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Magnetic moment for the negative parity $\Lambda \to \Sigma^0$ transition in light cone QCD sum rules

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ABSTRACT

The magnetic moment of the $\Lambda \to \Sigma^0$ transition between negative parity baryons is calculated in framework of the QCD sum rules approach by using the general form of the interpolating currents. The pollution arising from the positive-to-positive, and positive-to-negative parity baryons is eliminated by constructing the sum rules for different Lorentz structures. A comparison of our result with the predictions of the results of other approaches for the positive parity baryons is presented.

1. Introduction

Magnetic moment of baryons is one of the most important quantities in investigation of baryons' electromagnetic structure, and can provide essential information about the dynamics of the strong interaction at low energies. The magnetic moments of the octet baryons have already been calculated in various theoretical approaches. These calculations have the privilege that they can be checked against available precise experimental data. Additionally, the study of the $\Lambda \to \Sigma^0$ transition magnetic moment can play a critical role in investigation of the properties of the octet baryons.

In recent years the study of the negative parity baryons has become one of the most promising direction taken in connection with experiments conducted and planned at Jefferson Lab [1], and Mainz Microtron facility (MAMI) [2,3]. The magnetic moments of N^* are planned to be measured at MAMI [3,4]. In the present work we calculate the transition magnetic moment between the negative parity Λ^* and Σ^{0*} baryons within the QCD sum rules method (LCSR) (here and in further discussions, we denote the negative parity baryons as B^*). This method is based on operator product expansion (OPE) near light cone. The OPE is performed over the twist of operators rather than dimension, which is the case in the traditional QCD sum rules method. In this version all non-perturbative dynamics is encoded in light cone distribution amplitudes. These amplitudes appear when the matrix elements of the nonlocal operators are sandwiched between the vacuum and one-particle states (for the details of the LCSR, see [5]). The magnetic moment of the $\Lambda \to \Sigma^0$ transition for the positive parity baryons has already been calculated in framework of the traditional QCD sum rules [6], the external field method in the traditional QCD sum rules [7], and in the light cone version of the QCD sum rules method [8]. Note that the magnetic moments of the negative parity octet baryons, $J^P = \frac{3}{2}^-$ heavy baryons, as well as diagonal and transition magnetic moments of negative party heavy baryons are calculated within the same framework in [9,10] and [11], respectively.

The work is arranged as follows. In section 2 the LCSR for the magnetic moment of the $\Lambda^* \to \Sigma^{0*}$ transition is derived. In section 3 we numerically analyze these LCSR obtained for the transition magnetic moment. This section also contains concluding remarks.

2. Light cone QCD sum rules for the magnetic moment of the $\Lambda^* \to \Sigma^{0*}$ transition

The $\Lambda^* \to \Sigma^{0*}$ transition magnetic moment can be calculated by considering the following correlation function,

$$\Pi_\mu(p,q) = -\int d^4x \int d^4y e^{i(px+qy)} \times \left\langle 0 \left| \mathrm{T}\left\{ \eta_{\Sigma^0}(0) j_\mu^{el}(y) \bar{\eta}_\Lambda(x) \right\} \right| 0 \right\rangle , \quad (1)$$

where η_{Σ^0} and η_Λ are the interpolating currents of the Σ^0 and Λ baryons, respectively, $j_\mu^{el} = e_q \bar{q}\gamma_\mu q$ is the electromagnetic current, and e_q is the electric charge of the light quarks. The correlation function can be written in a more convenient form by introducing the plane wave electromagnetic background field,

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$$F_{\mu\nu} = -i\left(\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu\right) ,$$

which allows us to put the correlation function in the following form,

$$\Pi_\mu \varepsilon^\mu = i\int d^4x e^{ipx} \left\langle 0 \left| \mathrm{T}\left\{\eta_{\Sigma^0}(0)\bar{\eta}_\Lambda(x)\right\}\right| 0\right\rangle_F , \qquad (2)$$

where subscript F means that the vacuum expectation value is evaluated in presence of the background field $F_{\mu\nu}$. The correlation function (1) can be obtained from Eq. (2) by expanding it in powers of the background field and keeping only those terms that are linear in $F_{\mu\nu}$, corresponding to the photon radiation. The details of this procedure can be found in [12] (for a review about the background field method we refer the interested reader to an excellent work [13]).

According to the QCD sum rules strategy, the correlation function (1) is calculated in two different kinematical region. Firstly, on the phenomenological side, the calculation is carried out by saturating a tower of hadronic intermediate states carrying the same quantum numbers as the interpolating current, where $p^2 \simeq m^2_{\Sigma_0}$, and $(p+q)^2 \simeq m^2_\Lambda$. Secondly, on the QCD side, the same correlation function is expanded in terms of the photon distribution amplitudes (DAs) with increasing twist. The QCD sum rules are constructed by matching these two representations. It follows from Eqs. (1) and (2) that the interpolating currents are needed in calculation of the correlation function which are constructed from the quark fields with the same quantum numbers of the corresponding baryon. The general forms of the interpolating currents of Λ and Σ^0 baryons are given as [14]:

$$\begin{aligned}
\eta_\Lambda &= 2\sqrt{\frac{1}{6}}\varepsilon^{abc}\Big\{2(u^{aT}Cd^b)\gamma_5 s^c + (u^{aT}Cs^b)\gamma_5 d^c - (d^{aT}Cs^b)\gamma_5 u^c \\
&\quad + 2\beta(u^{aT}C\gamma_5 d^b)s^c + \beta(u^{aT}C\gamma_5 s^b)d^c - \beta(d^{aT}C\gamma_5 s^b)u^c\Big\} , \\
\eta_{\Sigma^0} &= \sqrt{2}\varepsilon^{abc}\Big\{(u^{aT}Cs^b)\gamma_5 d^c + (d^{aT}Cs^b)\gamma_5 u^c \\
&\quad + \beta(u^{aT}C\gamma_5 s^b)d^c + \beta(d^{aT}C\gamma_5 s^b)u^c\Big\} ,
\end{aligned} \qquad (3)$$

where a, b, c are the color indices, C is the charge conjugation operator, superscript T denotes the transpose operator, and β is an arbitrary parameter with $\beta = -1$ corresponding to the so-called Ioffe current.

Firstly we shall calculate the phenomenological part of the correlation function given in Eq. (1). Saturating the interpolating current with the intermediate hadronic states having the same quantum number as the interpolating currents, and isolating the ground state contributions we get,

$$\begin{aligned}
\varepsilon^\mu \Pi_\mu &= \varepsilon^\mu \frac{\left\langle 0\left|\eta_{\Sigma^0}\right|\Sigma^0(p_2)\right\rangle}{p_2^2 - m^2_{\Sigma^0}} \left\langle \Sigma^0(p_2)\left|j^{el}_\mu\right|\Lambda(p_1)\right\rangle \frac{\left\langle\Lambda(p_1)\left|\bar{\eta}_\Lambda\right|0\right\rangle}{p_1^2 - m^2_\Lambda} \\
&+ \varepsilon^\mu \frac{\left\langle 0\left|\eta_{\Sigma^{0*}}\right|\Sigma^{0*}(p_2)\right\rangle}{p_2^2 - m^2_{\Sigma^{0*}}} \left\langle \Sigma^{0*}(p_2)\left|j^{el}_\mu\right|\Lambda^*(p_1)\right\rangle \\
&\times \frac{\left\langle\Lambda^*(p_1)\left|\bar{\eta}_\Lambda\right|0\right\rangle}{p_1^2 - m^2_{\Lambda^*}} \\
&+ \varepsilon^\mu \frac{\left\langle 0\left|\eta_{\Sigma^0}\right|\Sigma^0(p_2)\right\rangle}{p_2^2 - m^2_{\Sigma^0}} \left\langle \Sigma^0(p_2)\left|j^{el}_\mu\right|\Lambda^*(p_1)\right\rangle \\
&\times \frac{\left\langle\Lambda^*(p_1)\left|\bar{\eta}_\Lambda\right|0\right\rangle}{p_1^2 - m^2_{\Lambda^*}} \\
&+ \varepsilon^\mu \frac{\left\langle 0\left|\eta_{\Sigma^{0*}}\right|\Sigma^{0*}(p_2)\right\rangle}{p_2^2 - m^2_{\Sigma^{0*}}} \left\langle \Sigma^{0*}(p_2)\left|j^{el}_\mu\right|\Lambda(p_1)\right\rangle \\
&\times \frac{\left\langle\Lambda(p_1)\left|\bar{\eta}_\Lambda\right|0\right\rangle}{p_1^2 - m^2_\Lambda} + \cdots ,
\end{aligned} \qquad (4)$$

where superscript * represents it is a negative parity baryon, and dots describe the contributions of higher states which carry the same quantum numbers as the ground state. The matrix elements in Eq. (4) are determined in the following way:

$$\begin{aligned}
&\left\langle 0\left|\eta\right|B(p)\right\rangle = \lambda_B u(p) , \\
&\left\langle 0\left|\eta\right|B^*(p)\right\rangle = \lambda_{B^*}\gamma_5 u(p) , \\
&\left\langle B_2(p_2)\left|j^{el}_\mu\right|B_1(p_1)\right\rangle \\
&\quad = e\bar{u}(p_2)\left[f_1\gamma_\mu - i\frac{\sigma_{\mu\nu}q^\nu}{m_{B_1}+m_{B_2}} f_2\right]u(p_1) , \\
&\left\langle B_2^*(p_2)\left|j^{el}_\mu\right|B_1^*(p_1)\right\rangle \\
&\quad = e\bar{u}(p_2)\left[f_1^*\gamma_\mu - i\frac{\sigma_{\mu\nu}q^\nu}{m_{B_1^*}+m_{B_2^*}} f_2^*\right]u(p_1) , \\
&\left\langle B_2^*(p_2)\left|j^{el}_\mu\right|B_1(p_1)\right\rangle \\
&\quad = e\bar{u}(p_2)\left[f_1^T\gamma_\mu - i\frac{\sigma_{\mu\nu}q^\nu}{m_{B_1}+m_{B_2^*}} f_2^T\right]\gamma_5 u(p_1) .
\end{aligned} \qquad (5)$$

Substituting these matrix elements into Eq. (4), and performing summation over the spins of the baryons we get,

$$\begin{aligned}
\Pi_\mu \varepsilon^\mu &= A'\left(\not{p}_2 + m_{\Sigma^0}\right)\not{\varepsilon}\left(\not{p}_1 + m_\Lambda\right) \\
&+ B'\left(\not{p}_2 - m_{\Sigma^{0*}}\right)\not{\varepsilon}\left(\not{p}_1 - m_{\Lambda^*}\right) \\
&+ C'\left(\not{p}_2 - m_{\Sigma^{0*}}\right)\not{\varepsilon}\left(\not{p}_1 + m_\Lambda\right) \\
&+ D'\left(\not{p}_2 + m_{\Sigma^0}\right)\not{\varepsilon}\left(\not{p}_1 - m_{\Lambda^*}\right) \\
&+ \text{other structures} ,
\end{aligned} \qquad (6)$$

where

$$\begin{aligned}
A' &= \frac{\lambda_{\Sigma^0}(\beta)\lambda_\Lambda(\beta)}{(m^2_{\Sigma^0} - p_2^2)(m^2_\Lambda - p_1^2)}(f_1 + f_2) , \\
B' &= \frac{\lambda_{\Sigma^{0*}}(\beta)\lambda_{\Lambda^*}(\beta)}{(m^2_{\Sigma^{0*}} - p_2^2)(m^2_{\Lambda^*} - p_1^2)}(f_1^* + f_2^*) , \\
C' &= \frac{\lambda_{\Sigma^{0*}}(\beta)\lambda_\Lambda(\beta)}{(m^2_{\Sigma^{0*}} - p_2^2)(m^2_\Lambda - p_1^2)}\left(f_1^T + \frac{m_{\Sigma^{0*}} - m_\Lambda}{m_{\Sigma^{0*}} + m_\Lambda} f_2^T\right) , \\
D' &= -\frac{\lambda_{\Sigma^0}(\beta)\lambda_{\Lambda^*}(\beta)}{(m^2_{\Sigma^0} - p_2^2)(m^2_{\Lambda^*} - p_1^2)}\left(f_1^T + \frac{m_{\Lambda^*} - m_{\Sigma^0}}{m_{\Lambda^*} + m_{\Sigma^0}} f_2^T\right) ,
\end{aligned} \qquad (7)$$

Among the terms in Eq. (5) $f_1 + f_2$, $f_1^* + f_2^*$, $f_1^T + \frac{m_{\Sigma^*0} - m_\Lambda}{m_{\Sigma^*0} - m_\Lambda} f_2^T$, and $f_1^T + \frac{m_{\Lambda^*} - m_{\Sigma^0}}{m_{\Lambda^*} + m_{\Sigma^0}} f_2^T$ that are proportional to γ_μ, the first two describe the magnetic moments of the positive-to-positive and negative-to-negative parity baryon transitions, while the third and last ones correspond to the transition magnetic moments between positive and negative parity baryons. In the present work our goal is to calculate the magnetic moment between the negative parity Λ^* and Σ^{*0} baryons, and therefore the contributions of other three terms should be removed. In order to determine the aforementioned magnetic moment four equations are needed, since we have four unknown magnetic moments.

In constructing these four equations we need four Lorentz structures. In the present work we choose the following structures $\not{p}\not{\varepsilon}\not{q}$, $\not{p}\not{\varepsilon}$, $\not{\varepsilon}\not{q}$, $\not{\varepsilon}$, and denote the corresponding invariant functions as Π_1, Π_2, Π_3 and Π_4, respectively, on the QCD side of the correlation function. The result for the correlation function from the QCD side is obtained from Eq. (2) by using the general form of the interpolating currents given in Eq. (3). It should be noted here that taking into account contributions of all four matrix elements to the magnetic moment of Λ^*–Σ^{0*} transition may give considerable uncertainty in the final result.

The sum rules for the $\Lambda^* \to \Sigma^{0^*}$ transition is derived by equating the coefficients of the structures $\not{p}\not{\varepsilon}\not{q}$, $\not{p}\not{\varepsilon}$, $\not{\varepsilon}\not{q}$, $\not{\varepsilon}$ of the correlation from the phenomenological and QCD side, and perform double Borel transformation over the variables $p_1^2 = (p+q)^2$ and $p_2^2 = p^2$, and then solve the system of algebraic equations. After carrying out these steps of calculations we get the following expression for the magnetic moment of the $\Lambda^* \to \Sigma^{0*}$ transition,

$$\begin{aligned}
\mu_{\Lambda^*\to\Sigma^{0*}} = {} & \frac{e^{m^2_{\Sigma^{0*}}/M^2}}{\lambda_{\Lambda^*}\lambda_{\Sigma^{0*}}(m_{\Sigma^0}+m_{\Sigma^{0*}})(m^2_{\Sigma^0}+3m^2_{\Sigma^{0*}})} \\
& \times \Big\{ \Big[m_{\Sigma^0}(m_{\Sigma^0}-m_{\Sigma^{0*}}) - 2m^2_{\Sigma^{0*}} \Big] \Pi_1^B \\
& - 2m_{\Sigma^0}(m_{\Sigma^0}+m_{\Sigma^{0*}})\Pi_2^B - (m_{\Sigma^0}-3m_{\Sigma^{0*}})\Pi_3^B \\
& - m_{\Sigma^0}(m_{\Sigma^0}+m_{\Sigma^{0*}})\Pi_4^B \Big\} , \qquad (8)
\end{aligned}$$

where we have used $M_1^2 - M_2^2 = 2M^2$ and $m_\Lambda \simeq m_{\Sigma^0}$, $m_{\Lambda^*} \simeq m_{\Sigma^{0*}}$. The residues λ_{Λ^*} and $\lambda_{\Sigma^{0*}}$ are calculated in [9].

The expressions of the invariant functions Π_i^B appearing in Eq. (8) are presented in the Appendix.

Here, few remarks about the calculation of the correlation function from the QCD side are in order. This correlation function contains three different contributions. a) Perturbative part, which corresponds to the case when photon interacts with quarks perturbatively, and all propagators of the free quarks are considered. b) Mixed part which corresponds to the case when photon interacts with quarks perturbatively, and at least one quark propagator is replaced by the corresponding condensates. c) Nonperturbative part. In this case photon interacts with the quarks at long distance. The expansion of the quark operators up to twist four is calculated in [15], which receives contributions from the two-particle $\bar{q}q$, three-particle $\bar{q}Gq$, and four-particle $\bar{q}GGq$, $\bar{q}q\bar{q}q$ nonlocal operators, where G is the gluon field strength tensor. In the present work we consider contribution coming from the two-particle $\bar{q}q$, three-particle $\bar{q}Gq$ operators. The contributions coming from the four-particle nonlocal operators are negligible, which is justified on the basis of an expansion in conformal spin (for the details about this issue, see [15]). Therefore, the long distance contributions are described by the matrix elements of the two or three-particle nonlocal operators between the vacuum and one-photon states, i.e.,

$$\left\langle \gamma(q) \left| \bar{q}\Gamma_i \left(G_{\mu\nu}\Gamma_i\right) q \right| 0 \right\rangle ,$$

where Γ_i are the relevant Dirac matrices. These matrix elements are parametrized in terms of the photon DAs. The definition of the above-mentioned matrix elements in terms of the photon DAs, as well as the explicit expressions of the photon DAs can be found in [12].

3. Numerical results

This section is devoted to the numerical analysis of the sum rules obtained for the Λ–Σ^0 transition magnetic moment of the negative parity baryons. The values of the input parameters entering to the sum rules are: $\langle\bar{u}u\rangle(1\ \text{GeV}) = \langle\bar{d}d\rangle(1\ \text{GeV}) = -(0.243)^3\ \text{GeV}^3$, $\langle\bar{s}s\rangle(1\ \text{GeV}) = 0.8\langle\bar{u}u\rangle(1\ \text{GeV})$, $m_0^2 = (0.8 \pm 0.2)\ \text{GeV}^2$ [16], $\Lambda = (0.5 \pm 0.1)$ GeV [17], $f_{3\gamma} = -0.039$ [12]. The value of the magnetic susceptibility is determined from the QCD sum rules analysis as $\chi(1\ \text{GeV}) = -(2.85 \pm 0.5)\ \text{GeV}^{-2}$ [18], and $m_s(2\ \text{GeV}) = (111 \pm 6)$ MeV [19]. Also, the expressions of the photon DAs, which are the main ingredients of the LCSR, are presented in [12].

As has already been noted, the sum rules for the magnetic moment of the Λ^*–Σ^{0*} transition contain three auxiliary parameters, namely, the Borel mass parameter M^2, the parameter β that enters to the expression of the interpolating current, and the continuum threshold s_0. It is clear that the transition magnetic moment should be independent of them. For this reason we must find such regions of these parameters where this condition is satisfied. This can be achieved with the help of the following three-step analysis. At the first stage, we try to find the "working region" of the Borel mass parameter M^2 at the fixed values of s_0 and β, where the magnetic moment exhibits good stability under its variation. The upper bound of M^2 is determined by requiring that, the higher states and continuum contributions are less than 40–50% of the contributions coming from the perturbative part. The lower bound of M^2 is obtained from the condition that, higher twist contributions should be less than that of the leading twist contributions. Our numerical analysis shows that, if M^2 varies in the domain $1.6\ \text{GeV}^2 \le M^2 \le 3.0\ \text{GeV}^2$ both aforementioned conditions are satisfied. This region is also obtained in [9], in analysis of the diagonal transition magnetic moments of the Λ^* and Σ^{0*} baryons. In Fig. 1 dependence of the transition magnetic moment $\mu_{\Lambda^*\to\Sigma^{0*}}$ on the Borel mass parameter M^2 is presented at four different values of the auxiliary parameter $\beta = -5; -3; -1; 1$, and at two fixed values of the continuum threshold $s_0 = 4.4; 4.8\ \text{GeV}^2$. It follows from this analysis that the transition magnetic moment $\mu_{\Lambda^*\to\Sigma^{0*}}$ exhibits good stability to the variation of M^2.

The second arbitrary parameter of the sum rules is the continuum threshold s_0. This parameter is related to the energy of the first state. Analysis of the various sum rules shows that the energy difference between the first and ground states ranges from 0.3 GeV to 0.8 GeV. In our calculations we use the average value $\sqrt{s_0} = \left[m_{ground} + 0.4(0.5)\right]$ GeV, where m_{ground} is the mass of the ground state baryon.

In Fig. 2 we present the dependence of the transition magnetic moment $\mu_{\Lambda^*\to\Sigma^{0*}}$ on the continuum threshold s_0, at four different values of the auxiliary parameter $\beta = -5; -3; -1; 1$, and at two fixed values of the Borel parameter $M^2 = 2.4; 3.0\ \text{GeV}^2$. We observe that the transition magnetic moment $\mu_{\Lambda^*\to\Sigma^{0*}}$ demonstrates rather good stability to the variations in s_0.

The final step in our analysis is to find the working region of β, which varies in the domain $-\infty \le \beta \le \infty$. It is more convenient and practical to map this infinitely large region into a more restricted domain by introducing $\tan\theta = \beta$, where θ runs in the domain $-\pi/2 \le \theta \le \pi/2$. Using this relation one can show that,

$$\cos\theta = \pm\frac{1}{\sqrt{1+\beta^2}} ,$$

and it is much more convenient drawing the transition magnetic moment as a function of $\cos\theta$, instead of on the auxiliary parameter β; and still carries the same information. In Fig. 3 we present the dependence of the transition magnetic moment $\mu_{\Lambda^*\to\Sigma^{0*}}$ on $\cos\theta$, at the fixed value of the continuum threshold $s_0 = 4.4\ \text{GeV}^2$, at three fixed values of the Borel mass, namely, $M^2 = 1.8\ \text{GeV}^2$, $M^2 = 2.4\ \text{GeV}^2$, and $M^2 = 3.0\ \text{GeV}^2$ which lie in the "working region" of M^2. We observe that the aforementioned transition magnetic moment is insensitive to variation in $\cos\theta$ in the domain $-0.40 \le \cos\theta \le -0.25$. We also have performed similar analysis at two other fixed values of the continuum threshold, $s_0 = 4.0\ \text{GeV}^2$ and $s_0 = 4.8\ \text{GeV}^2$. From this analysis we obtain that the results for the transition magnetic moment change at most about 4–5%.

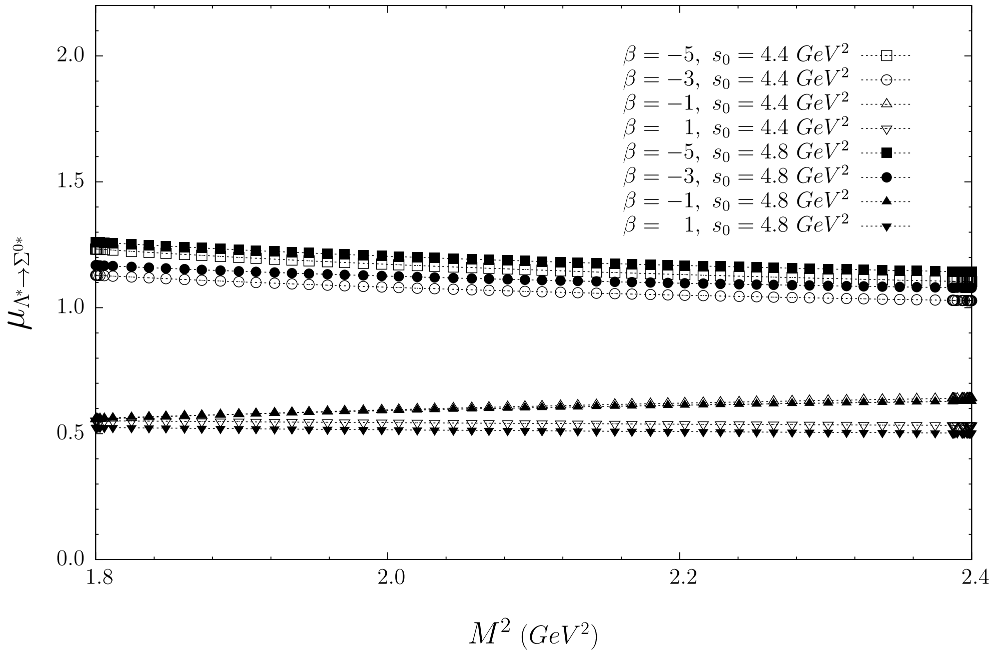

Fig. 1. Dependence of the transition magnetic moment $\mu_{\Lambda^* \to \Sigma^{0*}}$ on the Borel mass parameter M^2, at four different values of the auxiliary parameter $\beta = -5; -3; -1; 1$, and at two fixed values of the continuum threshold $s_0 = 4.4; 4.8$ GeV^2 in units of nuclear magneton μ_N.

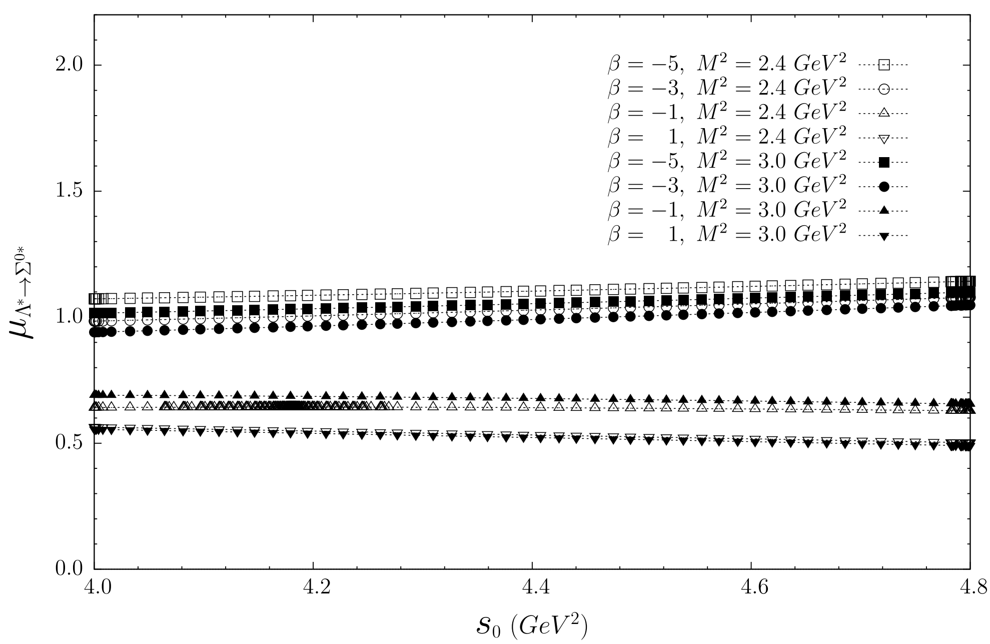

Fig. 2. Dependence of the transition magnetic moment $\mu_{\Lambda^* \to \Sigma^{0*}}$ on the continuum threshold s_0, at four different values of the auxiliary parameter $\beta = -5; -3; -1; 1$, and at two fixed values of the Borel parameter $M^2 = 2.4; 3.0$ GeV^2 in units of nuclear magneton μ_N.

As a result of our detailed numerical analysis, where we take into account the uncertainties in the input parameters, uncertainties entering into the photon DAs, as well as uncertainties due to the variations of M^2 and s_0, the transition magnetic moment $\mu_{\Lambda^* \to \Sigma^{0*}}$ is finally found to have the value,

$$\mu_{\Lambda^* \to \Sigma^{0*}} = (1.05 \pm 0.25)\mu_N \ .$$

Finally, we present our result on the transition magnetic moment $\mu_{\Lambda^* \to \Sigma^{0*}}$, and compare it with the predictions of the various approaches for the positive parity baryons, in Table 1.

When we compare the results given in Table 1, the magnetic moment for the negative parity Λ–Σ^0 transition is observed to be approximately m_N/m^* (where m^* is the averaged mass of the Σ^* and Λ^* baryons) times smaller compared to the results presented for the positive parity baryons predicted by the same approach cited in Table 1.

In conclusion, the magnetic moment of the Λ–Σ^0 transition for the negative parity baryons is estimated in framework of the LCSR. A comparison of our results with the predictions of the other approaches for the positive parity baryons is presented.

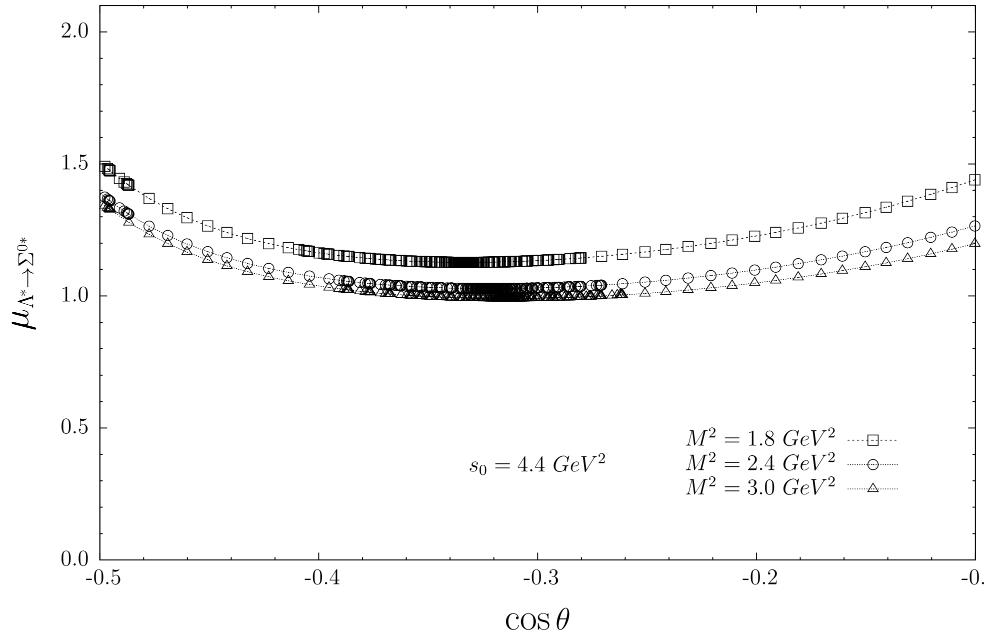

Fig. 3. Dependence of the transition magnetic moment $\mu_{\Lambda^* \to \Sigma^{0*}}$ on $\cos\theta$, at three fixed values of $M^2 = 1.8; 2.4; 3.0$ GeV^2, and at the fixed value of the continuum threshold $s_0 = 4.4$ GeV^2; in units of nuclear magneton μ_N.

Table 1
$\Lambda \to \Sigma^0$ transition magnetic moments of the negative parity (present work) and positive parity baryons (in units of nuclear magneton).

	present work	[6]	[7]	[8]	[20,21]	[22]	[23]	[24]
$\mu_{\Lambda \to \Sigma^0}$	1.05 ± 0.25	1.61	1.6	1.6	1.57	1.04	1.36	-1.48 ± 0.04

Appendix A

In this Appendix we present the expressions of the four invariant functions $\Pi_i^B(u,d,s)$ for the Λ^*–Σ^{0*} transition. In these expressions of the invariant functions $\Pi_i(u,d,s)$ masses of the light u and d quarks are set to zero.

1) Coefficient of the $\not{p}\not{\varepsilon}\not{q}$ structure

$$
\begin{aligned}
\Pi_1^B = {} & \frac{1}{64\sqrt{3}\pi^4}(1+\beta+\beta^2)(e_d - e_u)M^6 \\
& + \frac{1}{32\sqrt{3}\pi^2}(1-\beta)\beta m_s M^4 (\langle \bar{d}d\rangle e_d - e_u \langle \bar{u}u\rangle)\chi\varphi_\gamma(u_0) \\
& + \frac{1}{32\sqrt{3}\pi^2}(1-\beta)\beta m_s M^4 (\langle \bar{d}d\rangle e_d - e_u \langle \bar{u}u\rangle)\chi\varphi_\gamma(u_0) \\
& + \frac{1}{128\sqrt{3}\pi^2}(e_d - e_u) f_{3\gamma} M^4 \\
& \times \Big\{(1+4\beta+\beta^2)\Big[i_3(\mathcal{A},1) - 2i_3(\mathcal{A},v)\Big] \\
& - 2(1+\beta+\beta^2)\Big[i_3(\mathcal{V},1) + 2\psi^a(u_0)\Big]\Big\} \\
& + \frac{1}{64\sqrt{3}\pi^2} m_s M^2 \Big\{4(1-\beta)\langle \bar{d}d\rangle (e_s - e_u - 2\beta e_u) \\
& + 4(1+\beta+\beta^2)(e_d - e_u)\langle \bar{s}s\rangle \\
& + 4(1-\beta)(e_d + 2\beta e_d - e_s)\langle \bar{u}u\rangle - (1-\beta)(\langle \bar{d}d\rangle e_d - e_u\langle \bar{u}u\rangle) \\
& \times \Big[i_2(\widetilde{\mathcal{S}},1) + 2i_2(\mathcal{T}_1,1) - 2i_2(\mathcal{T}_2,1) + 3i_2(\mathcal{T}_3,1) \\
& - 3i_2(\mathcal{T}_4,1) - 2i_2(\widetilde{\mathcal{S}},v) + \beta\Big(\mathbb{A}(u_0) - 4i_2(\mathcal{S},1) - 3i_2(\widetilde{\mathcal{S}},1) \\
& + 6i_2(\mathcal{T}_1,1) - 6i_2(\mathcal{T}_2,1) - i_2(\mathcal{T}_3,1) + i_2(\mathcal{T}_4,1) + 2i_2(\widetilde{\mathcal{S}},v) \\
& + 4i_2(\mathcal{T}_3,v) - 4i_2(\mathcal{T}_4,v)\Big)\Big]\Big\} \\
& + \frac{1}{64\sqrt{3}\pi^2}(1-\beta)\left(\gamma_E - \ln\frac{\Lambda^2}{M^2}\right) m_s M^2 \\
& \times \Big\{4e_s(\langle \bar{d}d\rangle - \langle \bar{u}u\rangle) - (\langle \bar{d}d\rangle e_d - e_u\langle \bar{u}u\rangle) \\
& \times \Big[2i_2(\mathcal{S},1) - 2\beta i_2(\widetilde{\mathcal{S}},1) + (1+3\beta)i_2(\mathcal{T}_1,1) \\
& - (1+3\beta)i_2(\mathcal{T}_2,1) + (3+\beta)i_2(\mathcal{T}_3,1) - (3+\beta)i_2(\mathcal{T}_4,1)\Big]\Big\} \\
& + \frac{1}{384\sqrt{3}\pi^2}(1-\beta)\left(\gamma_E - \ln\frac{\Lambda^2}{M^2}\right) m_s \Big\{m_0^2\Big[-(2-\beta)\langle \bar{d}d\rangle e_s \\
& + 9(1+\beta)\langle \bar{d}d\rangle e_u - \Big(9(1+\beta)e_d + (-2+\beta)e_s\Big)\langle \bar{u}u\rangle\Big] \\
& + \beta\langle g_s^2 G^2\rangle(\langle \bar{d}d\rangle e_d - e_u\langle \bar{u}u\rangle)\chi\varphi_\gamma(u_0)\Big\} \\
& + \frac{1}{1536\sqrt{3}\pi^2 M^2}(1-\beta)\langle g_s^2 G^2\rangle m_s(\langle \bar{d}d\rangle e_d - e_u\langle \bar{u}u\rangle) \\
& \times \Big\{-2i_2(\mathcal{S},1) + i_2(\widetilde{\mathcal{S}},1) + i_2(\mathcal{T}_1,1) - i_2(\mathcal{T}_2,1) - 2i_2(\widetilde{\mathcal{S}},v) \\
& + \beta\Big[\mathbb{A}(u_0) - 4i_2(\mathcal{S},1) - i_2(\widetilde{\mathcal{S}},1) + 3i_2(\mathcal{T}_1,1) - 3i_2(\mathcal{T}_2,1) \\
& + 2\Big(-i_2(\mathcal{T}_3,1) + i_2(\mathcal{T}_4,1) + i_2(\widetilde{\mathcal{S}},v) + 2i_2(\mathcal{T}_3,v) \\
& - 2i_2(\mathcal{T}_4,v)\Big)\Big]\Big\} \\
& + \frac{1}{384\sqrt{3}\pi^2 M^2}(1-\beta)m_s(\langle \bar{d}d\rangle e_u - e_d\langle \bar{u}u\rangle)
\end{aligned}
$$

$$\times \Big[\langle g_s^2 G^2\rangle + 2\beta\langle g_s^2 G^2\rangle + 6(1+\beta) f_{3\gamma} m_0^2 \pi^2 \psi^a(u_0)\Big]$$
$$+ \frac{1}{4608\sqrt{3}\pi^2 M^4}(1-\beta)(1+2\beta)\langle g_s^2 G^2\rangle m_s(\langle\bar{d}d\rangle e_u - e_d\langle\bar{u}u\rangle)$$
$$\times \Big[3m_0^2 + 8 f_{3\gamma}\pi^2\psi^a(u_0)\Big]$$
$$+ \frac{1}{1152\sqrt{3}M^6}(1-\beta)(1+2\beta) e f_{3\gamma}\langle g_s^2 G^2\rangle m_0^2 m_s$$
$$\times (\langle\bar{d}d\rangle e_u - e_d\langle\bar{u}u\rangle)\psi^a(u_0)$$
$$+ \frac{1}{192\sqrt{3}}(e_d - e_u) f_{3\gamma} m_s\langle\bar{s}s\rangle\Big\{(5+14\beta+5\beta^2) i_3(\mathcal{A},1)$$
$$-(7+10\beta+7\beta^2) i_3(\mathcal{V},1)$$
$$+2\Big[-2(1+4\beta+\beta^2) i_3(\mathcal{A},v) + 3(1+\beta)^2 i_3(\mathcal{V},v)$$
$$-4(1+\beta+\beta^2)\psi^a(u_0)\Big]\Big\}$$
$$+ \frac{1}{576\sqrt{3}\pi^2}(1-\beta) m_s\Big\{-\beta\langle g_s^2 G^2\rangle(\langle\bar{d}d\rangle e_d - e_u\langle\bar{u}u\rangle)$$
$$\times \chi\varphi_\gamma(u_0)$$
$$+3(1+2\beta)(\langle\bar{d}d\rangle e_u - e_d\langle\bar{u}u\rangle)\Big[3m_0^2 + 8 f_{3\gamma}\pi^2\psi^a(u_0)\Big]\Big\}.$$

2) Coefficient of the $\not{p}\not{q}$ structure

$$\Pi_2^B = \frac{1}{768\sqrt{3}\pi^4}(1-\beta) M^6\Big\{3(3+\beta)(e_d - e_u) m_s$$
$$+4(1-\beta)\pi^2(\langle\bar{d}d\rangle e_d - e_u\langle\bar{u}u\rangle)\chi\varphi_\gamma'(u_0)\Big\}$$
$$+ \frac{1}{256\sqrt{3}\pi^2}(1-\beta) M^4(\langle\bar{d}d\rangle e_d - e_u\langle\bar{u}u\rangle)$$
$$\times\Big\{-2(1-\beta) i_3(\mathcal{S},1)$$
$$+2(1-\beta)\Big[i_3(\widetilde{\mathcal{S}},1) + i_3(\mathcal{T}_2,1) - 2i_3(\mathcal{T}_3,1) + i_3(\mathcal{T}_4,1)$$
$$-2\Big(i_3(\mathcal{T}_2,v) - 2i_3(\mathcal{T}_3,v) + i_3(\mathcal{T}_4,v)\Big)\Big]$$
$$+8(2+\beta)\widetilde{j}_1(h_\gamma) - (1-\beta)\mathbb{A}'(u_0)\Big\}$$
$$+ \frac{1}{128\sqrt{3}\pi^2}(1-\beta) M^4\Big\{2(3+\beta)\Big[\langle\bar{d}d\rangle(e_s - e_u)$$
$$-(e_d - e_u)\langle\bar{s}s\rangle + (e_d - e_s)\langle\bar{u}u\rangle$$
$$+(e_d - e_u) f_{3\gamma} m_s\psi^v(u_0)\Big] - (1+\beta)(e_d - e_u) f_{3\gamma} m_s\psi^{a\prime}(u_0)\Big\}$$
$$+ \frac{1}{3072\sqrt{3}\pi^4}(1-\beta) M^2\Big\{e_u\Big[-(3+\beta)\langle g_s^2 G^2\rangle m_s$$
$$+4(11+5\beta) m_0^2\pi^2(\langle\bar{d}d\rangle - \langle\bar{s}s\rangle)\Big]$$
$$+e_d\Big[(3+\beta)\langle g_s^2 G^2\rangle m_s + 4(11+5\beta) m_0^2\pi^2(\langle\bar{s}s\rangle - \langle\bar{u}u\rangle)\Big]$$
$$-4(11+5\beta) e_s m_0^2\pi^2(\langle\bar{d}d\rangle - \langle\bar{u}u\rangle)\Big\}$$
$$- \frac{1}{24\sqrt{3}}(1-\beta)(3+\beta) f_{3\gamma} M^2\Big[e_u(\langle\bar{d}d\rangle - \langle\bar{s}s\rangle)$$
$$+e_d(\langle\bar{s}s\rangle - \langle\bar{u}u\rangle) + e_s(\langle\bar{u}u\rangle - \langle\bar{d}d\rangle)\Big]\psi^v(u_0)$$
$$- \frac{1}{48\sqrt{3}}(1-\beta) f_{3\gamma} M^2\Big[\langle\bar{d}d\rangle(\beta e_u - e_s)$$
$$+(1+\beta)(e_u - e_d)\langle\bar{s}s\rangle - (\beta e_d - e_s)\langle\bar{u}u\rangle\Big]\psi^{a\prime}(u_0)$$
$$+ \frac{1}{1024\sqrt{3}\pi^4}(1-\beta)(3+\beta)(e_d - e_u)\left(\gamma_E - \ln\frac{\Lambda^2}{M^2}\right)$$
$$\times\langle g_s^2 G^2\rangle m_s M^2$$
$$+ \frac{1}{1536\sqrt{3}\pi^2}(1-\beta)(e_d - e_u)\left(\gamma_E - \ln\frac{\Lambda^2}{M^2}\right) f_{3\gamma}\langle g_s^2 G^2\rangle m_s$$
$$\times\Big[2(3+\beta)\psi^v(u_0) - (1+\beta)\psi^{a\prime}(u_0)\Big]$$
$$+ \frac{1}{1152\sqrt{3}\pi^2}(1-\beta) f_{3\gamma}\Big\{e_u\Big[(3+\beta)\langle g_s^2 G^2\rangle m_s$$
$$+2(11+5\beta) m_0^2\pi^2(\langle\bar{d}d\rangle - \langle\bar{s}s\rangle)\Big]$$
$$-e_d\Big[(3+\beta)\langle g_s^2 G^2\rangle m_s - 2(11+5\beta) m_0^2\pi^2(\langle\bar{s}s\rangle - \langle\bar{u}u\rangle)\Big]$$
$$-2(11+5\beta) e_s m_0^2\pi^2(\langle\bar{d}d\rangle - \langle\bar{u}u\rangle)\Big\}\psi^v(u_0)$$
$$+ \frac{1}{2304\sqrt{3}\pi^2}(1-\beta) f_{3\gamma}\Big\{e_u\Big[-(1+\beta)\langle g_s^2 G^2\rangle m_s$$
$$+2m_0^2\pi^2\Big((2+5\beta)\langle\bar{d}d\rangle + 4(1+\beta)\langle\bar{s}s\rangle\Big)\Big]$$
$$-2(2-\beta) e_s m_0^2\pi^2(\langle\bar{d}d\rangle - \langle\bar{u}u\rangle) + e_d\Big[(1+\beta)\langle g_s^2 G^2\rangle m_s$$
$$-2m_0^2\pi^2(4(1+\beta)\langle\bar{s}s\rangle$$
$$+(2+5\beta)\langle\bar{u}u\rangle)\Big]\Big\}\psi^{a\prime}(u_0).$$

3) Coefficient of the $\not{\varepsilon}\not{q}$ structure

$$\Pi_3^B = -\frac{1}{1536\sqrt{3}\pi^4}(1-\beta) M^6\Big\{3(13+11\beta)(e_d - e_u) m_s$$
$$-4\pi^2(\langle\bar{d}d\rangle e_d - e_u\langle\bar{u}u\rangle)\chi$$
$$\times\Big[2(7+11\beta)\varphi_\gamma(u_0) + (1-\beta)\varphi_\gamma'(u_0)\Big]\Big\}$$
$$- \frac{1}{512\sqrt{3}\pi^2}(1-\beta) M^4(\langle\bar{d}d\rangle e_d - e_u\langle\bar{u}u\rangle)$$
$$\times\Big\{2\Big[(5+7\beta)\mathbb{A}(u_0) + 6i_2(\mathcal{S},1) + 2i_2(\widetilde{\mathcal{S}},1)$$
$$+2i_2(\mathcal{T}_2,1) + 4i_2(\mathcal{T}_3,1) - 6i_2(\mathcal{T}_4,1) - 4i_2(\mathcal{T}_2,v)$$
$$-8i_2(\mathcal{T}_3,v) + 12i_2(\mathcal{T}_4,v) - i_3(\mathcal{S},1)$$
$$+i_3(\widetilde{\mathcal{S}},1) + i_3(\mathcal{T}_2,1) - 2i_3(\mathcal{T}_3,1) + i_3(\mathcal{T}_4,1)$$
$$-\beta\Big(6i_2(\mathcal{S},1) + 2i_2(\widetilde{\mathcal{S}},1) + 2i_2(\mathcal{T}_2,1)$$
$$+4i_2(\mathcal{T}_3,1) - 6i_2(\mathcal{T}_4,1 - 4i_2(\mathcal{T}_2,v) - 8i_2(\mathcal{T}_3,v)$$
$$+12i_2(\mathcal{T}_4,v) - i_3(\mathcal{S},1) + i_3(\widetilde{\mathcal{S}},1) + i_3(\mathcal{T}_2,1)$$
$$-2i_3(\mathcal{T}_3,1) + i_3(\mathcal{T}_4,1) - 2i_3(\mathcal{T}_2,v) - 4i_3(\mathcal{T}_3,v) + 2i_3(\mathcal{T}_4,v)$$
$$+4\widetilde{j}_1(h_\gamma) - 8\widetilde{j}_2(h_\gamma)\Big) - 2\Big(i_3(\mathcal{T}_2,v) - 2i_3(\mathcal{T}_3,v) + i_3(\mathcal{T}_4,v)$$
$$-4\widetilde{j}_1(h_\gamma) + 8\widetilde{j}_2(h_\gamma)\Big)\Big] - (1-\beta)\mathbb{A}'(u_0)\Big\}$$
$$- \frac{1}{256\sqrt{3}\pi^2}(1-\beta) M^4\Big\{2\langle\bar{d}d\rangle((-3+\beta) e_s + (-3+5\beta) e_u)$$
$$-2(9+7\beta)(e_d - e_u)\langle\bar{s}s\rangle + 2\Big[(3-5\beta) e_d + (3-\beta) e_s\Big]\langle\bar{u}u\rangle$$
$$+(e_d - e_u) f_{3\gamma} m_s\Big[2(5+7\beta) i_3(\mathcal{A},1) - 2(7+5\beta) i_3(\mathcal{V},1)$$
$$-8(1+\beta) i_3(\mathcal{A},v) + 4(3+\beta) i_3(\mathcal{V},v) - 4(3+\beta)\widetilde{j}_1(\psi^v)$$
$$-6(1+\beta)\psi^a(u_0) + 2(3+\beta)\psi^v(u_0) - (1+\beta)\psi^{a\prime}(u_0)\Big]\Big\}$$

$$
\begin{aligned}
&- \frac{1}{2048\sqrt{3}\pi^4}(1-\beta)(5+3\beta)(e_d-e_u)\left(\gamma_E - \ln\frac{\Lambda^2}{M^2}\right)\\
&\times \langle g_s^2 G^2\rangle m_s M^2\\
&- \frac{1}{64\sqrt{3}\pi^2}(1-\beta^2)(e_d-e_u)\left(\gamma_E - \ln\frac{\Lambda^2}{M^2}\right) f_{3\gamma} m_s M^4\\
&\times \Big[i_3(\mathcal{A},1) - i_3(\mathcal{V},1)\Big]\\
&- \frac{1}{3072\sqrt{3}\pi^2}(1-\beta)(e_d-e_u)\left(\gamma_E - \ln\frac{\Lambda^2}{M^2}\right) f_{3\gamma}\langle g_s^2 G^2\rangle m_s\\
&\times \Big[4(3+\beta)\widetilde{j}_1(\psi^v) - 2(1+\beta)\psi^a(u_0)\\
&- 2(3+\beta)\psi^v(u_0) + (1+\beta)\psi^{a\prime}(u_0)\Big]\\
&+ \frac{1}{3072\sqrt{3}\pi^2}(1-\beta)(e_d-e_u) f_{3\gamma}\langle g_s^2 G^2\rangle m_s\\
&\times \Big[(3+5\beta)i_3(\mathcal{A},1) - (5+3\beta)i_3(\mathcal{V},1)\\
&- 4(1+\beta)i_3(\mathcal{A},v) + 2(3+\beta)i_3(\mathcal{V},v)\Big]\\
&- \frac{1}{4608\sqrt{3}\pi^2}(1-\beta)(e_d-e_u) f_{3\gamma}\langle g_s^2 G^2\rangle m_s\\
&\times \Big\{4(3+\beta)\widetilde{j}_1(\psi^v) + 4(1+\beta)\psi^a(u_0)\\
&- 2(3+\beta)\psi^v(u_0) + (1+\beta)\psi^{a\prime}(u_0)\Big\}\\
&+ \frac{1}{1152\sqrt{3}}(1-\beta)(e_d-e_u) f_{3\gamma} m_0^2 ss\\
&\times \Big\{2(11+5\beta)\widetilde{j}_1(\psi^v) - 4(1+\beta)\psi^a(u_0)\\
&- (11+5\beta)\psi^v(u_0) + 2(1+\beta)\psi^{a\prime}(u_0)\Big\}\\
&- \frac{1}{2304\sqrt{3}}(1-\beta) f_{3\gamma} m_0^2\langle\bar{u}u\rangle\Big\{4(11+5\beta)(e_d-e_s)\widetilde{j}_1(\psi^v)\\
&+ 2\Big[(2+5\beta)e_d - (2-\beta)e_s\Big]\psi^a(u_0)\\
&- 2(11+5\beta)(e_d-e_s)\psi^v(u_0)\\
&- \Big[(2+5\beta)e_d - (2-\beta)e_s\Big]\psi^{a\prime}(u_0)\Big\}\\
&- \frac{1}{2304\sqrt{3}}(1-\beta)\langle\bar{d}d\rangle f_{3\gamma} m_0^2\Big\{4(11+5\beta)(e_s-e_u)\widetilde{j}_1(\psi^v)\\
&+ 2\Big[(2-\beta)e_s - (2+5\beta)e_u\Big]\psi^a(u_0)\\
&- 2(11+5\beta)(e_s-e_u)\psi^v(u_0)\\
&- \Big[(2-\beta)e_s - (2+5\beta)e_u\Big]\psi^{a\prime}(u_0)\Big\}\,.
\end{aligned}
$$

4) Coefficient of the $\not{\varepsilon}$ structure

$$
\begin{aligned}
\Pi_4^B = &- \frac{1}{64\sqrt{3}\pi^4}(1+\beta+\beta^2)(e_d-e_u)M^8\\
&- \frac{\sqrt{3}}{128\pi^2}(1-\beta^2)m_s M^6(\langle\bar{d}d\rangle e_d - e_u\langle\bar{u}u\rangle)\chi\varphi_\gamma'(u_0)\\
&- \frac{1}{128\sqrt{3}\pi^2}(e_d-e_u) f_{3\gamma} M^6\Big\{(1+4\beta+\beta^2)i_4(\mathcal{A},v)\\
&+ 2(1+\beta+\beta^2)\Big[i_4(\mathcal{V},v) + 4\psi^v(u_0) - \psi^{a\prime}(u_0)\Big]\Big\}\\
&- \frac{1}{256\sqrt{3}\pi^2} m_s M^4\Big\{8(2-\beta-\beta^2)\langle\bar{d}d\rangle(e_s-e_u)\\
&+ 16(1+\beta+\beta^2)(e_d-e_u)\langle\bar{s}s\rangle + 8(2-\beta-\beta^2)(e_d-e_s)\langle\bar{u}u\rangle\\
&+ (1-\beta)(\langle\bar{d}d\rangle e_d - e_u\langle\bar{u}u\rangle)\Big[2(7-\beta)i_3(\mathcal{S},1)\\
&+ 2(1-7\beta)i_3(\widetilde{\mathcal{S}},1) + 12 i_3(\mathcal{T}_1,1) + 2i_3(\mathcal{T}_2,1) - 14 i_3(\mathcal{T}_4,1)\\
&- 16 i_3(\mathcal{S},v) - 4i_3(\widetilde{\mathcal{S}},v) - 12 i_3(\mathcal{T}_2,v) + 12 i_3(\mathcal{T}_3,v)\\
&+ 24\widetilde{j}_1(h_\gamma) + 2\beta\Big(6i_3(\mathcal{T}_1,1) - 7i_3(\mathcal{T}_2,1) + i_3(\mathcal{T}_4,1)\\
&- 4i_3(\mathcal{S},v) + 2i_3(\widetilde{\mathcal{S}},v) - 2i_3(\mathcal{T}_2,v) + 6i_3(\mathcal{T}_3,v)\\
&- 4i_3(\mathcal{T}_4,v) + 4\widetilde{j}_1(h_\gamma)\Big) - 3(1+\beta)\mathbb{A}'(u_0)\Big]\Big\}\\
&+ \frac{1}{768\sqrt{3}\pi^2}(1-\beta)m_s M^2\Big\{6m_0^2\Big[(3+2\beta)\langle\bar{d}d\rangle e_s\\
&+ 3(1+\beta)\langle\bar{d}d\rangle e_u - \Big(3(1+\beta)e_d + (3+2\beta)e_s\Big)\langle\bar{u}u\rangle\Big]\\
&+ 64(2+\beta) f_{3\gamma}\pi^2(\langle\bar{d}d\rangle e_u - e_d\langle\bar{u}u\rangle)\psi^v(u_0)\\
&+ (1+\beta)\langle g_s^2 G^2\rangle(\langle\bar{d}d\rangle e_d - e_u\langle\bar{u}u\rangle)\chi\varphi_\gamma'(u_0)\\
&- 16(1+2\beta) f_{3\gamma}\pi^2(\langle\bar{d}d\rangle e_u - e_d\langle\bar{u}u\rangle)\psi^{a\prime}(u_0)\Big\}\\
&+ \frac{1}{384\sqrt{3}\pi^2}(e_d-e_u)m_s M^2\langle\bar{s}s\rangle\Big\{(5+8\beta+5\beta^2)m_0^2\\
&+ 2f_{3\gamma}\pi^2\Big[(1-\beta)^2 i_4(\mathcal{A},v) - (1-\beta)^2 i_4(\mathcal{V},v)\\
&- 4(1+\beta+\beta^2)(4\psi^v(u_0) - \psi^{a\prime}(u_0))\Big]\Big\}\\
&- \frac{1}{512\sqrt{3}\pi^2}(1-\beta^2)\left(\gamma_E - \ln\frac{\Lambda^2}{M^2}\right)\langle g_s^2 G^2\rangle m_s M^2\\
&\times (\langle\bar{d}d\rangle e_d - e_u\langle\bar{u}u\rangle)\chi\varphi_\gamma'(u_0)\\
&- \frac{1}{128\sqrt{3}\pi^2}(1-\beta)\left(\gamma_E - \ln\frac{\Lambda^2}{M^2}\right)m_s M^4(\langle\bar{d}d\rangle e_d - e_u\langle\bar{u}u\rangle)\\
&\times \Big\{(5+\beta)\Big[i_3(\mathcal{S},1) - i_3(\mathcal{T}_4,1)\Big]\\
&- (1+5\beta)\Big[i_3(\widetilde{\mathcal{S}},1) + i_3(\mathcal{T}_2,1)\Big]\\
&+ 3(1+\beta)\Big[i_3(\mathcal{T}_1,1) + i_3(\mathcal{T}_3,1)\Big]\Big\}\\
&- \frac{1}{384\sqrt{3}\pi^2}(2-\beta-\beta^2)\left(\gamma_E - \ln\frac{\Lambda^2}{M^2}\right)\langle g_s^2 G^2\rangle m_s\\
&\times (\langle\bar{d}d\rangle e_u - e_d\langle\bar{u}u\rangle)\\
&- \frac{1}{2304\sqrt{3}\pi^2 M^2}(1-\beta)\langle g_s^2 G^2\rangle m_s(\langle\bar{d}d\rangle e_u - e_d\langle\bar{u}u\rangle)\\
&\times \Big\{3(2+\beta)m_0^2 - 2f_{3\gamma}\pi^2\Big[4(2+\beta)\psi^v(u_0)\\
&- (1+2\beta)\psi^{a\prime}(u_0)\Big]\Big\}\\
&+ \frac{1}{2304\sqrt{3}M^4}(1-\beta) f_{3\gamma}\langle g_s^2 G^2\rangle m_0^2 m_s(\langle\bar{d}d\rangle e_u - e_d\langle\bar{u}u\rangle)\\
&\times \Big[4(2+\beta)\psi^v(u_0) - (1+2\beta)\psi^{a\prime}(u_0)\Big]\\
&- \frac{1}{6144\sqrt{3}\pi^2}(1-\beta)\langle g_s^2 G^2\rangle m_s(\langle\bar{d}d\rangle e_d - e_u\langle\bar{u}u\rangle)\\
&\times \Big\{-4(1-\beta)i_3(\mathcal{S},1) + \Big[-4(1-\beta)i_3(\widetilde{\mathcal{S}},1)\\
&- 6(1+\beta)i_3(\mathcal{T}_1,1) - 4i_3(\mathcal{T}_2,1) + 6i_3(\mathcal{T}_3,1)\\
&+ 4\Big(i_3(\mathcal{T}_4,1) + 4i_3(\mathcal{S},v) + i_3(\widetilde{\mathcal{S}},v) + 3i_3(\mathcal{T}_2,v)
\end{aligned}
$$

$$
\begin{aligned}
& - 3 i_3(\mathcal{T}_3, v) - 6\widetilde{j}_1(h_\gamma)\Big) + 2\beta\Big(2 i_3(\mathcal{T}_2, 1) + 3 i_3(\mathcal{T}_3, 1) \\
& - 2 i_3(\mathcal{T}_4, 1) + 4 i_3(\mathcal{S}, v) - 2 i_3(\widetilde{\mathcal{S}}, v) + 2 i_3(\mathcal{T}_2, v) \\
& - 6 i_3(\mathcal{T}_3, v) + 4 i_3(\mathcal{T}_4, v) - 4\widetilde{j}_1(h_\gamma)\Big)\Big] + 3(1+\beta)\mathbb{A}'(u_0)\Big\} \\
& - \frac{1}{1152\sqrt{3}\pi^2}(1-\beta) m_s(\langle \bar{d}d\rangle e_u - e_d\langle \bar{u}u\rangle)\Big[(2+\beta)\langle g_s^2 G^2\rangle \\
& - 9(1+\beta) f_{3\gamma} m_0^2 \pi^2 \Big(4\psi^v(u_0) - \psi^{a\prime}(u_0)\Big)\Big] .
\end{aligned}
$$

The functions i_n $(n = 1, 2)$, and $\widetilde{j}_1(f(u))$ are defined as:

$$i_0(\phi, f(v)) = \int \mathcal{D}\alpha_i \int_0^1 dv \phi(\alpha_{\bar{q}}, \alpha_q, \alpha_g) f(v)(k-u_0)\theta(k-u_0) ,$$

$$i_1(\phi, f(v)) = \int \mathcal{D}\alpha_i \int_0^1 dv \phi(\alpha_{\bar{q}}, \alpha_q, \alpha_g) f(v)\theta(k-u_0) ,$$

$$i_2(\phi, f(v)) = \int \mathcal{D}\alpha_i \int_0^1 dv \phi(\alpha_{\bar{q}}, \alpha_q, \alpha_g) f(v)\delta(k-u_0) ,$$

$$i_3(\phi, f(v)) = \int \mathcal{D}\alpha_i \int_0^1 dv \phi(\alpha_{\bar{q}}, \alpha_q, \alpha_g) f(v)\delta'(k-u_0) ,$$

$$i_4(\phi, f(v)) = \int \mathcal{D}\alpha_i \int_0^1 dv \phi(\alpha_{\bar{q}}, \alpha_q, \alpha_g) f(v)\delta''(k-u_0) ,$$

$$\widetilde{j}_1(f(u)) = \int_{u_0}^1 du f(u) ,$$

$$\widetilde{j}_2(f(u)) = \int_{u_0}^1 du (u-u_0) f(u) ,$$

where

$$k = \alpha_q + \alpha_g \bar{v} , \qquad u_0 = \frac{M_1^2}{M_1^2 + M_2^2} , \qquad M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2} .$$

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