$, respectively.

Recently, a first investigation of the coupled fluctuations of dark matter, neutrinos, baryons and the cosmon field has been performed for the models within the linear approximation [4]. For a specific model with a present average neutrino mass of 2.1 eV, the neutrino fluctuations grow nonlinear at a redshift around one. The typical size of these fluctuations is large, in the range of superclusters and beyond. A further investigation of the fate of these neutrino lumps will have to follow their collapse due to the scalar-mediated attractive interaction and gravity. This should generate the distribution of the integration constants of the present solution, like the characteristic mass and size of the lumps.

Our study demonstrates that the presence of instabilities in quintessence cosmologies with a variable neutrino mass may have interesting astrophysical consequences. After a sufficiently long time, these instabilities may lead to the formation of stable bound neutrino lumps. Their radius and mass within the family of al-

lowed solutions (for given m_ν) depend on the details of the dynamical formation mechanism. Since in the models of [1,2] the neutrinos remain free streaming until a rather recent cosmological epoch (say, $z = 5$), one may expect a large typical size of the neutrino lumps (more than 100 Mpc). At the present stage of the investigations it is not clear if such lumps have already decoupled from the cosmological expansion—for this, the perturbations have to grow nonlinear—or if this will happen only in the future. In the extreme case of an early formation of a population of lumps with subgalactic size, they could even play the role of dark matter. The detection of lumps could proceed directly through their gravitational potential, or indirectly through their attraction for baryons. Quintessence cosmologies may provide surprises for structures on very large scales.

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