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A new trigonometric shear deformation theory for isotropic, laminated composite and sandwich plates

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ABSTRACT

A new trigonometric shear deformation theory for isotropic and composite laminated and sandwich plates, is developed. The new displacement field depends on a parameter "*m*", whose value is determined so as to give results closest to the 3D elasticity bending solutions. The theory accounts for adequate distribution of the transverse shear strains through the plate thickness and tangential stress-free boundary conditions on the plate boundary surface, thus a shear correction factor is not required. Plate governing equations and boundary conditions are derived by employing the principle of virtual work. The Navier-type exact solutions for static bending analysis are presented for sinusoidally and uniformly distributed loads. The accuracy of the present theory is ascertained by comparing it with various available results in the literature. The results show that the present model performs as good as the Reddy's and Touratier's shear deformation theories for analyzing the static behavior of isotropic and composite laminated and sandwich plates.

1. Introduction

Laminated composite and sandwich structures are extensively used in many fields such as aerospace, naval, automotive and civil industries, for their superior performance and reliability. In last decades the use of sandwich construction grew rapidly around the world. Its advantages, the development of new materials and the necessity of high performance under static, dynamic and thermal loads guarantee that the sandwich structures will be in demand for many years (Vinson, 1999, 2001, 2005). With the increased use of sandwich structures, there is a tremendous need to develop efficient manufacturing techniques, economical and effective repair techniques, and analysis methods to predict the short and long-term behavior of the multilayer composite materials under a variety of loading and environmental conditions.

In the literature, different models were proposed in order to study multilayered composite structures such as equivalent single layer, quasi-layerwise and layerwise models (Demasi, 2009a,b). The heterogeneous laminated plates and shells are treated as a statically equivalent single layer, thus reducing the 3D problem to a 2D one. Another important point in the analysis of composite structures is the variational statement used in the analysis to derive the necessary governing equations and the boundary conditions. Deformation theories can either be developed using displacement-based theories (when the principle of virtual displacement is used), stress based theories or displacement-stress-based theories (see Carrera, 2000, 2001; Demasi, 2006, 2009a,b,c,d,e). The well-described unified formulation, initially presented by Carrera (2003a) and extended by Demasi (2008, 2009a,b,c,d,e), describes precisely and clearly the models, types and class of theories. Recently, Carrera and Petrolo (2011a) using Carrera unified formulation (CUF) and an asymptotic expansion method have discussed the effectiveness of higher-order terms in refined beam theories. In a similar fashion, Carrera et al. (2011b) by using CUF have found the so-called *best plate theory*, but this time considering an axiomatic hypothesis method. More detailed information and applications of CUF can be found in the very recent books authored by Carrera et al. (2011c,d).

According to the generalized unified formulation (GUF) proposed by Demasi (2008, 2009a,b,c,d,e), among equivalent single layer theories, there are many classes of theories. However, there are mainly three well-known major theories; namely the classical lamination theory (CLT), which is based on the assumptions of Kirchhoff's plate theory (for example see Reissner and Stavsky, 1961; Whitney and Leissa, 1970; Ashton, 1970) which neglects the interlaminar shear deformation, the first order shear deformation theory (FSDT) (Dong and Tso, 1972; Chou and Carleone, 1973; Reissner, 1975) assumes constant transverse shear deformation through the entire thickness of the laminate and violates stress free boundary conditions at the top and bottom surfaces of the plate, and more accurate theories such as higher order theories (HSDT) assume quadratic, cubic or higher (also non-polynomial, e.g. trigonometric) variations of surface-parallel displacements through the

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entire thickness of the laminates to model the behavior of the structure for thick to thin regions (Murthy, 1981; Reddy and Liu, 1985; Murakami, 1986; Kant and Swaminathan, 2002; Swaminathan and Ragounadin, 2004).

In fact, few higher order shear deformation theories containing non-polynomial shape strain functions were developed. For example, the unknowns in generalized formulations presented by Carrera (2003a,b), Carrera et al. (2011b) and Demasi (2008, 2009a,b,c,d,e) are expanded along the thickness by using chosen polynomial functions. Demasi (2009a) mentioned that a series of trigonometric functions of the thickness coordinate z could be also used in his generalized unified formulation (GUF). However, according to the authors knowledge, regarding to equivalent single layer theories, there is only one trigonometric shear deformation theory in which the sinus function was firstly developed by Levy (1877), corroborate and assessed by Stein (1986) (after almost one century) and later extensively used by Touratier (1991) and coworkers, see Ghugal (2010) for more details. Therefore, it is still important to exploit the behavior of other trigonometric functions in the implementation of new shear deformation theories, as proposed in this paper.

Equivalent single layer (ESL) theories give a sufficiently accurate description of the global laminate response. However, one important issue that needs to be mentioned in the context of the above mentioned (Ambartsumian, 1958; Reddy and Liu, 1985; Touratier, 1991; Soldatos, 1992; Karama et al., 2009) and present theory is that while the in-plane strains are continuous across the plate thickness, the corresponding stresses have jumps at the layer interfaces. This type of behavior can only be captured by the zig-zag type theories (Seide, 1980; Chaudhuri and Seide, 1987; Chaudhuri, 2005, 2008). Combination of the present approach with the zig-zag theory would yield more accurate results (Carrera, 2004; Demasi, 2004; Brischetto et al., 2009). Surveys of such theories can be found in the works of Murakami (1986), Carrera (2001, 2003b, 2004) and Demasi (2004, 2009d). Furthermore, a layerwise shear deformation theory can also be considered by employing the present theory. Layerwise theories may provide a better representation of interlaminar stresses (continuous transverse stresses at layer interfaces) and moderate to severe cross-sectional warping, thus they allow to analyze the local behavior of laminated structures when needed (e.g. modeling damage, impact, non-linear effects), but they may be computationally too expensive, for more details, see Carrera (2001, 2003a), Carrera and Demasi (2002) and Demasi (2009b).

In the present work, a new trigonometric higher order shear deformation theory in which the displacement of the middle surface expanded as tangential trigonometric functions of the thickness coordinate and the transverse displacement taken to be constant through the thickness is proposed. Necessary equilibrium equations and boundary conditions are derived by employing the principle of virtual work. The theory accounts for adequate distribution of the transverse shear strains through the plate thickness and the tangential stress-free boundary conditions on the plate boundary surface, therefore a shear correction factor is not required.

Exact solutions for deflections and stresses of simply supported sandwich plates are presented by using Navier type solution technique. The accuracy of the present theory is ascertained by comparing it with Pagano (1970), Pagano and Hatfield (1972) and Srinivas (1973), and also with various numerical calculations such as finite element (see Pandya and Kant, 1988; Ferreira and Barbosa, 2000) and meshless methods (Ferreira et al., 2003, 2005; Xiang et al., 2009).

2. Higher order displacement theory

The main purpose of this theory is to developed a simple and accurate new trigonometric shear deformation theory which can also be easily implemented in a new layerwise finite element code. Therefore, for example, other shear deformation theories presented by Karama et al. (2009) and Mantari et al. (2011a,b) are not considered for comparison purposes. An example of what the authors want to achieve can be found in the shear deformation theory developed by Touratier (1991). This model has been used in several higher order layerwise and zig-zag shear deformation theories (Shimpi and Ghugal, 1999, 2001; Arya et al., 2002). Moreover, advanced numerical calculations such as finite element (Shimpi and Aynapure, 2001; Vidal and Polit, 2008, 2011) and meshless methods (Ferreira et al., 2003, 2005, 2011a,b; Xiang et al., 2009; Roque et al., 2005) were also implemented by using a sinus function in the expansion of the HSDT. Therefore, it can be said that there are evidences of the demand of trigonometric shear deformation theories, mainly because they are richer than polynomial functions, simple, more accurate, and the free surface boundary conditions can be guaranteed *a priori*.

For the development of the present shear deformation theory, the following displacement field is assumed:

$$\begin{aligned} \bar{u}(x,y,z) &= u(x,y) - z\frac{\partial w}{\partial x} + f(z)\theta_1(x,y), \\ \bar{v}(x,y,z) &= v(x,y) - z\frac{\partial w}{\partial y} + f(z)\theta_2(x,y), \\ \bar{w}(x,y,z) &= w(x,y), \end{aligned} \tag{1a – c}$$

where $u(x,y)$, $v(x,y)$, $w(x,y)$, $\theta_1(x,y)$ and $\theta_2(x,y)$ are the five unknown functions of middle surface of the plate as given in Fig. 1, while "$f(z)$" represents the shape functions determining the distribution of the transverse shear strains and stresses along the thickness.

Basset (1890) appears to have been the first to suggest that the displacements can be expanded in power series of the thickness coordinate. Following Basset's approach, several HSDTs have been implemented. Surveys of various higher-order shear deformation theories may be found in the works of Idibi et al. (1997), Karama et al. (2003, 2009) and Aydogdu (2009). The shape functions derived by different researchers are chronologically described as follows:

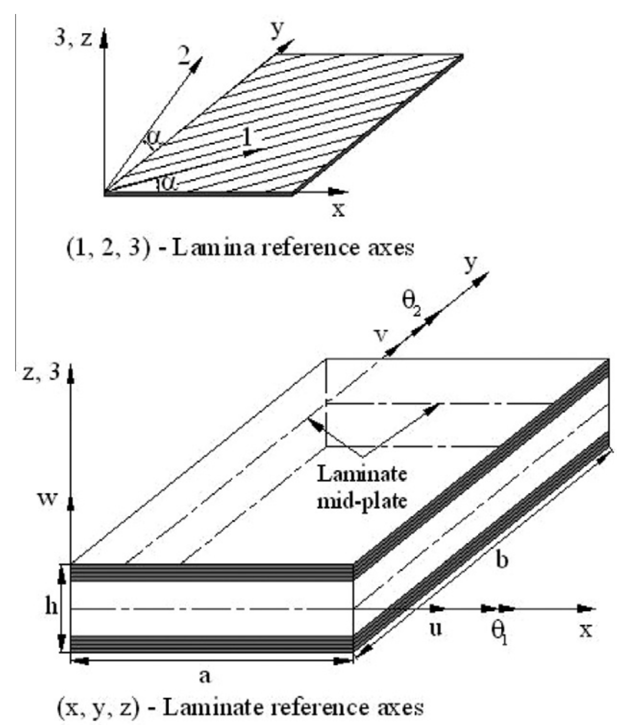


Fig. 1. Laminate geometry with positive set of lamina/laminate reference axes, displacement components and fiber orientation.

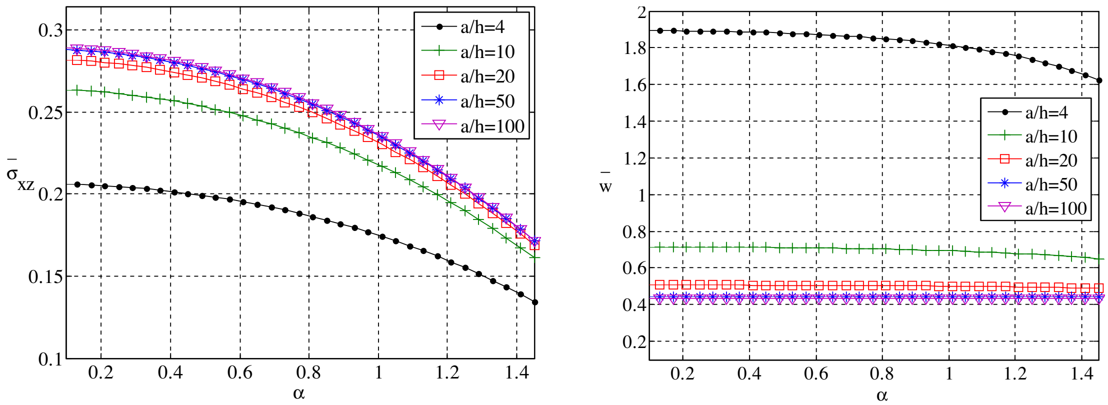


Fig. 2. Variation of shear stress $\bar{\sigma}_{xz}$ at $(0, b/2, 0)$ and center deflection at $(a/2, b/2, 0)$ with parameter "α" for different a/h ratios.

Ambartsumian (1958),

$$f(z) = \frac{z}{2}\left[\frac{h^2}{4} - \frac{z^2}{3}\right], \tag{2}$$

Kaczkowski (1968), Panc (1975) and Reissner (1975),

$$f(z) = \frac{5z}{4}\left[1 - \frac{4z^2}{3h^2}\right], \tag{3}$$

Levinson (1980), Murthy (1981) and Reddy (1984a),

$$f(z) = z\left[1 - \frac{4z^2}{3h^2}\right], \tag{4}$$

Levy (1877), Stein (1986), Touratier (1991),

$$f(z) = \frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right), \tag{5}$$

Soldatos (1992),

$$f(z) = h\sinh\left(\frac{z}{h}\right) - z\cosh\left(\frac{1}{2}\right), \tag{6}$$

Karama et al. (2009) and Aydogdu (2009),

$$f(z) = ze^{-2(z/h)^2} = z\alpha^{-2(z/h)^2/\ln\alpha}, \quad \forall\alpha > 0, \tag{7}$$

Mantari et al. (2011a,b),

$$f(z) = \sin\left(\frac{\pi z}{h}\right)e^{\frac{1}{2}\cos\left(\frac{\pi z}{h}\right)} + \frac{\pi}{2h}z; \quad f(z) = z2.85^{-2(z/h)^2} + y^* z, \quad y^* = \{0, 0.028\}. \tag{8}$$

Following similar procedures as presented by Reddy and Liu (1985) or the generalized procedure developed by Soldatos (1992), the new displacement field presented in here that guaranties the stress free boundary conditions at the top and bottom surfaces of the plate, is given as follows:

$$\begin{aligned}
\bar{u}(x,y,z) &= u(x,y) + z\left[-m\sec^2(m\frac{h}{2})\theta_1 - \frac{\partial w}{\partial x}\right] + \tan mz\theta_1, \\
\bar{v}(x,y,z) &= v(x,y) + z\left[-m\sec^2(m\frac{h}{2})\theta_2 - \frac{\partial w}{\partial y}\right] + \tan mz\theta_2, \\
\bar{w}(x,y,z) &= w.
\end{aligned} \tag{9a–c}$$

Therefore, by comparing the Eqs. (9a,b) with (1a,b), $f(z)$ becomes,

$$f(z) = \tan mz + y^* z, \quad m \geqslant 0, \quad y^* = -m\sec^2(\alpha), \quad \alpha = m\frac{h}{2}. \tag{10}$$

Table 1
Square isotropic plate under uniform load.

Method	a/h	$\bar{w}$ $(a/2, b/2, 0)$	$\bar{\sigma}_{xx}$ $(a/2, b/2, h/2)$
Exact (Reddy, 1984c)	10	4.791	0.276
Present		4.666	0.289
Reddy (1993)		4.770	0.289
Ferreira et al. (2003) ($N = 21$)		4.787	0.278
Ferreira et al. (2005) ($N = 21$)		4.788	0.278
Xiang et al. (Karama) (2009)		4.610	0.288
Xiang et al. (Touratier) (2009)		4.609	0.287
Xiang et al. (Reddy) (2009)		4.612	0.288
Exact (Reddy, 1984c)	20	4.570	0.268
Present		4.494	0.288
Reddy (1993)		4.570	0.268
Ferreira et al. (2003) ($N = 21$)		4.613	0.276
Ferreira et al. (2005) ($N = 21$)		4.616	0.277
Xiang et al. (Karama) (2009)		4.433	0.287
Xiang et al. (Touratier) (2009)		4.442	0.286
Xiang et al. (Reddy) (2009)		4.440	0.285
Exact (Reddy, 1984c)	50	4.496	0.267
Present		4.445	0.287
Reddy (1993)		4.496	0.266
Ferreira et al. (2003) ($N = 21$)		4.575	0.276
Ferreira et al. (2005) ($N = 21$)		4.578	0.276
Xiang et al. (Karama) (2009)		4.391	0.286
Xiang et al. (Touratier) (2009)		4.396	0.292
Xiang et al. (Reddy) (2009)		4.352	0.281

The function $f(z)$ in Eq. (10) is m dependent and therefore it needs to be selected or calculated. Therefore, it is obtained after several computations of the plate governing Eq. (17a–e) to reach: (a) the shear stress "σ_{xz}" at $(0, b/2, 0)$ and (b) center plate deflection $\bar{w}$ at $(a/2, b/2, 0)$ which gives closest results to 3D elasticity bending solutions. Details will be discussed after finding the plate equations.

The linear strain expressions derived from the displacement model of Eq. (9a–c), valid for thin, moderately thick and thick plate under consideration are as follows:

$$\begin{aligned}
\varepsilon_{xx} &= \varepsilon_{xx}^0 + z\varepsilon_{xx}^1 + \tan(mz)\varepsilon_{xx}^2, \\
\varepsilon_{yy} &= \varepsilon_{yy}^0 + z\varepsilon_{yy}^1 + \tan(mz)\varepsilon_{yy}^2, \\
\varepsilon_{yz} &= \varepsilon_{yz}^0 + \sec^2(mz)\varepsilon_{yz}^3, \\
\varepsilon_{xz} &= \varepsilon_{xz}^0 + \sec^2(mz)\varepsilon_{xz}^3, \\
\varepsilon_{xy} &= \varepsilon_{xy}^0 + z\varepsilon_{xy}^1 + \tan(mz)\varepsilon_{xy}^2,
\end{aligned} \tag{11a–e}$$

where

Table 2
Non-dimensionalized deflections and stresses in three-layer (0°/90°/0°) square laminates under sinusoidal load ($b = a$).

Method	a/h	$\overline{w}$ $(a/2,b/2,0)$	$\bar{\sigma}_{xx}$ $(a/2,b/2,h/2)$		$\overline{\sigma}_{yy}$ $(a/2,b/2,h/6)$		$\bar{\sigma}_{xy}$ $(0,0,h/2)$		$\bar{\sigma}_{xz}$ $(0,b/2,0)$		$\bar{\sigma}_{yz}$ $(a/2,0,0)$	
3D-elasticity (Srinivas, 1973)	4	–	0.755	Diff. (%)	0.556	Diff. (%)	−0.051	Diff. (%)	0.282	Diff. (%)	0.217	Diff. (%)
Present		1.9222	0.733	−2.9	0.502	−9.7	−0.050	−1.5	0.202	−28.3	0.183	−15.7
Reddy and Liu (1985)		1.9218	0.734	−2.8	–	–	–	–	–	–	0.183	−15.7
3D-elasticity (Srinivas, 1973)	10	–	0.590	Diff. (%)	0.288	Diff. (%)	−0.029	Diff. (%)	0.357	Diff. (%)	0.123	Diff. (%)
Present		0.7131	0.568	−3.7	0.269	−6.7	−0.028	−4.1	0.244	−31.6	0.103	−15.9
Reddy and Liu (1985)		0.7125	0.568	−3.7	–	–	–	–	–	–	0.103	−15.9
3D-elasticity (Srinivas, 1973)	20	–	0.552	Diff. (%)	0.210	Diff. (%)	−0.023	Diff. (%)	0.385	Diff. (%)	0.094	Diff. (%)
Present		0.5049	0.546	−1.1	0.204	−2.8	−0.023	−1.6	0.254	−33.9	0.08	−12.0
3D-elasticity (Srinivas, 1973)	50	–	0.541	Diff. (%)	0.185	Diff. (%)	−0.022	Diff. (%)	0.393	Diff. (%)	0.084	Diff. (%)
Present		0.4439	0.540	−0.2	0.184	−0.8	−0.022	0.0	0.258	−34.8	0.076	−9.7
3D-elasticity (Srinivas, 1973)	100	–	0.539	Diff. (%)	0.181	Diff. (%)	−0.021	Diff. (%)	0.395	Diff. (%)	0.083	Diff. (%)
Present		0.4351	0.539	0.0	0.181	−0.3	−0.021	0.4	0.258	−34.7	0.075	−9.4
Reddy and Liu (1985)		0.4342	0.539	0.0	–	–	–	–	–	–	0.075	−9.4

$$\varepsilon_{xx}^0 = \frac{\partial u}{\partial x},\quad \varepsilon_{xx}^1 = y^*\frac{\partial \theta_1}{\partial x} - \frac{\partial^2 w}{\partial x^2},\quad \varepsilon_{xx}^2 = \frac{\partial \theta_1}{\partial x},$$
$$\varepsilon_{yy}^0 = \frac{\partial v}{\partial y},\quad \varepsilon_{yy}^1 = y^*\frac{\partial \theta_2}{\partial y} - \frac{\partial^2 w}{\partial y^2},\quad \varepsilon_{yy}^2 = \frac{\partial \theta_2}{\partial y},$$
$$\varepsilon_{xy}^0 = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y},\quad \varepsilon_{xy}^1 = y^*\frac{\partial \theta_2}{\partial x} + y^*\frac{\partial \theta_1}{\partial y} - 2\frac{\partial^2 w}{\partial x \partial y},\quad \varepsilon_{xy}^2 = \frac{\partial \theta_2}{\partial x} + \frac{\partial \theta_1}{\partial y},$$
$$\varepsilon_{xz}^0 = y^*\theta_1,\quad \varepsilon_{xz}^3 = \theta_1.$$
$$\varepsilon_{yz}^0 = y^*\theta_2,\quad \varepsilon_{yz}^3 = \theta_2. \tag{12a–j}$$

By performing the transformation rule of stresses/strain between the lamina and the laminate coordinate system, the stress–strain relations in the global x–y–z coordinate system can be obtained as,

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} & 0 & 0 \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} & 0 & 0 \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \overline{Q}_{55} & \overline{Q}_{54} \\ 0 & 0 & 0 & \overline{Q}_{45} & \overline{Q}_{44} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yx} \end{Bmatrix},$$
$$\{\sigma\} = [\overline{Q}^k]\{\varepsilon\} \tag{13a, b}$$

in which, $\sigma = \{\sigma_{xx}, \sigma_{yy}, \tau_{xy}, \tau_{xz}, \tau_{yz}\}^T$ and $\varepsilon = \{\varepsilon_{xx}, \varepsilon_{yy}, \gamma_{xy}, \gamma_{xz}, \gamma_{yz}\}^T$ are the stresses and the strain vectors with respect to the laminate coordinate system.

Considering the static version of the principle of virtual work, the following expressions can be obtained

$$0 = \left[\int_{-h/2}^{h/2}\left\{\int_{\Omega}^{(k)}\left[\sigma_{xx}\delta\varepsilon_{xx}^{(k)} + \sigma_{yy}\delta\varepsilon_{yy}^{(k)} + \sigma_{xy}\delta\varepsilon_{xy}^{(k)} + \sigma_{yz}\delta\varepsilon_{yz}^{(k)} + \sigma_{xz}\delta\varepsilon_{xz}^{(k)}\right]dxdy\right\}dz\right] - \left[\int_{\Omega} q\delta w dxdy\right], \tag{14}$$

$$0 = \int_{\Omega}\Big(N_1\delta\varepsilon_{xx}^0 + M_1\delta\varepsilon_{xx}^1 + P_1\delta\varepsilon_{xx}^2 + N_2\delta\varepsilon_{yy}^0 + M_2\delta\varepsilon_{yy}^1 + P_2\delta\varepsilon_{yy}^2 + N_6\delta\varepsilon_{xy}^0 + M_6\delta\varepsilon_{xy}^1 + P_6\delta\varepsilon_{xy}^2 + Q_2\delta\varepsilon_{yz}^0 + K_2\delta\varepsilon_{yz}^3 + Q_1\delta\varepsilon_{xz}^0 + K_1\delta\varepsilon_{xz}^3 - q\delta w\Big)dxdy, \tag{15}$$

where q is the load term, N_i, M_i, P_i, Q_i and K_i are the resultants of the following integrations,

Table 3
Non-dimensionalized deflections and stresses in three-layer (0°/90°/0°) square laminates under sinusoidal load ($b = 3a$).

Method	a/h	$\bar{w}$ $(a/2,b/2,0)$		$\bar{\sigma}_{xx}$ $(a/2,b/2,h/2)$		$\bar{\sigma}_{yy}$ $(a/2,b/2,h/6)$		$\bar{\sigma}_{xy}$ $(0,0,h/2)$		$\bar{\sigma}_{xz}$ $(0,b/2,0)$		$\bar{\sigma}_{yz}$ $(a/2,0,0)$		Total average error
3D-elasticity (Srinivas, 1973)	4	2.820	Diff. (%)	1.100	Diff. (%)	0.119	Diff. (%)	−0.0281	Diff. (%)	0.387	Diff. (%)	0.033	Diff. (%)	
Present		2.6415	−6.3	1.034	−6.0	0.103	−13.8	−0.0263	−6.4	0.272	−29.7	0.035	5.5	11.3
Touratier (1991)		2.6657	−5.5	1.067	−3.0	0.103	−13.1	−0.0268	−4.6	0.285	−26.3	0.036	7.6	10.0
Reddy and Liu (1985)		2.6411	−6.3	1.036	−5.8	0.103	−13.6	−0.0263	−6.4	0.272	−29.6	0.035	5.5	11.2
3D-elasticity (Srinivas, 1973)	10	0.9190	Diff. (%)	0.725	Diff. (%)	0.044	Diff. (%)	−0.0123	Diff. (%)	0.420	Diff. (%)	0.015	Diff. (%)	0
Present		0.8631	−6.1	0.692	−4.6	0.040	−8.6	−0.0115	−6.3	0.285	−32.0	0.017	13.2	11.8
Touratier (1991)		0.8698	−5.4	0.698	−3.7	0.040	−7.8	−0.0116	−5.7	0.302	−28.1	0.017	14.7	10.9
Reddy and Liu (1985)		0.8622	−6.2	0.692	−4.5	0.040	−8.5	−0.0115	−6.5	0.286	−31.9	0.017	13.3	11.8
3D-elasticity (Srinivas, 1973)	20	0.6100	Diff. (%)	0.650	Diff. (%)	0.030	Diff. (%)	−0.0093	Diff. (%)	0.434	Diff. (%)	0.012	Diff. (%)	0
Present		0.5948	−2.5	0.641	−1.5	0.029	−3.4	−0.0091	−1.9	0.287	−33.8	0.014	16.3	9.9
Touratier (1991)		0.5958	−2.3	0.642	−1.2	0.029	−3.0	−0.0091	−2.2	0.305	−29.8	0.014	17.5	9.3
Reddy and Liu (1985)		0.5937	−2.7	0.641	−1.4	0.029	−3.3	−0.0091	−2.2	0.288	−33.6	0.014	15.8	9.8
3D-elasticity (Srinivas, 1973)	50	0.5200	Diff. (%)	0.628	Diff. (%)	0.026	Diff. (%)	−0.0084	Diff. (%)	0.439	Diff. (%)	0.011	Diff. (%)	0
Present		0.5190	−0.2	0.626	−0.3	0.026	−0.6	−0.0084	0.4	0.288	−34.4	0.013	18.9	9.1
3D-elasticity (Srinivas, 1973)	100	0.5080	Diff. (%)	0.624	Diff. (%)	0.025	Diff. (%)	−0.0083	Diff. (%)	0.439	Diff. (%)	0.011	Diff. (%)	0
Present		0.5081	0.0	0.624	0.0	0.025	0.0	−0.0083	0.4	0.288	−34.4	0.013	20.0	9.1
Reddy and Liu (1985)		0.5070	0.2	0.624	0.0	0.025	0.0	−0.0083	0.4	0.289	−34.3	0.013	19.4	9.1

$$(N_i, M_i, P_i) = \sum_{k=1}^{N} \int_{z^{(k-1)}}^{z^{(k)}} \sigma_i^{(k)}(1, z, \tan mz)dz, \quad (i = 1, 2, 6)$$
$$(Q_1, K_1) = \sum_{k=1}^{N} \int_{z^{(k-1)}}^{z^{(k)}} \sigma_5^{(k)}(1, m\sec^2 mz)dz, \tag{16a—c}$$
$$(Q_2, K_2) = \sum_{k=1}^{N} \int_{z^{(k-1)}}^{z^{(k)}} \sigma_4^{(k)}(1, m\sec^2 mz)dz.$$

The static governing equations are derived from Eq. (15) by integrating the displacement gradients by parts and setting the coefficients of δu, δv, δw, $\delta\theta_1$, $\delta\theta_2$ to zero separately. The equations obtained are as follows:

$$\begin{aligned}
\delta u &: \quad \frac{\partial N_1}{\partial x} + \frac{\partial N_6}{\partial y} = 0,\\
\delta v &: \quad \frac{\partial N_2}{\partial x} + \frac{\partial N_6}{\partial y} = 0,\\
\delta w &: \quad \frac{\partial^2 M_1}{\partial x^2} + \frac{\partial^2 M_2}{\partial y^2} + 2\frac{\partial^2 M_6}{\partial x \partial y} + p = 0,\\
\delta\theta_1 &: \quad y^* \frac{\partial M_1}{\partial x} + y^* \frac{\partial M_6}{\partial y} + \frac{\partial P_1}{\partial x} + \frac{\partial P_6}{\partial y} + y^* Q_1 - K_1 = 0,\\
\delta\theta_2 &: \quad y^* \frac{\partial M_2}{\partial y} + y^* \frac{\partial M_6}{\partial x} + \frac{\partial P_2}{\partial y} + \frac{\partial P_6}{\partial x} + y^* Q_2 - K_2 = 0.
\end{aligned} \tag{17a—e}$$

By substituting the stress–strain relations into the Eq. (16a–c) the following equations are obtained:

$$\begin{Bmatrix} N_1\\ N_2\\ N_6\\ M_1\\ M_2\\ M_6\\ P_1\\ P_2\\ P_6 \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} & E_{11} & E_{12} & E_{16}\\ A_{21} & A_{22} & A_{26} & B_{21} & B_{22} & B_{26} & E_{21} & E_{22} & E_{26}\\ A_{61} & A_{62} & A_{66} & B_{61} & B_{62} & B_{66} & E_{61} & E_{62} & E_{66}\\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & F_{11} & F_{12} & F_{16}\\ B_{21} & B_{22} & B_{26} & D_{21} & D_{22} & D_{26} & F_{21} & F_{22} & F_{26}\\ B_{61} & B_{62} & B_{66} & D_{61} & D_{62} & D_{66} & F_{61} & F_{62} & F_{66}\\ E_{11} & E_{12} & E_{16} & F_{11} & F_{12} & F_{16} & H_{11} & H_{12} & H_{16}\\ E_{21} & E_{22} & E_{26} & F_{21} & F_{22} & F_{26} & H_{21} & H_{22} & H_{26}\\ E_{61} & E_{62} & E_{66} & F_{61} & F_{62} & F_{66} & H_{61} & H_{62} & H_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^0\\ \varepsilon_{yy}^0\\ \varepsilon_{xy}^0\\ \varepsilon_{xx}^1\\ \varepsilon_{yy}^1\\ \varepsilon_{xy}^1\\ \varepsilon_{xx}^2\\ \varepsilon_{yy}^2\\ \varepsilon_{xy}^2 \end{Bmatrix}, \tag{18}$$

$$\begin{Bmatrix} Q_1\\ Q_2\\ K_1\\ K_2 \end{Bmatrix} = \begin{bmatrix} A_{55} & A_{54} & J_{11} & J_{12}\\ A_{45} & A_{44} & J_{21} & J_{22}\\ J_{11} & J_{12} & L_{11} & L_{12}\\ J_{21} & J_{22} & L_{21} & L_{22} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xz}^0\\ \varepsilon_{yz}^0\\ \varepsilon_{xz}^3\\ \varepsilon_{yz}^3 \end{Bmatrix}. \tag{19}$$

where

$$A_{ij} = \int_{-h/2}^{h/2} Q_{ij}^{(k)} dz, \quad (i = 1, 2, 4, 5, 6) \tag{20a}$$

$$(B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij}) = \int_{-h/2}^{h/2} Q_{ij}^{(k)}(z, z^2, \tan mz, z\tan mz, \tan^2 mz)dz, \quad (i = 1, 2, 6) \tag{20b}$$

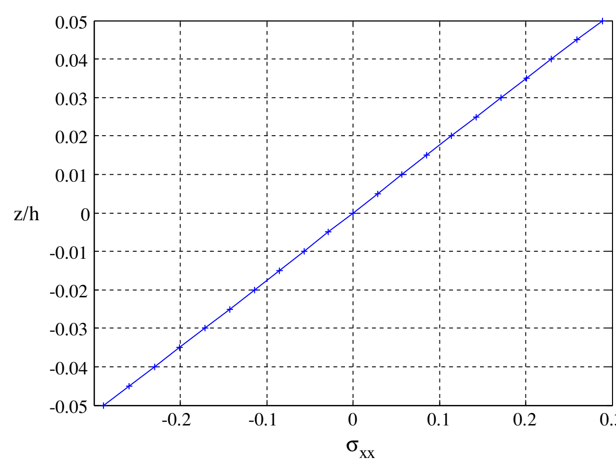
$$\left.\begin{aligned} J_{ij} &= \textstyle\int_{-h/2}^{h/2} Q_{ij}^{(k)} m\sec^2 mz dz,\\ L_{ij} &= \textstyle\int_{-h/2}^{h/2} Q_{ij}^{(k)} m^2 \sec^4 mz dz. \end{aligned}\right\} \quad (i = 4, 5) \tag{20c}$$

In what follows, the simply supported boundary conditions are prescribed at all four edges:

$$\begin{aligned}
&u(x,0) = u(x,b) = v(0,y) = v(a,y) = 0,\\
&w(x,0) = w(x,b) = w(0,y) = w(a,y) = 0,\\
&N_2(x,0) = N_2(x,b) = N_1(0,y) = N_1(a,y) = 0,\\
&M_2(x,0) = M_2(x,b) = M_1(0,y) = M_1(a,y) = 0,\\
&P_2(x,0) = P_2(x,b) = P_1(0,y) = P_1(a,y) = 0,\\
&\theta_1(x,0) = \theta_1(x,b) = \theta_2(0,y) = \theta_2(a,y) = 0.
\end{aligned} \tag{21}$$

3. Solution procedure

Exact solutions of the partial differential Eq. (17a–e) on arbitrary domains and for general boundary conditions are difficult. More general boundary conditions would require solution strategies involving, e.g., boundary discontinuous double Fourier series approach (see Chaudhuri, 1989, 1990, 2002). Some higher order shear deformation theory based results are currently available for laminated flat/curved panels (see Oktem and Chaudhuri, 2007a,b,c,d, 2008a,b, 2009a,b; Oktem and Guedes Soares, 2011). However, for simply supported panels, the abovementioned constitutive equations can be solved exactly, when lamination scheme is of antisymmetric cross-ply [0°/90°/0°/90°···] or symmetric cross-ply [0°/90°···]$_s$ type (Reddy and Liu, 1985). The Navier solution exists if the following stiffnesses are zero (Reddy, 1984a,b):



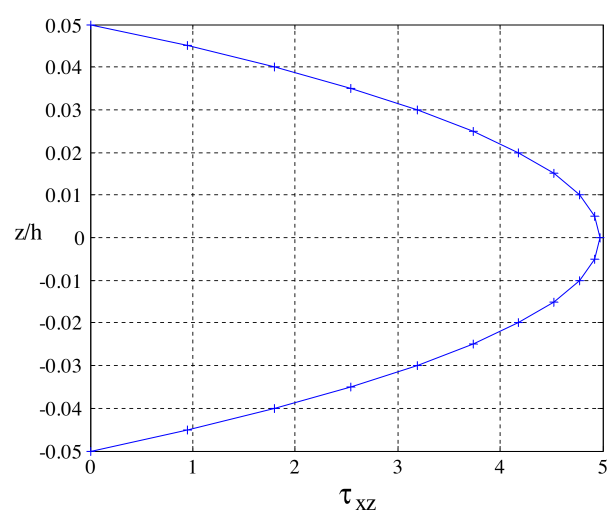


Fig. 3. Variation of normalized in-plane stress $\bar{\sigma}_{xx}$ and shear stress $\bar{\sigma}_{xz}$ through the thickness of an isotropic plate.

Table 4
Non-dimensionalized deflections and stresses in four-layer (0°/90°/90°/0°) square laminates under sinusoidal load ($b = a$).

Method	a/h	$\bar{w}$ $(a/2,b/2,0)$		$\bar{\sigma}_{xx}$ $(a/2,b/2,h/2)$		$\bar{\sigma}_{yy}$ $(a/2,b/2,h/4)$		$\bar{\sigma}_{xy}$ $(0,0,h/2)$		$\bar{\sigma}_{xz}$ $(0,b/2,0)$		$\bar{\sigma}_{yz}$ $(a/2,0,0)$		Total average error
3D-elasticity (Srinivas, 1973)	4	1.954	Diff. (%)	0.720	Diff. (%)	0.663	Diff. (%)	0.047	Diff. (%)	0.219	Diff. (%)	0.291	Diff. (%)	
Present		1.894	−3.063	0.664	−7.777	0.631	−4.759	0.044	−5.649	0.206	−5.883	0.239	−17.993	7.5
Reddy and Liu (1985)		1.893	−3.122	0.665	−7.625	0.632	−4.646	0.044	−5.782	0.206	−5.753	0.239	−17.904	7.5
3D-elasticity (Srinivas, 1973)	10	0.743	Diff. (%)	0.559	Diff. (%)	0.401	Diff. (%)	0.028	Diff. (%)	0.301	Diff. (%)	0.196	Diff. (%)	–
Present		0.715	−3.712	0.545	−2.452	0.388	−3.149	0.027	−2.576	0.264	−12.433	0.153	−21.997	7.7
Reddy and Liu (1985)		0.715	−3.809	0.546	−2.397	0.389	−3.042	0.027	−2.545	0.264	−12.292	0.153	−21.888	7.7
3D-elasticity (Srinivas, 1973)	20	0.517	Diff. (%)	0.543	Diff. (%)	0.308	Diff. (%)	0.023	Diff. (%)	0.328	Diff. (%)	0.156	Diff. (%)	–
Present		0.507	−1.965	0.539	−0.717	0.304	−1.265	0.023	−0.695	0.282	−14.030	0.123	−20.940	6.6
Reddy and Liu (1985)		0.506	−2.128	0.539	−0.681	0.304	−1.201	0.023	−0.870	0.283	−13.872	0.123	−20.897	6.6
3D-elasticity (Srinivas, 1973)	100	0.439	Diff. (%)	0.539	Diff. (%)	0.276	Diff. (%)	0.022	Diff. (%)	0.337	Diff. (%)	0.141	Diff. (%)	–
Present		0.435	−0.761	0.539	−0.081	0.271	−1.902	0.021	−0.987	0.289	−14.188	0.112	−20.819	6.5
Reddy and Liu (1985)		0.434	−0.958	0.538	−0.148	0.270	−2.174	0.021	−1.389	0.290	−14.036	0.112	−20.780	6.6

$$A_{i6} = B_{i6} = D_{i6} = E_{i6} = F_{i6} = H_{i6} = 0, \quad (i = 1, 2) \tag{22a}$$
$$A_{45} = J_{45} = L_{45} = 0. \tag{22b}$$

Solution functions to the partial differential Eq. (17a–e) of a cross-ply plate for simply supported boundary conditions given by Eq. (21) are assumed as follows:

$$u(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} U_{mn}\cos(\alpha x)\sin(\beta y), \tag{23a}$$
$$v(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} V_{mn}\sin(\alpha x)\cos(\beta y), \tag{23b}$$
$$w(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{mn}\sin(\alpha x)\sin(\beta y), \tag{23c}$$
$$\theta_1(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \theta^1_{mn}\cos(\alpha x)\sin(\beta y), \tag{23d}$$
$$\theta_2(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \theta^2_{mn}\sin(\alpha x)\cos(\beta y), \tag{23e}$$

where

$$\alpha = \frac{m\pi}{a}, \quad \beta = \frac{n\pi}{b}. \tag{24}$$

Substituting Eqs. (23a–e) into Eq. (17a–e), the following equations are obtained,

$$K_{ij}d_j = F_j \quad (i,j = 1,\ldots\ldots,5) \text{ and } (K_{ij} = K_{ji}). \tag{25a}$$

Elements of K_{ij} in Eq. (25a) are given in Appendix A

$$\{d_j\}^T = \{U_{mn} \quad V_{mn} \quad W_{mn} \quad \theta^1_{mn} \quad \theta^2_{mn}\}, \tag{25b}$$
$$\{F_j\}^T = \{0 \quad 0 \quad Q_{mn} \quad 0 \quad 0\}, \tag{25c}$$

where Q_{mn} are the coefficients in the double Fourier expansion of the transverse load,

$$q(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} Q_{mn}\sin(\alpha x)\sin(\beta y). \tag{26}$$

4. Numerical results and discussion

4.1. Selection of the parameter "m" in f(z) shape strain function

In Section 2, the governing equations are formulated with "y", and therefore they are "m" parameter dependent. From Eq. (10), it can be noticed than "m" is directly proportional to "α". For convenience, "α" is used to get the f(z) shape strain function. The unknown parameter "α" of the present higher order theory is obtained by providing (a) the shear stress $\bar{\sigma}_{xz}$ at $(0,b/2,0)$ and (b) center plate deflection at $(a/2,b/2,0)$ which produces relatively close results to 3D elasticity bending solutions provided by Pagano and Hatfield (1972). The transverse shear deformation distribution is approximately parabolic, and it exactly satisfies the zero shear stress conditions on the upper and lower plate surfaces.

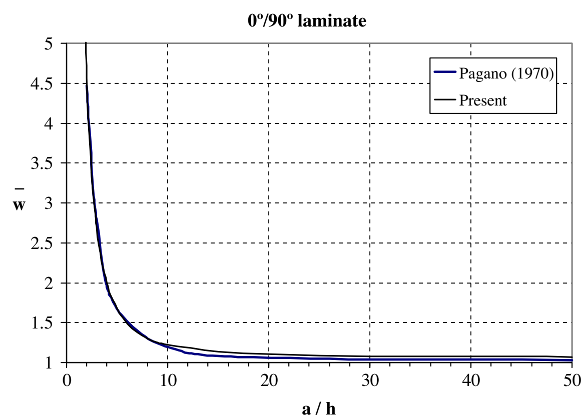


Fig. 4. Maximum central plane deflection in two-layer (0°/90°) square laminates under sinusoidal load as a function of span-to-depth ratio ($a = b$).

Increasing number of layers provides faster convergence to the respective classical plate results (Pagano and Hatfield, 1972), therefore in determination of "α" parameter, a four layer cross-ply composite plate is used as given in Fig. 2, same as the one used by Pagano and Hatfield (1972). The case "α = 0.1" produces the shear stress and the center deflection (at the abovementioned positions) in composite cross-ply plates which are closest to 3D exact solution provided by Pagano and Hatfield (1972) for $a/h = \{4, 10, 20, 50, 100\}$, see Fig. 2 and Table 4. Therefore, the errors between 3-D and 2-D solutions are lowered, and the results are in good agreement with Reddy's and Touratier's HSDTs as they are presented in Tables 1–3. However, it is important to note that there still exist considerable errors between 3-D and 2-D solutions for transverse shear stresses, as given in other HSDTs.

Substituting the value of "α" in Eq. (10), the parameter "m" and the shape strain function are obtained and they are given in Eq. (27). Following the determination of the shear strain shape function "f(z)", example problems for laminated composites are solved in order to show the validity and accuracy of present shear deformation theory

$$f(z) = \tan mz + m\sec^2 m\frac{h}{2}, \qquad m = \frac{1}{5h}. \tag{27}$$

The exact solutions of isotropic as well as composite symmetric and antisymmetric cross-ply plates are calculated by the present and various higher order shear deformation theories under sinusoidally and transversely distributed loads for a simply supported plate on all edges. The results are then discussed in the following sections.

4.2. Bending analysis of isotropic plates

Simply supported square isotropic plate ($a = b$) under sinusoidal and uniform load q is considered. The modulus of elasticity E and the Poisons' ratio ν are taken as 1 and 0.3, respectively, see Xiang et al. (2009). The non-dimensional maximum deflection, normal and shear stresses are obtained by Eqs. (28a–f). Plates of several ratios of a/h = 10, 20, 50 are used to examine the accuracy of the present HSDT for isotropic plates.

Table 1 lists the non-dimensional maximum deflection and normal stress of the simply supported square isotropic plate under uniform load with various shear deformation theories and for various a/h ratios. It is found that the present results are in good agreement with various shear deformation theories and they are in good agreement with the exact results provided by Reddy (1984c, 1993), Ferreira et al. (2003, 2005) and Xiang et al. (2009). In Table 1, comparison between the results provided by both trigonometric HSDTs, the present and the one used by Xiang et al. (2009), is made. With respect to the center plate deflection $\bar{w}$, the superiority of the present HSDT can be noticed for moderate thick to thin plates $a/h = \{10, 20, 50\}$. However, it can not be said the same for $\bar{\sigma}_{xx}$, except for thin isotropic plates $a/h = \{50\}$. Fig. 3 shows the variation of normalized stresses σ_{xx} and τ_{xz} through the thickness of the isotropic square plate, with N = 101 (number of terms in Fourier solution) for a/h = 10, respectively. It is interesting to note a very smooth evolution for the normal stress "$\bar{\sigma}_{xx}$" and the secant (trigonometric function) evolution of "$\bar{\tau}_{xz}$".

4.3. Bending analysis of cross-ply laminated composite plates

4.3.1. Three-layer symmetric cross-ply (0°/90°/0°) plates under sinusoidal load

Pagano (1970) provided exact solutions for one square ($b = a$) and one rectangle ($b = 3a$) plate for three-layer symmetric cross-ply (0°/90°/0°) lamination and simply supported on all edges.

The mechanical properties of each layer are as follows Pagano (1970):

$$E_1 = 174.6\ \text{GPa},\quad E_2 = 7\ \text{GPa},\quad G_{12} = G_{13} = 3.5\ \text{GPa},$$
$$G_{23} = 1.4\ \text{GPa},\quad \nu_{12} = \nu_{13} = 0.25.$$

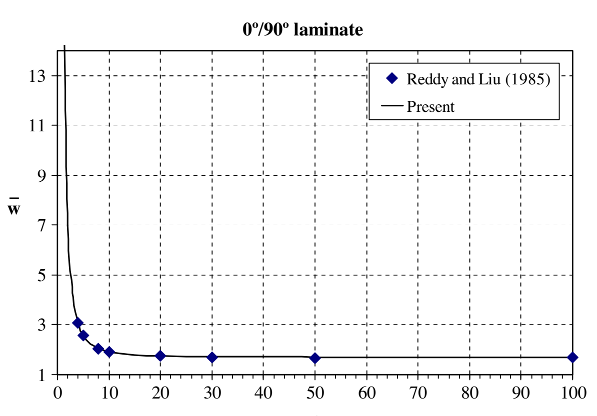


Fig. 5. Maximum central plane deflection in two-layer (0°/90°) square laminates under uniform load as a function of span-to-depth ratio ($a = b$).

The following normalized quantities are defined for deflection and stresses:

$$\begin{aligned}
&\bar{w} = w\left(\frac{a}{2},\frac{b}{2},0\right)\frac{10^2 E_2 h^3}{q_0 a^4},\quad \bar{\sigma}_{xx} = \sigma_{xx}\left(\frac{a}{2},\frac{b}{2},\frac{h}{2}\right)\frac{h^2}{q_0 a^2},\\
&\bar{\sigma}_{yy} = \sigma_{yy}\left(\frac{a}{2},\frac{b}{2},\frac{h}{6}\right)\frac{h^2}{q_0 a^2},\\
&\bar{\sigma}_{xy} = \sigma_{xy}\left(0,0,\frac{h}{2}\right)\frac{h^2}{q_0 a^2},\quad \bar{\tau}_{xz} = \tau_{xz}\left(0,\frac{b}{2},0\right)\frac{h}{q_0 a},\\
&\bar{\tau}_{yz} = \tau_{yz}\left(\frac{a}{2},0,0\right)\frac{h}{q_0 a}.
\end{aligned} \tag{28a–f}$$

The results of the present theory and other theories such as the one proposed by Reddy and Liu (1985) and Touratier (1991) are compared with the three-dimensional elasticity results given by Pagano (1970). The performance of the present theory is evaluated by calculating particular and global errors. The global average error of each theory is calculated by averaging the absolute values of all the particulars errors presented for each a/h ratio, and they are presented in the last columns of Tables 3 and 4.

The results for square plates are presented in Table 2. It is evident from Table 2 that the present method gives better results for normal stresses than in shear stresses for thick and thin plates. Results for the rectangular plates ($b = 3a$) are given in Table 3. Similar conclusions, compared with the square plate, can be inferred. From Table 3 it can be noticed that, for moderate to thick plate $a/h = \{4, 10, 20\}$, Touratier's shear deformation theory is better in deflection, shear and normal stresses than the present HSDT, except in the case of $\bar{\sigma}_{yz}$ (for moderate to thick plate $a/h = \{4, 10, 20\}$) and $\bar{\sigma}_{xy}$ (for moderate thick plate $a/h = \{20\}$). The global error indicates that the present HSDT is in good agreement with Reddy's HSDT (Table 2) and Touratier's HSDT (Table 3) results for $a/h = \{4, 10, 20, 50, 100\}$.

4.3.2. Four-layer symmetric cross-ply (0°/90°/90°/0°) plates under sinusoidal load

In this section, four layer symmetric (0°/90°/90°/0°) cross-ply square plates ($b = a$) are considered. The mechanical properties of each layer are the same as in Section 4.3.1. Table 4 contains the non-dimensionalized deflection and stresses as defined in Section 4.3.1, except for normalized σ_{yy} that is calculated at $z = h/4$. Exact 3D solution is also obtained from Pagano and Hatfield (1972). Results provided by Reddy and Liu (1985) are compared with the three-dimensional elasticity theory given by Pagano and Hatfield (1972). Normally, the efficiency of the different models can be checked in several cases such as thick laminates (Karama et al., 2003). In this section, the results are presented in Table 4. It shows that the present results are in good agreement with 3D-elasticity solution in deflection, and normal stresses. The global error indicates that the present HSDT performs as good as Reddy's HSDT. However, there is a considerable difference with 3D-elasticity solution for normalized "τ_{yz}" stress.

4.3.3. Two-layer symmetric cross-ply (0°/90°) plates under sinusoidal and uniform load

This section presents the results of the two-layer antisymmetric cross-ply (0°/90°) plates under sinusoidal load are also calculated for $a/h = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100\}$, and the results are presented in Table 5. Same material properties and normalization scheme are used as in Section 4.3.1. The deflections are in good agreement with Pagano (1970) for sinusoidally distributed load as given in Fig. 4. They are also in good agreement with Reddy and Liu (1985) for uniformly distributed load as it is shown in Fig. 5.

Table 5
Non-dimensionalized deflections and stresses in two-layer (0°/90°) square laminates under sinusoidal load ($b = a$).

a/h	$\bar{w}$ $(a/2,b/2,0)$	$\bar{\sigma}_{xx}$ $(a/2,b/2,h/2)$	$\bar{\sigma}_{yy}$ $(a/2,b/2,h/6)$	$\bar{\sigma}_{xy}$ $(0,0,h/2)$	$\bar{\sigma}_{xz}$ $(0,b/2,0)$	$\bar{\sigma}_{yz}$ $(a/2,0,0)$
1	12.216	0.141	−0.772	−0.114	0.233	0.093
2	4.568	0.102	−0.359	−0.072	0.293	0.117
3	2.699	0.093	−0.258	−0.062	0.307	0.123
4	2.001	0.089	−0.220	−0.058	0.313	0.125
5	1.669	0.087	−0.202	−0.056	0.315	0.126
6	1.486	0.087	−0.192	−0.055	0.317	0.127
7	1.375	0.086	−0.186	−0.054	0.318	0.127
8	1.303	0.086	−0.183	−0.054	0.318	0.127
9	1.253	0.085	−0.180	−0.054	0.319	0.127
10	1.218	0.085	−0.178	−0.053	0.319	0.128
15	1.133	0.085	−0.173	−0.053	0.319	0.128
20	1.103	0.085	−0.172	−0.053	0.320	0.128
25	1.089	0.085	−0.171	−0.053	0.320	0.128
30	1.082	0.085	−0.171	−0.053	0.320	0.128
35	1.077	0.085	−0.170	−0.053	0.320	0.128
40	1.074	0.084	−0.170	−0.053	0.320	0.128
45	1.072	0.084	−0.170	−0.053	0.320	0.128
50	1.071	0.084	−0.170	−0.053	0.320	0.128
55	1.070	0.084	−0.170	−0.053	0.320	0.128
60	1.069	0.084	−0.170	−0.053	0.320	0.128
65	1.069	0.084	−0.170	−0.053	0.320	0.128
70	1.068	0.084	−0.170	−0.053	0.320	0.128
75	1.068	0.084	−0.170	−0.053	0.320	0.128
80	1.067	0.084	−0.170	−0.053	0.320	0.128
85	1.067	0.084	−0.170	−0.053	0.320	0.128
90	1.067	0.084	−0.170	−0.053	0.320	0.128
95	1.067	0.084	−0.170	−0.053	0.320	0.128
100	1.066	0.084	−0.170	−0.053	0.320	0.128

4.3.4. Three layer square sandwich plate under uniform load

A simply supported square sandwich plate is considered under uniform load. The a/h ratio is taken as 10. The sandwich laminate is composed of two outer layers (skins) of thickness $h_1 = h_3 = 0.1h$ and one inner layer (core) of thickness $h_2 = 0.8h$. The skin orthotropic properties are obtained by multiplying the core orthotropic properties with an integer R, and they are given below:

$$\overline{Q}^{core} = \begin{bmatrix} 0.999781 & 0.231192 & 0 & 0 & 0 \\ 0.231192 & 0.524886 & 0 & 0 & 0 \\ 0 & 0 & 0.262931 & 0 & 0 \\ 0 & 0 & 0 & 0.266810 & 0 \\ 0 & 0 & 0 & 0 & 0.159914 \end{bmatrix}. \tag{29}$$

And the skin properties are obtained by,

$$\overline{Q}^{skin} = R\overline{Q}^{core}. \tag{30}$$

Sandwich results are compared with the exact solution of Srinivas (1973), finite element results of Pandya and Kant (1988), classical plate theory (CPT), shell non-linear finite element formulation by Ferreira and Barbosa (2000), higher order shear formulation with multiquadrics by Ferreira et al. (2003), trigonometric shear deformation theory and multiquadrics by Ferreira et al. (2005). The present model is also compared with a meshless method and a various shear deformation theories by Xiang et al. (2009), which consider several shear deformations theories, as the one proposed by Reddy and Liu (1985), Touratier (1991) and Karama et al. (2003, 2009).

The normalized quantities used in Tables 6–8 are defined for deflection and stresses given below:

$$\bar{w} = w\left(\frac{a}{2},\frac{a}{2},0\right)\frac{0.999781}{hq}, \quad \bar{\sigma}^1_{xx} = \sigma^1_{xx}\left(\frac{a}{2},\frac{b}{2},-\frac{h}{2}\right)\frac{1}{q},$$
$$\bar{\sigma}^2_{xx} = \sigma^1_{xx}\left(\frac{a}{2},\frac{b}{2},-\frac{2h}{5}\right)\frac{1}{q}, \quad \bar{\sigma}^3_{xx} = \sigma^2_{xx}\left(\frac{a}{2},\frac{b}{2},-\frac{2h}{5}\right)\frac{1}{q},$$
$$\bar{\sigma}^1_{yy} = \sigma^1_{yy}\left(\frac{a}{2},\frac{b}{2},-\frac{h}{2}\right)\frac{1}{q}, \quad \bar{\sigma}^2_{yy} = \sigma^1_{yy}\left(\frac{a}{2},\frac{b}{2},-\frac{2h}{5}\right)\frac{1}{q},$$
$$\bar{\sigma}^3_{yy} = \sigma^2_{yy}\left(\frac{a}{2},\frac{b}{2},-\frac{2h}{5}\right)\frac{1}{q}, \quad \bar{\tau}^1_{xz} = \tau^2_{xz}\left(0,\frac{b}{2},0\right)\frac{1}{q}. \tag{31}$$

Tables 6–8 lists the present non-dimensional maximum deflection, normal and shear stresses results of the simply supported square sandwich plate under uniform load with various higher order

Table 6
Maximum deflection and stresses of a simply supported square sandwich plate under uniform load ($R = 5$).

Method	$\bar{w}$ $(a/2,b/2,0)$	$\bar{\sigma}^1_{xx}$ $(a/2,b/2,h/2)$	$\bar{\sigma}^2_{xx}$ $(a/2,b/2,2h/5)$	$\bar{\sigma}^3_{xx}$ $(a/2,b/2,2h/5)$	$\bar{\sigma}^1_{yy}$ $(a/2,b/2,h/2)$	$\bar{\sigma}^2_{yy}$ $(a/2,b/2,2h/5)$	$\bar{\sigma}^3_{yy}$ $(a/2,b/2,2h/5)$	$\bar{\tau}^1_{xz}$ $(0,b/2,0)$
3D-elasticity (Srinivas, 1973)	258.970	60.353	46.623	9.340	38.491	30.097	6.161	4.364
Pandya and Kant HSDT (1988)	258.740	62.380	46.910	9.382	38.930	30.330	6.065	3.089
Pandya and Kant FSDT (1988)	236.100	61.870	49.500	9.899	36.650	29.320	5.864	3.313
CPT	216.940	61.141	48.623	9.783	36.622	29.297	5.860	4.590
Ferreira and Barbosa (2000)	258.740	59.210	45.610	9.122	37.880	29.590	5.918	3.593
Ferreira et al. (HSDT) (2003)	257.110	60.366	47.003	9.401	38.456	30.242	6.048	4.548
Ferreira et al. (HSDT) (2005)	257.000	60.396	46.971	9.394	38.460	30.222	6.044	4.553
Xiang et al. (Karama) (2009)	253.724	59.950	46.655	9.331	38.191	30.018	6.003	3.637
Xiang et al. (Touratier) (2009)	253.989	60.123	47.097	9.419	38.249	30.187	6.037	3.707
Xiang et al. (Reddy) (2009)	253.638	60.124	46.703	9.340	38.242	30.020	6.004	3.764
Present	256.966	60.348	46.989	9.398	38.435	30.226	6.045	4.712

Table 7
Maximum deflection and stresses of a simply supported square sandwich plate under uniform load (R = 10).

Method	$\bar{w}$ $(a/2,b/2,0)$	$\bar{\sigma}^1_{xx}$ $(a/2,b/2,h/2)$	$\bar{\sigma}^2_{xx}$ $(a/2,b/2,2h/5)$	$\bar{\sigma}^3_{xx}$ $(a/2,b/2,2h/5)$	$\bar{\sigma}^1_{yy}$ $(a/2,b/2,h/2)$	$\bar{\sigma}^2_{yy}$ $(a/2,b/2,2h/5)$	$\bar{\sigma}^3_{yy}$ $(a/2,b/2,2h/5)$	$\bar{\tau}^1_{xz}$ $(0,b/2,0)$
3D-elasticity (Srinivas, 1973)	159.380	65.332	48.857	4.903	43.566	33.413	3.500	4.096
Pandya and Kant HSDT (1988)	152.330	64.650	51.310	5.131	42.830	33.970	3.397	3.147
Pandya and Kant FSDT (1988)	131.095	67.800	54.240	4.424	40.100	32.080	3.208	3.152
CPT	118.870	65.332	48.857	5.356	40.099	32.079	3.208	4.367
Ferreira and Barbosa (2000)	159.402	64.160	47.720	4.772	42.970	42.900	3.290	3.518
Ferreira et al. (HSDT) (2003)	154.658	65.381	49.973	4.997	43.240	33.637	3.364	3.528
Ferreira et al. (HSDT) (2005)	155.030	65.370	49.823	4.982	43.273	33.601	3.361	4.284
Xiang et al. (Karama) (2009)	152.664	65.008	49.684	4.968	42.945	33.394	3.339	3.450
Xiang et al. (Touratier) (2009)	153.139	65.050	50.206	5.020	43.015	33.653	3.365	3.641
Xiang et al. (Reddy) (2009)	153.357	65.100	49.499	4.949	43.059	33.379	3.337	3.843
Present	154.480	65.335	49.939	4.994	43.198	33.605	3.360	4.252

Table 8
Maximum deflection and stresses of a simply supported square sandwich plate under uniform load (R = 15).

Method	$\bar{w}$ $(a/2,b/2,0)$	$\bar{\sigma}^1_{xx}$ $(a/2,b/2,h/2)$	$\bar{\sigma}^2_{xx}$ $(a/2,b/2,2h/5)$	$\bar{\sigma}^3_{xx}$ $(a/2,b/2,2h/5)$	$\bar{\sigma}^1_{yy}$ $(a/2,b/2,h/2)$	$\bar{\sigma}^2_{yy}$ $(a/2,b/2,2h/5)$	$\bar{\sigma}^3_{yy}$ $(a/2,b/2,2h/5)$	$\bar{\tau}^1_{xz}$ $(0,b/2,0)$
3D-elasticity (Srinivas (1973))	121.720	66.787	48.299	3.238	46.424	34.955	2.494	3.964
Pandya and Kant HSDT (1988)	110.430	66.620	51.970	3.465	44.920	35.410	2.361	3.035
Pandya and Kant FSDT (1988)	90.850	70.040	56.030	3.753	41.390	33.110	2.208	3.091
CPT	81.768	69.135	55.308	3.687	41.410	33.128	2.209	4.283
Ferreira and Barbosa (2000)	121.821	65.650	47.090	3.140	45.850	34.420	2.294	3.466
Ferreira et al. (HSDT) (2003)	114.644	66.920	50.323	3.355	45.623	35.170	2.345	3.021
Ferreira et al. (HSDT) (2005)	115.460	66.870	50.041	3.336	45.724	35.150	2.343	4.177
Xiang et al. (Karama) (2009)	113.088	66.539	50.043	3.336	45.293	34.903	2.326	3.254
Xiang et al. (Touratier) (2009)	113.964	66.544	50.679	3.378	45.431	35.278	2.351	3.472
Xiang et al. (Reddy) (2009)	114.585	66.621	49.663	3.310	45.546	34.919	2.327	3.706
Present	114.466	66.858	50.279	3.352	45.566	35.127	2.342	3.904

formulations, for different values of R (5, 10 and 15). It is found that the present results are in good agreement with Srinivas (1973), and also with various numerical calculations such as finite element (Pandya and Kant, 1988; Ferreira and Barbosa, 2000) and meshless methods (Ferreira et al., 2003, 2005; Xiang et al., 2009). As mentioned above, Xiang et al. (2009) provided meshless numerical results by using several HSDTs (including Touratier's HSDT). Comparative results, between this HSDT and the present one, show the superiority of the present HSDT except in the case of $\bar{\sigma}_{yy}$ for R (5 and 15), see Tables 6–8.

5. Conclusions

A new trigonometric higher order shear deformation theory for isotropic and laminated composite and sandwich plates is presented. The theory accounts for adequate distribution of the transverse shear strains through the plate thickness and tangential stress-free boundary conditions on the plate boundary surface, therefore a shear correction factor is not required. The accuracy of the present theory is ascertained by comparing it with various available results in the literature. The results show that the present model is in close agreement with Reddy's and Touratier's shear deformation theories for analyzing the static behavior of isotropic and composite laminated and sandwich plates.

The results may be further improved by considering the continuity of transverse shear stresses between the layer interfaces by combination of the present approach with a zig-zag approach. It is considered that the present theory will be extended to cover multilayered shell structures, higher order layer-wise shear deformation theories and advanced numerical calculations, such as finite element and meshless methods.

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Appendix A. Definition of Constants in Eq. (25a)

The following proposed simple technique to calculate the K_{ij} element matrices (which comes from the governing Eq. (17a–e)) is perhaps more convenient and simple than the others, (e.g. Reddy and Liu, 1985). The advantage of the present technique is that all the abovementioned shear deformation theories can be calculated using the same following matrices; only "y^*" should be changed. In the present case $y^* = -m\sec^2\left(m\frac{h}{2}\right)$ (see Eq. (10)).

A.1. Calculation of N, M and P

$$\begin{bmatrix} (N_1^c, M_1^c, P_1^c) \\ (N_2^c, M_2^c, P_2^c) \\ (N_6^c, M_6^c, P_6^c) \end{bmatrix} = (A_{ij}, B_{ij}, E_{ij}) \begin{bmatrix} -\alpha & 0 & 0 & 0 & 0 \\ 0 & -\beta & 0 & 0 & 0 \\ \beta & \alpha & 0 & 0 & 0 \end{bmatrix} + (B_{ij}, D_{ij}, F_{ij}) \begin{bmatrix} 0 & 0 & \alpha^2 & -y^*\alpha & 0 \\ 0 & 0 & \beta^2 & 0 & -y^*\beta \\ 0 & 0 & -2\alpha\beta & y^*\beta & y^*\alpha \end{bmatrix} + (E_{ij}, F_{ij}, H_{ij}) \begin{bmatrix} 0 & 0 & 0 & -\alpha & 0 \\ 0 & 0 & 0 & 0 & -\beta \\ 0 & 0 & 0 & \beta & \alpha \end{bmatrix}, \quad (A1)$$

where i,j = 1,2,6.

First derivative of N, M and P with respect to x:

$$\begin{bmatrix} \frac{\partial\left(N_1^c,M_1^c,P_1^c\right)}{\partial x} \\ \frac{\partial\left(N_2^c,M_2^c,P_2^c\right)}{\partial x} \\ \frac{\partial\left(N_6^c,M_6^c,P_6^c\right)}{\partial x} \end{bmatrix} = (A_{ij},B_{ij},E_{ij})\begin{bmatrix} -\alpha^2 & 0 & 0 & 0 & 0 \\ 0 & -\alpha\beta & 0 & 0 & 0 \\ -\alpha\beta & -\alpha^2 & 0 & 0 & 0 \end{bmatrix} + (B_{ij},D_{ij},F_{ij})\begin{bmatrix} 0 & 0 & \alpha^3 & -y^*\alpha^2 & 0 \\ 0 & 0 & \alpha\beta^2 & 0 & -y^*\alpha\beta \\ 0 & 0 & 2\alpha^2\beta & -y^*\alpha\beta & -y^*\alpha^2 \end{bmatrix} + (E_{ij},F_{ij},H_{ij})\begin{bmatrix} 0 & 0 & 0 & -\alpha^2 & 0 \\ 0 & 0 & 0 & 0 & -\alpha\beta \\ 0 & 0 & 0 & -\alpha\beta & -\alpha^2 \end{bmatrix}. \quad \text{(A2)}$$

First derivative of N, M and P with respect to y:

$$\begin{bmatrix} \frac{\partial\left(N_1^c,M_1^c,P_1^c\right)}{\partial y} \\ \frac{\partial\left(N_2^c,M_2^c,P_2^c\right)}{\partial y} \\ \frac{\partial\left(N_6^c,M_6^c,P_6^c\right)}{\partial y} \end{bmatrix} = (A_{ij},B_{ij},E_{ij})\begin{bmatrix} -\alpha\beta & 0 & 0 & 0 & 0 \\ 0 & -\beta^2 & 0 & 0 & 0 \\ -\beta^2 & -\alpha\beta & 0 & 0 & 0 \end{bmatrix} + (B_{ij},D_{ij},F_{ij})\begin{bmatrix} 0 & 0 & \alpha^2\beta & -y^*\alpha\beta & 0 \\ 0 & 0 & \beta^3 & 0 & -y^*\beta^2 \\ 0 & 0 & 2\alpha\beta^2 & -y^*\beta^2 & -y^*\alpha\beta \end{bmatrix} + (E_{ij},F_{ij},H_{ij})\begin{bmatrix} 0 & 0 & 0 & -\alpha\beta & 0 \\ 0 & 0 & 0 & 0 & -\beta^2 \\ 0 & 0 & 0 & -\beta^2 & -\alpha\beta \end{bmatrix}. \quad \text{(A3)}$$

Second partial derivative of N, M and P with respect to x:

$$\begin{bmatrix} \frac{\partial^2\left(N_1^c,M_1^c,P_1^c\right)}{\partial x^2} \\ \frac{\partial^2\left(N_2^c,M_2^c,P_2^c\right)}{\partial x^2} \\ \frac{\partial^2\left(N_6^c,M_6^c,P_6^c\right)}{\partial x^2} \end{bmatrix} = (A_{ij},B_{ij},E_{ij})\begin{bmatrix} \alpha^3 & 0 & 0 & 0 & 0 \\ 0 & \alpha^2\beta & 0 & 0 & 0 \\ -\alpha^2\beta & -\alpha^3 & 0 & 0 & 0 \end{bmatrix} + (B_{ij},D_{ij},F_{ij})\begin{bmatrix} 0 & 0 & -\alpha^4 & y^*\alpha^3 & 0 \\ 0 & 0 & -\alpha^2\beta^2 & 0 & y^*\alpha^2\beta \\ 0 & 0 & 2\alpha^3\beta & -y^*\alpha^2\beta & -y^*\alpha^3 \end{bmatrix} + (E_{ij},F_{ij},H_{ij})\begin{bmatrix} 0 & 0 & 0 & \alpha^3 & 0 \\ 0 & 0 & 0 & 0 & \alpha^2\beta \\ 0 & 0 & 0 & -\alpha^2\beta & -\alpha^3 \end{bmatrix}. \quad \text{(A4)}$$

Second partial derivative of N, M and P with respect to y:

$$\begin{bmatrix} \frac{\partial^2\left(N_1^c,M_1^c,P_1^c\right)}{\partial y^2} \\ \frac{\partial^2\left(N_2^c,M_2^c,P_2^c\right)}{\partial y^2} \\ \frac{\partial^2\left(N_6^c,M_6^c,P_6^c\right)}{\partial y^2} \end{bmatrix} = (A_{ij},B_{ij},E_{ij})\begin{bmatrix} \alpha\beta^2 & 0 & 0 & 0 & 0 \\ 0 & \beta^3 & 0 & 0 & 0 \\ -\beta^3 & -\alpha\beta^2 & 0 & 0 & 0 \end{bmatrix} + (B_{ij},D_{ij},F_{ij})\begin{bmatrix} 0 & 0 & -\alpha^2\beta^2 & y^*\alpha\beta^2 & 0 \\ 0 & 0 & -\beta^4 & 0 & y^*\alpha\beta^3 \\ 0 & 0 & 2\alpha\beta^3 & -y^*\alpha\beta^3 & -y^*\alpha\beta^2 \end{bmatrix} + (E_{ij},F_{ij},H_{ij})\begin{bmatrix} 0 & 0 & 0 & \alpha\beta^2 & 0 \\ 0 & 0 & 0 & 0 & \beta^3 \\ 0 & 0 & 0 & -\beta^3 & -\alpha\beta^2 \end{bmatrix}. \quad \text{(A5)}$$

Second partial derivative of N, M and P with respect to x and y:

$$\begin{bmatrix} \frac{\partial^2\left(N_1^c,M_1^c,P_1^c\right)}{\partial x\partial y} \\ \frac{\partial^2\left(N_2^c,M_2^c,P_2^c\right)}{\partial x\partial y} \\ \frac{\partial^2\left(N_6^c,M_6^c,P_6^c\right)}{\partial x\partial y} \end{bmatrix} = (A_{ij},B_{ij},E_{ij})\begin{bmatrix} -\alpha^2\beta & 0 & 0 & 0 & 0 \\ 0 & -\alpha\beta^2 & 0 & 0 & 0 \\ \alpha\beta^2 & \alpha^2\beta & 0 & 0 & 0 \end{bmatrix} + (B_{ij},D_{ij},F_{ij})\begin{bmatrix} 0 & 0 & \alpha^3\beta & -y^*\alpha^2\beta & 0 \\ 0 & 0 & \alpha\beta^3 & 0 & -y^*\alpha\beta^2 \\ 0 & 0 & -2\alpha^2\beta^2 & y^*\alpha\beta^2 & y^*\alpha^2\beta \end{bmatrix} + (E_{ij},F_{ij},H_{ij})\begin{bmatrix} 0 & 0 & 0 & -\alpha^2\beta & 0 \\ 0 & 0 & 0 & 0 & -\alpha\beta^2 \\ 0 & 0 & 0 & \alpha^2\beta & \alpha^2\beta \end{bmatrix}. \quad \text{(A6)}$$

Example to obtain $K(1,j)$ in Eq. (25a).

From the Equations (A1) and (A2), $\frac{\partial N_1^c}{\partial x}$ and $\frac{\partial N_6^c}{\partial y}$ can be easily obtained and substituted in Eq. (A7)

$$K(1,j) = \frac{\partial N_1^c}{\partial x} + \frac{\partial N_6^c}{\partial y}, \quad \text{where } j = 1,2,\ldots,5. \quad \text{(A7)}$$

Following the same technique the coefficients associated with Q and K can be obtained.

Note: N_1 or N_2 are not present in the governing equations, but they were expressed here for homogeneity purposes.

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