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Colliding branes and formation of spacetime singularities

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Abstract

We construct a class of analytic solutions with two free parameters to the five-dimensional Einstein field equations, which represents the collision of two timelike 3-branes. We study the local and global properties of the spacetime, and find that spacelike singularities generically develop after the collision, due to the mutual focus of the two branes. Non-singular spacetime can be constructed only in the case where both of the two branes violate the energy conditions.

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1. Introduction

Branes in string/M-Theory are fundamental constituents [1], and of particular relevance to cosmology [2,3]. These substances can move freely in bulk, collide, recoil, reconnect, and whereby form a brane gas in the early universe [4], or create an ekpyrotic/cyclic universe [5]. Understanding these processes is fundamental to both string/M-Theory and their applications to cosmology.

Recently, Maeda and his collaborators numerically studied the collision of two branes in a five-dimensional bulk, and found that the formation of a spacelike singularity after the collision is generic [6] (see also [7]). This is a very important result, as it implies that a low-energy description of colliding branes breaks down at some point, and a complete predictability is lost, without the complete theory of quantum gravity. Similar conclusions were obtained from the studies of two colliding orbifold branes [8]. However, lately it was argued that, from the point of view of the higher dimensional spacetime where the low effective action was derived, these singularities are very mild and can be easily regularised [9].

In this Letter, we present a class of analytic solutions to the five-dimensional Einstein field equations, which represents the collision of two timelike 3-branes in a five-dimensional vacuum bulk, and show explicitly that a spacelike singularity always develops after the collision due to the mutual focus of the two branes, when both of them satisfy the energy conditions. If only one of them satisfies the energy conditions, spacetime singularities always exist too, but these singularities may appear either before or after the collision. Non-singular spacetimes can be constructed only in the case where both of the two branes violate the energy conditions. Specifically, the Letter is organized as follows: in Section 2 we first present such solutions, and then study their local and global properties, while in Section 3 we present our main conclusions and remarks.

2. Colliding timelike 3-branes

Let us consider the solutions,

$$ds_5^2 = A^{-2/3}(t,y)\big(dt^2 - dy^2\big) - A^{2/3}(t,y)\,d\Sigma_0^2, \tag{2.1}$$

where $d\Sigma_0^2 \equiv (dx^2)^2 + (dx^3)^2 + (dx^4)^2$, $x^A = \{t, y, x^i\}$ $(i = 2, 3, 4)$, and

$$A(t,y) = a(t+by)H(t+by) + b(t-ay)H(t-ay) + A_0, \tag{2.2}$$

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with a, b and A_0 being arbitrary constants, and $H(x)$ the Heavside function, defined as

$$H(x) = \begin{cases} 1, & x > 0, \\ 0, & x < 0. \end{cases} \tag{2.3}$$

Without loss of generality, we assume $a \neq -b$ and $A_0 > 0$. Then, it can be shown that the corresponding spacetime is vacuum, except on the hypersurfaces $t = ay$ and $t = -by$, where the non-vanishing components of the Einstein tensor are given by

$$\begin{aligned} G_{00} &= -ab\left(\frac{a\delta(t-ay)}{A} + \frac{b\delta(t+by)}{A}\right), \\ G_{01} &= ab\left(\frac{\delta(t-ay)}{A} - \frac{\delta(t+by)}{A}\right), \\ G_{11} &= -\left(\frac{b\delta(t-ay)}{A} + \frac{a\delta(t+by)}{A}\right), \\ G_{ij} &= \frac{1}{3}A^{1/3}\delta_{ij}\big(b\big(a^2-1\big)\delta(t-ay) \\ &\quad + a\big(b^2-1\big)\delta(t+by)\big), \end{aligned} \tag{2.4}$$

where $\delta(x)$ denotes the Dirac delta function. As we will see in the following, with the proper choice of the free parameters a and b, on each of these two hypersurfaces the spacetime represents a 3-brane filled with a perfect fluid.

The normal vector to the surfaces $t - ay = 0$ and $t + by = 0$ are given, respectively, by

$$\begin{aligned} n_A &\equiv \frac{\partial(t-ay)}{\partial x^A} = \delta^t_A - a\delta^y_A, \\ l_A &\equiv \frac{\partial(t+by)}{\partial x^A} = \delta^t_A + b\delta^y_A, \end{aligned} \tag{2.5}$$

for which we find

$$\begin{aligned} n_A n^A &= -A^{2/3}\big(a^2-1\big), \\ l_A l^A &= -A^{2/3}\big(b^2-1\big). \end{aligned} \tag{2.6}$$

Thus, in order to have these surfaces be timelike, we must choose a and b such that

$$a^2 > 1, \qquad b^2 > 1. \tag{2.7}$$

Introducing the timelike vectors u_A and v_A along each of the two 3-branes by

$$\begin{aligned} u_A &= \frac{1}{A_u^{1/3}(t)(a^2-1)^{1/2}}\big(a\delta^t_A - \delta^y_A\big), \\ v_A &= \frac{1}{A_v^{1/3}(t)(b^2-1)^{1/2}}\big(b\delta^t_A + \delta^y_A\big), \end{aligned} \tag{2.8}$$

where

$$\begin{aligned} A_u(t) &\equiv A(t,y)|_{y=t/a} = (a+b)tH\left(t + \frac{b}{a}t\right) + A_0, \\ A_v(t) &\equiv A(t,y)|_{y=-t/b} = (a+b)tH\left(t + \frac{a}{b}t\right) + A_0, \end{aligned} \tag{2.9}$$

we find $u_A n^A = 0 = v_A l^A$. From the five-dimensional Einstein field equations, $G_{AB} = \kappa T_{AB}$, we obtain

$$T_{AB} = A_u^{1/3} T^{(u)}_{AB}\delta(t-ay) + A_v^{1/3} T^{(v)}_{AB}\delta(t+by), \tag{2.10}$$

where

$$\begin{aligned} T^{(u)}_{AB} &= \rho_u u_A u_B + p_u \sum_{i=2}^{4} X^{(i,u)}_A X^{(i,u)}_B, \\ T^{(v)}_{AB} &= \rho_v v_A v_B + p_v \sum_{i=2}^{4} X^{(i,v)}_A X^{(i,v)}_B, \end{aligned} \tag{2.11}$$

$X^{(i,a)}_A$ are unit vectors, defined as $X^{(i,a)}_A \equiv A_a^{1/3}\delta^i_A$ ($i = 2, 3, 4$; $a = u, v$), and

$$\begin{aligned} \rho_u &= -3p_u = -\frac{b(a^2-1)}{\kappa A_u^{2/3}(t)}, \\ \rho_v &= -3p_v = -\frac{a(b^2-1)}{\kappa A_v^{2/3}(t)}. \end{aligned} \tag{2.12}$$

Therefore, the solutions in the present case represent the collision of two timelike 3-branes, moving along, respectively, the line $t - ay = 0$ and the one $t + by = 0$. Each of the two 3-branes supports a perfect fluid. They approach each other as t increases, and collide at point $(t, y) = (0, 0)$, and then move apart. Depending on the specific values of the free parameters a and b, we have three distinguishable cases: (a) $a, b < -1$; (b) $a > 1, b < -1$; and (c) $a, b > 1$. The case $a < -1, b > 1$ can be obtained from case (b) by exchanging the two free parameters. In the following let us consider them separately.

2.1. $a < -1, b < -1$

In this subcase, from Eq. (2.12) we can see that the perfect fluids on both of the two branes satisfy all three energy conditions, weak, strong, and dominant [10]. To study the solutions further, we divide the spacetime into four regions, region I: $t < 0, t/|b| < y < t/|a|$, region II: $y > 0, -|a|y < t < |a|y$, region III: $y < 0, |b|y < t < -|a|y$, and region IV: $t > 0, -t/|a| < y < t/|b|$, as shown in Fig. 1, with the two 3-branes as their boundaries, where we denote them, respectively, as, $\Sigma_u \equiv \{x^A\colon\ t - ay = 0\}$ and $\Sigma_v \equiv \{x^A\colon\ t + by = 0\}$.

Along the hypersurface Σ_v, we find

$$\begin{aligned} ds^2|_{t=|b|y} &= \frac{b^2-1}{b^2 A_v^{2/3}(t)}dt^2 - A_v^{2/3}(t)\,d\Sigma_0^2 \\ &= d\tau^2 - a_v^2(\tau)\,d\Sigma_0^2, \end{aligned} \tag{2.13}$$

where

$$\begin{aligned} A_v(t) &= \begin{cases} A_0 - (|a|+|b|)t, & t \geqslant 0, \\ A_0, & t < 0, \end{cases} \\ d\tau &= \frac{\sqrt{b^2-1}}{|b|A_v^{1/3}(t)}dt, \\ a_v(\tau) &= \begin{cases} a_v^0(\tau_0 - \tau)^{1/2}, & t \geqslant 0, \\ A_0^{1/3}, & t < 0, \end{cases} \end{aligned} \tag{2.14}$$

with $\tau_0 = \tau_0(a, b, A_0)$, and $a_v^0 \equiv A_0^{1/3}\tau_0^{-1/2}$. Exchanging the free parameters a and b we can get the corresponding expressions for the brane located on the hypersurface $t - ay = 0$. From these expressions and Eq. (2.12) we can see that the two

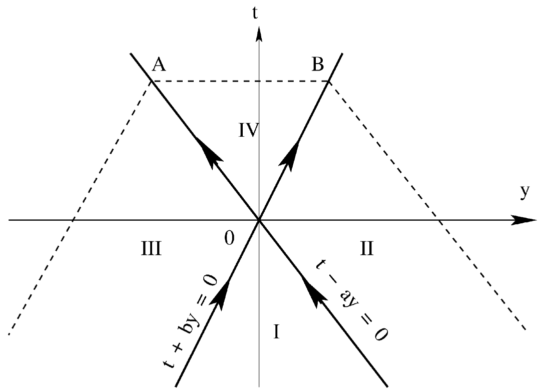

Fig. 1. The five-dimensional spacetime in the (t, y)-plane for $a < -1, b < -1$. The two 3-branes approach each other from $t = -\infty$ and collide at $(t, y) = (0, 0)$. Due to their gravitational mutual focus, the spacetime ends up at a spacelike singularity on the hypersurface $A_0 + (a + b)t = 0$ in region IV, denoted by the horizontal dashed line. The spacetime is also singular along the line $A_0 - |a|(t - |b|y) = 0$ ($A_0 - |b|(t + |a|y) = 0$) in region III (II), which is parallel to the 3-brane located on the hypersurface $t + by = 0$ ($t - ay = 0$).

3-branes come from $t = -\infty$ with constant energy densities and pressures, for which the spacetime on each of the branes is Minkowski. After they collide at the point $(t, y) = (0, 0)$, they focus each other, where $\dot{a}_{v,u}(\tau) < 0$, and finally end up at a singularity where $a_{v,u}(\tau) = 0$, denoted, respectively, by the point A and B in Fig. 1.

The spacetime outside the two 3-branes are vacuum, and the function $A(t, y)$ is given by

$$A(t, y) = \begin{cases} A_0 - (|a| + |b|)t, & \text{IV}, \\ A_0 - |a|(t - |b|y), & \text{III}, \\ A_0 - |b|(t + |a|y), & \text{II}, \\ A_0, & \text{I}. \end{cases} \tag{2.15}$$

From this expression we can see that the spacetime is Minkowski in region I and the function $A(t, y)$ vanishes on the hypersurfaces $A_0 - (|a| + |b|)t = 0$ in region IV, $A_0 - |a|(t - |b|y) = 0$ in region III, and $A_0 - |b|(t + |a|y) = 0$ in region II, denoted by the dashed lines in Fig. 1. These hypersurfaces actually represent the spacetime singularities. This can be seen clearly from the Kretschmann scalar,

$$I \equiv R_{ABCD}R^{ABCD} = \frac{8}{9A^{8/3}} \times \begin{cases} (a + b)^4, & \text{IV}, \\ a^4(b^2 - 1)^2, & \text{III}, \\ b^4(a^2 - 1)^2, & \text{II}, \\ 0, & \text{I}. \end{cases} \tag{2.16}$$

The above analysis shows clearly that, when the matter fields on the two branes satisfy the energy conditions, due to their mutual gravitational focus, a spacelike singularity is always formed after the collision. This is similar to the conclusions obtained numerically in [6,7].

2.2. $a > 1, b < -1$

In this case, Eq. (2.12) shows that the perfect fluid on the brane $t = ay$ satisfies all the three energy conditions, while the one on the brane $t = -by$ does not. To study these solutions further, it is found convenient to consider the two cases $a > |b| > 1$ and $|b| > a > 1$ separately.

Case 2.1. $a > |b| > 1$: In this case, the two colliding branes divide the whole spacetime into the following four regions,

$$\begin{aligned} &\text{I:} \quad t = \begin{cases} < ay, & y < 0, \\ < |b|y, & y > 0, \end{cases} \\ &\text{II:} \quad y < 0, \quad ay < t < |b|y, \\ &\text{III:} \quad y > 0, \quad |b|y < t < ay, \\ &\text{IV:} \quad t = \begin{cases} > ay, & y > 0, \\ > |b|y, & y < 0, \end{cases} \end{aligned} \tag{2.17}$$

as shown in Fig. 2. Then, we find that

$$A(t, y) = \begin{cases} A_0 + (a - |b|)t, & \text{IV}, \\ A_0 + a(t - |b|y), & \text{III}, \\ A_0 - |b|(t - ay), & \text{II}, \\ A_0, & \text{I}. \end{cases} \tag{2.18}$$

Clearly, the spacetime is again Minkowski in region I, but the function $A(t, y)$ now vanishes only on the hypersurfaces $A_0 + (a - |b|)t = 0$ in region IV, and $A_0 - |b|(t - ay) = 0$ in region II, denoted by the dashed lines in Fig. 2. Similar to the last case, the Kretschmann scalar blows up on these surfaces, so they actually represent the spacetime singularities. As a result, the region $A_0/|b| + ay < t < -A_0/(a - |b|), y < 0$, denoted by D in Fig. 2, is not part of the whole spacetime. In region III we have $A(t, y) > 0$, and no any kind of spacetime singularities appears in this region.

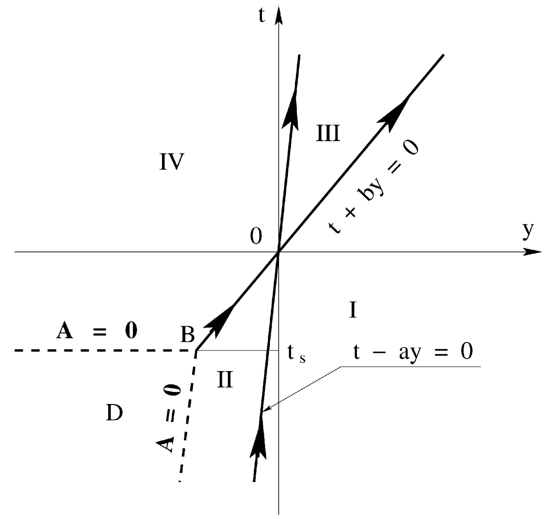

Fig. 2. The spacetime in the (t, y)-plane for $a > |b| > 1, b < -1$. It is singular along the two half dashed lines, $t = -A_0/(a - |b|), y < -A_0/[|b|(a - |b|)]$, and $A_0 - |b|(t - ay) = 0, t < -A_0/(a - |b|)$. The 3-brane located on the hypersurface $t + by = 0$ starts to expand from the singular point B, $t = -A_0/(a - |b|)$ and $y = -A_0/[|b|(a - |b|)]$, until the point $(t, y) = (0, 0)$, where it collides with the other brane moving in along the hypersurface $t - ay = 0$. After the collision, it continuously moves forward but with constant energy density and pressure, and the spacetime on the brane becomes flat. The spacetime on the 3-brane located on the hypersurface $t - ay = 0$ is flat before the collision, but starts to expand as $a_u(\eta) \propto (\eta + \eta_0)^{1/2}$ after the collision. This 3-brane is free of any kind of spacetime singularities.

Along the hypersurface $t+by=0$, the metric takes the same form as that given by Eq. (2.13) but now with

$$A_v(t)=\begin{cases}A_0, & t\geqslant 0,\\ A_0+(a-|b|)t, & t<0,\end{cases}$$
$$a_v(\tau)=\begin{cases}A_0^{1/3}, & t\geqslant 0,\\ a_v^0(\tau+\tau_s)^{1/2}, & t<0,\end{cases} \tag{2.19}$$

where $t=t_s\equiv -A_0/(a-|b|)$ corresponds to $\tau=\tau_s$ and $t=0$ to $\tau=\tau_0$, with $\tau_0\equiv(b^2-1)^{1/2}A_0^{2/3}/[2|b|(a-|b|)]$, and $a_v^0=A_0^{1/3}(\tau_0+\tau_s)^{-1/2}$. Thus, in this case the 3-brane located on the hypersurface $t+by=0$ starts to expand from the singular point $\tau=\tau_s$ and collides with the other incoming 3-brane at the point $(t,y)=(0,0)$. After the collision, the 3-brane transfers part of its energy to the one moving along the hypersurface $t-ay=0$, so that its energy density and pressure remain constant, and whereby the spacetime on this 3-brane becomes Minkowski.

Along the hypersurface $t-ay=0$, the metric takes the form

$$ds^2|_{t=ay}=\frac{a^2-1}{a^2A_u^{2/3}(t)}dt^2-A_u^{2/3}(t)\,d\Sigma_0^2$$
$$=d\eta^2-a_u^2(\eta)\,d\Sigma_0^2, \tag{2.20}$$

where

$$A_u(t)=\begin{cases}A_0+(a-|b|)t, & t\geqslant 0,\\ A_0, & t<0,\end{cases}$$
$$d\eta=\sqrt{\frac{a^2-1}{a^2A_u^{2/3}(t)}}dt,$$
$$a_u(\eta)=\begin{cases}a_u^0(\eta+\eta_0)^{1/2}, & t\geqslant 0,\\ A_0^{1/3}, & t<0,\end{cases} \tag{2.21}$$

where $t=0$ corresponds to $\eta=0$ and $\eta_0\equiv 3(a^2-1)^{1/2}A_0^{2/3}/[2a(a-|b|)]>0$. Thus, in the present case the brane located on the hypersurface $t-ay=0$ comes from $t=-\infty$ with constant energy density and pressure $\rho_u=-3p_u=|b|(a^2-1)/(\kappa A_0^{2/3})>0$, which satisfies all the three energy conditions. The spacetime on this brane is flat before the collision. After the collision, the spacetime of the brane starts to expand as $(\eta+\eta_0)^{1/2}$ without the big-bang type of singularities. The expansion rate is the same as that of a radiation-dominated universe in Einstein's theory of 4D gravity, where $a(\eta)\propto\eta^{1/2}$. But its energy density and pressure now decreases as $\rho_u=-3p_u\propto(\eta+\eta_0)^{-1}$, in contrast to $\rho=3p\propto\eta^{-2}$ in Einstein's 4D gravity [10].

Case 2.2. $|b|>a>1$: In this case, the two colliding branes divide the whole spacetime into the four regions,

$$\text{I:}\quad t=\begin{cases}<|b|y, & y<0,\\ <ay, & y>0,\end{cases}$$
$$\text{II:}\quad y<0,\quad |b|y<t<ay,$$
$$\text{III:}\quad y>0,\quad ay<t<|b|y,$$
$$\text{IV:}\quad t=\begin{cases}>|b|y, & y>0,\\ >ay, & y<0,\end{cases} \tag{2.22}$$

as shown in Fig. 3.

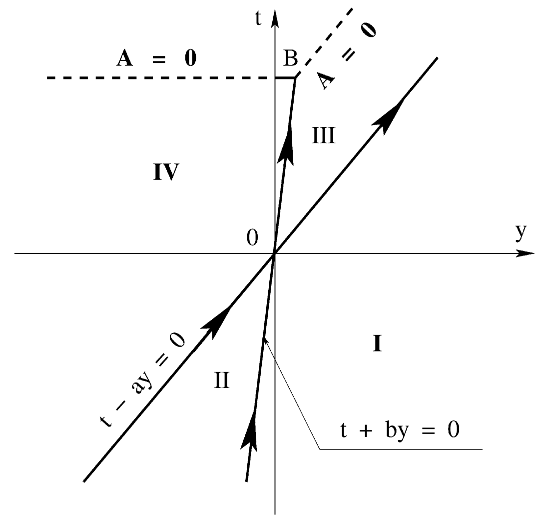

Fig. 3. The spacetime in the (t,y)-plane for $|b|>a>1$, $b<-1$. It is singular along the two half dashed lines where $A=0$. The spacetime of the 3-brane along $t+by=0$ is flat before the collision, but collapses to form a spacetime singularity at the point B. The spacetime of the 3-brane along $t-ay=0$ is contracting from $t=-\infty$ before the collision, but becomes flat after the collision. At the colliding point $(t,y)=(0,0)$ no any kind of spacetime singularities exists.

Following a similar analysis as we did in the last subcase one can show that the spacetime now is singular on the half lines $t=A_0/(|b|-a)$, $y<A_0/[|b|(|b|-a)]$ in region IV, and $t=A_0/|b|+ay>A_0/(|b|-a)$ in region III, denoted by the dashed lines in Fig. 3.

Along the hypersurface $t-ay=0$, the metric takes the form of Eq. (2.20) but now with

$$A_u(t)=\begin{cases}A_0, & t\geqslant 0,\\ A_0-(|b|-a)t, & t<0,\end{cases}$$
$$a_u(\eta)=\begin{cases}A_0^{1/3}, & t\geqslant 0,\\ a_u^0(\eta_0-\eta)^{1/2}, & t<0,\end{cases} \tag{2.23}$$

where $t\leqslant 0$ corresponds to $\eta\leqslant 0$ with $\eta_0\equiv 3(a^2-1)^{1/2}A_0^{2/3}/[2a(|b|-a)]>0$. Thus, in the present case the brane located on the hypersurface $t-ay=0$ comes from $t=-\infty$ with energy density and pressure $\rho_u=-3p_u\propto(\eta_0-\eta)^{-1}$, which satisfies all the three energy conditions. The spacetime on this brane is non-flat before the collision and becomes flat after the collision.

Along the line $t+by=0$, the metric takes the same form as that given by Eq. (2.13) but now with

$$A_v(t)=\begin{cases}A_0-(|b|-a)t, & t\geqslant 0,\\ A_0, & t<0,\end{cases}$$
$$a_v(\tau)=\begin{cases}a_v^0(\tau_s-\tau)^{1/2}, & t\geqslant 0,\\ A_0^{1/3}, & t<0,\end{cases} \tag{2.24}$$

where $t=t_s\equiv A_0/(|b|-a)$ corresponds to $\tau=\tau_s$ and $t=0$ to $\tau=\tau_0$, with $\tau_0\equiv(b^2-1)^{1/2}A_0^{2/3}/[2|b|(|b|-a)]$. Thus, in this case the 3-brane located on the hypersurface $t+by=0$ moves in from $t=-\infty$ and has constant energy density and pressure before the collision. After the collision, it collapses to a singularity at $\tau=\tau_s$.

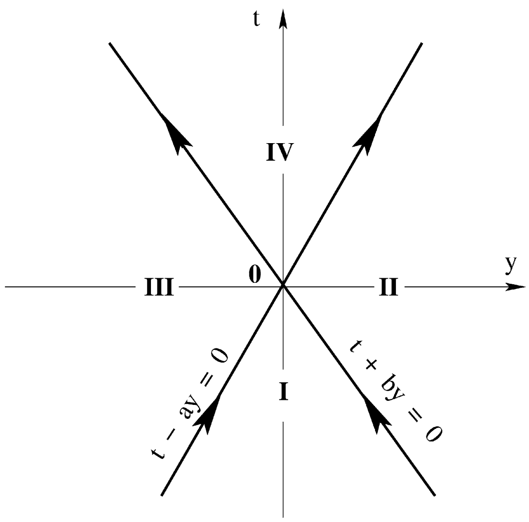

Fig. 4. The spacetime in the (t, y)-plane for $a > 1, b > 1$. It is free of any kind of spacetime singularities in the whole spacetime, including the two hypersurfaces of the 3-branes. The two 3-branes all come from $t = -\infty$ with constant energy density and pressure. They remain so until the moment right before collision. After the collision, the spacetime on each of the 3-branes is expanding like $a(\tau) \propto \tau^{1/2}$, while their energy densities and pressures decrease like $\rho = -3p \propto \tau^{-1}$.

2.3. $a > 1, b > 1$

In this subcase, from Eq. (2.12) we can see that both of the two branes violate all the three energy conditions [10]. Dividing the spacetime into the following four regions,

$$\begin{aligned}
&\text{I:} && t < 0, && \frac{t}{a} < y < -\frac{t}{b},\\
&\text{II:} && y > 0, && -by < t < ay,\\
&\text{III:} && y < 0, && ay < t < -by,\\
&\text{IV:} && t > 0, && -\frac{t}{b} < y < \frac{t}{a},
\end{aligned} \tag{2.25}$$

as shown in Fig. 4, we find that

$$A(t, y) = \begin{cases} A_0 + (a+b)t, & \text{IV},\\ A_0 + b(t - ay), & \text{III},\\ A_0 + a(t + by), & \text{II},\\ A_0, & \text{I}, \end{cases}$$
$$A_u(t) = \begin{cases} A_0 + (a+b)t, & t \geqslant 0,\\ A_0, & t < 0, \end{cases}$$
$$A_v(t) = \begin{cases} A_0 + (a+b)t, & t \geqslant 0,\\ A_0, & t < 0, \end{cases} \tag{2.26}$$

which are non-zero in the whole spacetime. Thus, in the present case the spacetime is free of any kind of spacetime singularities, and flat in region I. Before the collision the two branes move in from $t = -\infty$ all with constant energy density and pressure. After the collision, their energy densities and pressures all decrease like τ^{-1}, while the spacetime on these two branes is expanding like $a(\tau) \propto \tau^{1/2}$, where τ is the proper time on each of the two branes, and $a(\tau)$ their expansion factor.

3. Conclusions

In this Letter, we have studied the collision of branes and the formation of spacetime singularities. We have constructed a class of analytic solutions to the five-dimensional Einstein field equations, which represents such a collision, and found that when both of the two 3-branes satisfy the energy conditions, a spacelike singularity is always developed after the collision, due to their mutual gravitational focus. This is consistent with the results obtained numerically in [6,7]. When only one of the two branes satisfies the energy conditions, the other brane either starts to expands from a singular point [cf. Fig. 2], or comes from $t = -\infty$ and then focuses to a singular point after the collision [cf. Fig. 3]. It is interesting to note that in all these three cases the spacetime in region IV is locally Kasner. As a result, the power-law singularity developed after the brane collision is that of Kasner type. However, if both of the two colliding 3-branes violate the weak energy condition, no spacetime singularities exist at all in the whole spacetime. Before the collision, the two branes approach each other in a flat background with constant energy densities and pressures. After they collide at $(t, y) = (0, 0)$, they start to expand as $a(\tau) \propto \tau^{1/2}$, where $a(\tau)$ denotes their expansion factor, and τ their proper time. As the branes are expanding, their energy densities and pressures decrease as $\rho, p \propto \tau^{-1}$, in contrast to that of $\rho, p \propto \tau^{-2}$ in the four-dimensional FRW model. Region IV in this case is also locally Kasner, but the Kasner spacetime singularity is not part of this region.

As argued in [9], these singularities may become very mild when the five-dimensional spacetime is left to higher dimensional spacetimes, ten dimensions in string theory and eleven in M-Theory, a question that is under our current investigation.

Finally, we would like to notice that the solution presented here is purely gravitational, and hence there is no charge associated with the colliding 3-branes. If charges are included, one may wonder whether these 3-branes are stable and spacetime singularities are still formed after the collision. We would like to address all these issues in other occasions.

Acknowledgements

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