



ELSEVIER

Available online at www.sciencedirect.com

SCIENCE @ DIRECT®

Physics Letters B 558 (2003) 1–8

PHYSICS LETTERS B

www.elsevier.com/locate/npe

The ultimate fate of life in an accelerating universe

Katherine Freese^{a,b}, William H. Kinney^b

^a *Michigan Center for Theoretical Physics, University of Michigan, Ann Arbor, MI 48109, USA*

^b *Institute for Strings, Cosmology and Astroparticle Physics, Columbia University, 550 W. 120th St., New York, NY 10027, USA*

Received 25 May 2002; received in revised form 27 January 2003; accepted 27 January 2003

Editor: J. Frieman

Abstract

The ultimate fate of life in a universe with accelerated expansion is considered. Previous work [J.D. Barrow, F. Tipler, *The Anthropic Cosmological Principle*, Oxford Univ. Press, Oxford, 1986; L.M. Krauss, G.D. Starkman, *Astrophys. J.* 531 (2000) 22] showed that life cannot go on indefinitely in a universe dominated by a cosmological constant. In this Letter we consider instead other models of acceleration (including quintessence and Cardassian expansion). We find that it is possible in these cosmologies for life to persist indefinitely. As an example we study potentials of the form $V \propto \phi^n$ and find the requirement $n < -2$.

© 2003 Elsevier Science B.V. Open access under [CC BY license](https://creativecommons.org/licenses/by/4.0/).

1. Introduction

The question of the ultimate fate of life in an expanding universe has been a subject of debate in the physics community for over two decades [1]. In 1979, Dyson [2] proposed a framework within which to discuss whether or not some form of life, material or otherwise, can go on. At the time of his work the universe was assumed to be decelerating. However, in the light of recent evidence that the universe is accelerating, the conclusions of Dyson's original work deserve reinvestigation. Observations of Type IA Supernovae [3,4] as well as concordance with other observations (including the microwave background and

galaxy power spectra) indicate that the universe is accelerating. Hence the question of the future of life in our universe deserves another look in the context of this acceleration. Barrow and Tipler [5] and Krauss and Starkman [6] followed the basic approach outlined by Dyson to consider life in a universe dominated by a cosmological constant. They concluded that life is inevitably doomed to oblivion in such a universe. Any lifeform would eventually fry to death in the bath of thermal Hawking radiation produced by the de Sitter vacuum [6]. Beings of any kind generate heat by the process of living and eventually are unable to dissipate their heat in the background of this thermal bath.

In this Letter we consider the consequences of other explanations for the acceleration of our universe. Other than a cosmological constant, alternatives include a decaying vacuum energy [7,8], quintessence [9–18], and Cardassian expansion [20] as possible ex-

E-mail addresses: ktfreese@umich.edu (K. Freese), kinney@physics.columbia.edu (W.H. Kinney).

planations for such an acceleration. Quintessence has a time dependent vacuum energy given by a rolling scalar field. Cardassian expansion is a model with matter and radiation alone (no vacuum at all) in which acceleration is driven by a modified Friedmann–Robertson–Walker (FRW) equation. The crucial difference between these cases and that of a cosmological constant is that the temperature of the cosmological Hawking radiation decreases in time, in many cases quickly enough to allow life to continue indefinitely despite the presence of the thermal bath. We consider two cases: (1) a constant equation of state $p = w\rho$ with $-1 \leq w < -1/3$ (which includes the case of Cardassian expansion for constant exponent n defined below), and (2) a time-varying equation of state generated by a “quintessence” potential of the form $V(\phi) \propto \phi^n$ with $n < 0$. In the case of constant equation of state, we find that any equation of state *except* a cosmological constant ($w = -1$) allows for the indefinite continuation of life. In the quintessence case, we find that any potential $V \propto \phi^n$ with $n < -2$ is consistent with the indefinite continuation of life. We note at the outset that our arguments assume the existence of Hawking radiation in generic accelerating spacetimes; while such Hawking radiation is plausible (see, e.g., [21]), its existence has never been proven.¹ If there is no such radiation, then these issues of life burning up in the Hawking radiation become irrelevant, and it is even easier for some type of lifeform to continue to exist.

In the conclusions, we discuss speculative scenarios in which life might avoid inevitable extinction even in the case of a universe dominated by a cosmological constant, including quantum computation, oscillating universes, wormholes, and laboratory-created universes. We also comment on the argument of Krauss and Starkman that the presence of a quantum-mechanical ground state for the system renders Dyson’s argument invalid in general. We argue that the inclusion of a cosmological Hawking temperature in Dyson’s classical argument correctly captures the quantum nature of the system and that therefore Dyson’s conclusions are in fact valid.

2. The premise set out by Dyson

Dyson introduced the “biological scaling” hypothesis to estimate the rate at which an organism in an environment of temperature T can perform computation. We refer the reader to Ref. [2] for a definition and detailed discussion of the scaling hypothesis, and instead concentrate here on its relevant consequences. The important consequence of Dyson’s scaling hypothesis is the notion of “subjective time”, i.e., the appropriate measure of time as experienced by a living creature is the quantity

$$u(t) = f \int_{t_0}^t T(t') dt', \quad (1)$$

where $T(t)$ is the temperature of the creature and $f = (300 \text{ deg s})^{-1}$ is introduced so as to make u dimensionless.² One can think of the quantity u as the number of possible computations in a time t . We can define one “computation” as some change of state in a quantum system. Then a single computation, from the energy/time uncertainty relation, takes place over a characteristic timescale

$$\Delta t \geq \frac{\hbar}{\Delta E} \sim \frac{\hbar}{kT}, \quad (2)$$

where k is Boltzmann’s constant. Then the total number of computations Δu over time Δt is then given by

$$\frac{\Delta u}{\Delta t} \propto \frac{kT}{\hbar}, \quad (3)$$

which leads directly to the definition of subjective time in Eq. (1). (Later we comment on the possibility of using quantum computation to alter this notion of subjective time.) The continuation of life requires the possibility of an infinite number of computations in a system with only a finite amount of energy.

Dyson points out a second consequence of the scaling law: any creature is characterized by a quantity Q which measures its rate of entropy production per unit of subjective time, $dS = Q du$, in some sense the

¹ We thank an anonymous referee as well as Gary Gibbons on this point.

² The value of f suggested by Dyson is motivated by the fact that human life takes place at 300 K with each moment of consciousness lasting about a second; the precise value of f is immaterial to the arguments.

“complexity” of the creature. Dyson estimates that, for a human dissipating about 200 W of power at 300 K, $Q \sim 10^{23}$ bits. Krauss and Starkman estimate that the uncertainty in this number suggests that a civilization of conscious beings requires $\log Q > 50$ –100. Any creature in the process of living and computing will dissipate energy. A lifeform with given Q and given temperature T will dissipate energy at a rate

$$m \equiv \frac{dE}{dt} = kT \frac{dS}{dt} = kfQT^2, \quad (4)$$

where m is the metabolic rate measured in ergs per second. The total energy consumed by the creature is then

$$E = kfQ \int_{t_0}^t T^2(t') dt'. \quad (5)$$

Since the rate of computation scales as T while the rate of energy consumption scales at T^2 , it at first appears possible that an organism can perform an infinite number of computations using a finite amount of energy, as long as the operating temperature of the organism continuously decreases in time, $T(t) \propto t^{-\alpha}$, with $1/2 < \alpha \leq 1$. We will refer to this as *Dyson’s condition*: life can be considered “infinite” if the number of computations, or “subjective time”, (1) can be infinite while the total energy consumed (5) is finite.

This naive analysis assumes that the organism is completely free to choose its temperature $T(t)$ so as to satisfy Dyson’s condition. Several constraints restrict this temperature. The creature must be able to get rid of the heat E generated by the computations it performs (4). However, Dyson estimates an upper limit to the rate at which waste heat can be radiated as

$$I(t) < 2.84 \frac{N_e e^2}{m_e \hbar^2 c^3} (kT)^3. \quad (6)$$

The creature will fry to death unless it can dissipate the heat E that it creates; dissipation by radiation implies a lower limit on the operating temperature for the organism:

$$T(t) > T_{\min} = (Q/N_e) \times 10^{-12} \text{ K}. \quad (7)$$

Since the ratio (Q/N_e) between the complexity of the society and the number of electrons at its disposal cannot be made arbitrarily small, there must be a

finite minimum temperature for which computation is possible. Therefore, Dyson’s condition cannot be satisfied, and the creature or society cannot survive indefinitely.

However, Dyson proposes a strategy to avoid this sad conclusion: hibernation. Life may find a way to metabolize intermittently, yet continue to radiate waste heat into space during its periods of hibernation. The society can remain active for a fraction $g(t)$ of its time while hibernating for the remaining $1 - g(t)$ fraction of the time. During these periods of hibernation, metabolism can be effectively stopped while radiation of waste heat continues. Then the total subjective time is modified to

$$u(t) = f \int_0^t g(t') T(t') dt', \quad (8)$$

while the average rate of heat production by the organism becomes

$$m = kfQgT^2. \quad (9)$$

Therefore, the temperature of the organism can drop below T_{\min} in Eq. (7) and the heat generated by the computation can still be dissipated: the condition (7) becomes

$$T(t) > T_{\min} \equiv \frac{Q}{N_e} g(t) \times 10^{-12} \text{ K}. \quad (10)$$

As long as the operating temperature of the organism is above this limit, it can dispose of waste heat. The total energy consumed is

$$E = kfQ \int_{t_0}^t g(t') T^2(t') dt'. \quad (11)$$

The organism is free to choose $g(t)$ and $T(t)$ to satisfy Dyson’s condition. We will assume (consistent with other authors) that $g(t) \propto T(t) \propto t^{-p}$, the minimum amount of hibernation consistent with the energy dissipation condition (11). Then the subjective time is given by

$$u(t) \propto \int_{t_0}^t (t')^{-2p} dt', \quad (12)$$

and the total energy consumed scales as

$$E(t) \propto \int_{t_0}^t (t')^{-3p} dt'. \quad (13)$$

Dyson's condition

$$u(t \rightarrow \infty) \rightarrow \infty, \quad E(t \rightarrow \infty) \rightarrow \text{const} \quad (14)$$

is then satisfied for

$$\frac{1}{3} < p \leq \frac{1}{2}. \quad (15)$$

An additional constraint is generated by the fact that the creatures, even if hibernating, cannot cool off any faster than the background universe. Hence, if the universe temperature scales as

$$T_u(t) \propto t^{-q}, \quad (16)$$

then this second constraint requires that

$$p < q. \quad (17)$$

It is clear that Eqs. (15) and (17) can both be simultaneously satisfied in a (decelerating) Cold Dark Matter-dominated cosmology. There the temperature of the background universe is given by the Cosmic Microwave Background temperature, which scales in a matter-dominated cosmology at $T_{\text{CMB}} \propto t^{-2/3}$, i.e., $q = 2/3$. Hence the background temperature indeed drops more quickly than the temperature required for the organism to satisfy Dyson's bound.

3. Cosmological constant dominated universe

Krauss and Starkman considered modifications to these questions in the context of a universe dominated by a cosmological constant Λ . In de Sitter space, Hawking radiation creates a thermal bath of particles at the de Sitter temperature,

$$T_{\text{deS}} = \sqrt{\frac{\Lambda}{12\pi^2}} = \text{const}. \quad (18)$$

Hence the universe itself has a fundamental minimum temperature, with $q = 0$. Then the constraint in Eq. (10) is replaced by

$$T(t) > T_{\text{min}} \equiv \max \left[T_{\text{deS}}, \frac{Q}{N} g(t) \times 10^{-12} \text{ K} \right]. \quad (19)$$

Therefore, no hibernation strategy will be sufficient to satisfy Dyson's condition. The first term in Eq. (19) eventually dominates, and then Eq. (17) cannot be satisfied with $q = 0$. Eventually thermal equilibrium will be reached with everything at the Hawking temperature of the thermal bath, and further computation will be an impossibility. Life in a universe with a cosmological constant is doomed to extinction.

4. Dark energy

The "dark energy" driving the acceleration of the universe, however, need not be a cosmological constant. In a more general scenario, the energy density driving the acceleration can be variable in time, or, equivalently, have an equation of state $p > -\rho$. Acceleration takes place for any equation of state $p < -1/3\rho$. In this section, we examine the case of a more general equation of state and show that Dyson's condition for infinite computation can be met for a wide range of accelerating cosmologies.

4.1. Constant equation of state

We first consider the simple case of equation of state $p = w\rho$, with w constant in time. Any accelerating cosmology evolves toward flatness at late time, so we can assume a flat cosmology. From the Friedmann equation

$$H^2 \equiv \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi}{3m_{\text{Pl}}^2} \rho \quad (20)$$

and the Raychaudhuri equation

$$\left(\frac{\ddot{a}}{a} \right) = -\frac{4\pi}{3m_{\text{Pl}}^2} (\rho + 3p), \quad (21)$$

we have, for $w = \text{const}$,

$$\frac{dH}{dt} = -\frac{3}{2}(1+w)H^2. \quad (22)$$

For the case of a cosmological constant, $w = -1$ and $H = \text{const}$, so that $q = 0$ and one can never satisfy Eq. (17). Therefore, Dyson's condition is violated. However, for $w > -1$, we have $H \propto t^{-1}$. The Hawking temperature of the space (the generalization of the de Sitter temperature in de Sitter space) therefore also decreases as $T_{\text{H}} \propto H \propto t^{-1}$, i.e., $q = 1$. Then,

in Eq. (17), the first term ($\propto t^{-1}$) drops more rapidly than the second ($\propto t^{-p}$), and one is back to Dyson’s original condition in Eq. (10). As long as Eq. (15) is satisfied, the Dyson condition that an infinite amount of computation be possible with finite total energy expended can be met in any cosmology with $w = \text{const}$ *except* the special case of a cosmological constant, $w = -1$.

4.2. Time-varying equation of state

In general, however, the equation of state of the dark energy need not remain constant. We cannot comment in general upon all time-varying equations of state. We here concentrate upon the particular case of “quintessence” models, in which the dark energy consists of a slowly rolling scalar field. The time-dependence of the equation of state depends on the form of the potential for the quintessence field ϕ . The equation of motion for a scalar field in a cosmological background is

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0. \quad (23)$$

The case of an exponential potential, $V(\phi) \propto e^{\phi/M}$ is just that of a constant equation of state considered above, since

$$a(t) \propto t^{1/\epsilon}, \quad (24)$$

with $\epsilon = \text{const} < 1$ corresponding to accelerated expansion. The equation of state is

$$w = \frac{2}{3}\epsilon - 1 = \text{const}. \quad (25)$$

This example can be generalized to an arbitrary potential as follows. For a slowly rolling scalar field ϕ , the equation of motion (23) is approximately

$$3H\dot{\phi} \simeq -V'(\phi), \quad (26)$$

and the Friedmann equation is

$$H^2 = \frac{8\pi}{3m_{\text{Pl}}^2} \left[\frac{1}{2}\dot{\phi}^2 + V(\phi) \right] \simeq \frac{8\pi}{3m_{\text{Pl}}^2} V(\phi). \quad (27)$$

Then

$$\frac{dH}{dt} = \frac{1}{2}H \left(\frac{V'(\phi)}{V(\phi)} \right) \dot{\phi} = -\epsilon H^2, \quad (28)$$

where the slow-roll parameter ϵ is

$$\epsilon = \frac{m_{\text{Pl}}^2}{16\pi} \left(\frac{V'(\phi)}{V(\phi)} \right)^2. \quad (29)$$

The exponential potential then has $\epsilon = \text{const}$ as above, but ϵ varies with time for arbitrary potential. The equation of state

$$w \simeq \frac{2}{3}\epsilon - 1, \quad (30)$$

therefore varies in time as well. We consider the class of potentials

$$V(\phi) \propto \phi^n. \quad (31)$$

We first consider quintessence models with $n < 0$, so that the universe is accelerating in the late-time limit. The slow-roll parameter ϵ is then

$$\epsilon = \frac{n^2 m_{\text{Pl}}^2}{16\pi} \left(\frac{1}{\phi^2} \right) \propto H^{-4/n}. \quad (32)$$

The equation of motion for the Hubble parameter is

$$\frac{dH}{dt} \propto -H^{2(n-2)/n}, \quad (33)$$

with solution

$$H(t) \propto \left[\left(\frac{n-4}{n} \right) t + \text{const} \right]^{n/(4-n)}. \quad (34)$$

We are interested in the solution at late times, so that for the quintessence case of $n < 0$, the Hubble parameter and the Hawking temperature evolve as

$$T_{\text{H}} \propto H \propto t^{-|n/(4-n)|} \quad (35)$$

so that $q = |n/(4-n)|$. The Dyson condition requires $p < q$ as $t \rightarrow \infty$, where we are allowed to choose p anywhere in the range $1/3 < p \leq 1/2$. Taking the *slowest* rate of falloff for T_{min} , we have the condition

$$\left| \frac{n}{4-n} \right| > p > \frac{1}{3}, \quad (36)$$

so that Dyson’s condition can be satisfied for quintessence models with $n < -2$.

Some quintessence models, for example, those based on light pseudo Nambu–Goldstone bosons [19], take $n > 0$. In these models, the minimum of the potential is at zero or negative energy and the accelerated expansion generically ends at late times. In such a situation it is a simple matter to satisfy Dyson’s condition, and we do not consider this case further here.

5. Modified FRW equations

An alternative way to drive acceleration of the universe is modification of the Friedmann–Robertson–Walker equations. In Cardassian expansion [20], the FRW equations become

$$H^2 = \frac{8\pi}{3m_{\text{pl}}^2} \rho + B\rho^n \quad (37)$$

with $n < 2/3$. In this model there is no vacuum term at all, and the energy density ρ is simply given by ordinary matter and radiation. The second term becomes more important as time goes on and for redshifts $z < 1/2$ drives acceleration of the universe. The universe is thus flat, matter dominated, and accelerating in this model. An alternate way to modify the FRW equations has been studied by [22]. For constant coefficient n in the Cardassian model, the background evolution of the universe behaves dynamically the same as a constant equation of state $w = n - 1$ so that the conclusion in the previous section implies that life can persist in a universe with $n > 0$.

6. Conclusion and discussion

We have found that life can go on indefinitely in an accelerating universe, depending on what energy density drives the acceleration. Previous authors [5,6] showed that the (time-independent) de Sitter radiation in a cosmological constant driven expansion destroys all life eventually. But in other cases we considered, including quintessence and Cardassian expansion, we found that the de Sitter radiation cools off just rapidly enough to allow life to survive. In particular, for any constant effective equation of state $w = p/\rho > -1$, and any constant Cardassian exponent $n > 0$, life can persist. In addition, we considered time-varying equations of state for the case of a quintessence with potential $V \propto \phi^n$, and found successful futures for life if $n < -2$.

As mentioned in the Introduction, our arguments have assumed the existence of Hawking radiation in generic accelerating spacetimes. If there is no such radiation, then these issues of life burning up in the Hawking radiation become irrelevant, and it is even easier for some type of lifeform to continue to exist.

In this work we followed the basic premise set up by Dyson. Currently we understand that there is a disagreement between Dyson on the one hand and Krauss and Starkman (KS) on the other hand as to whether or not there is a flaw in the premise. KS argue that any system in which computation is an irreversible process must eventually reach a quantum-mechanical ground state, beyond which further metabolism will not be possible. If such a system is finite, it must necessarily reach the ground state in finite time. We argue that this line of reasoning is valid only in the limit of a *static* (i.e., de Sitter) spacetime. Consider the phase space available for quantum modes in an accelerating spacetime. The horizon size $d_H \sim H^{-1}$ provides an infrared cutoff, since modes with momentum $p < H$ have wavelength longer than the horizon size, at which point they become classical perturbations. Therefore, the horizon size defines an effective ground state for quantum modes in the spacetime, $E > H$. However, this is exactly the physics which leads to the Hawking temperature $T_{\text{des}} \sim H$! In the case of exact de Sitter space, the ground state energy is constant in time, and therefore the argument of KS that any finite system must relax to its ground state in finite subjective time is valid. However, in backgrounds where the horizon size is increasing in time, the “ground state” energy defined by the infrared cutoff is decreasing in time and the system continuously has new, lower-energy states made available to it. Classically, this behavior is manifest in the time-dependence of the Hawking temperature. Thus the system can continue to radiate waste heat and reaches a ground state only after infinite subjective time, exactly as suggested by the classical calculation. This argument is obviously speculative, and it would be desirable to frame it in a more quantitative way. In particular, it is not clear that a system with these properties can truly be considered “finite”.

One might wonder if quantum computing would allow us to modify the Dyson condition in a useful way. Then the number of computations (the “subjective time”) given in Eq. (1) will be much larger for a given rate of energy dissipation. A lifeform may clearly continue to live or compute for a much longer time period with the same energy consumption. However, it is straightforward to show that including quantum computation as a possibility does not affect our conclusions about the ultimate fate of life. The above discussion was based on the thermodynamics of a con-

ventional computer, which uses energy at a rate of $m = fQkT^2$ to “flip” Q bits at temperature T . We can make an optimistic estimate of the increase in efficiency afforded by quantum computing by supposing that any operation performed on Q bits by a conventional computer can be performed on a superposition of 2^Q entangled quantum states by a quantum computer, with identical energy consumption. Thus a classical system with complexity Q can be built as a quantum system with complexity $\log_2(Q)$, which dissipates energy at a rate

$$m_{\text{quant}} = f \log_2(Q)kT^2. \quad (38)$$

However, this improvement in efficiency alters the energy integral (11) by a multiplicative factor, and has no effect on whether the total energy consumed is finite or infinite. Therefore, our arguments apply equally well to quantum as well as classical computers. However, we note that an organism of a given complexity Q can live exponentially longer in subjective time by adopting quantum computing as a strategy.

We note that a finite system, while it may be capable of an infinite amount of computation, is only capable of storing a *finite* number of memories. As long as the expansion of the universe is accelerating, any system which is initially finite must remain so, since any additional material with which to build new “memory” has redshifted beyond the horizon and is therefore unavailable. We thus reach the apparently inescapable conclusion that, while life itself may be immortal, any individual is doomed to mortality.

There are certainly limitations to Dyson’s premise. One alternative cosmology which would violate Dyson’s premise would be if the universe oscillates [23–26] or is cyclic [27]. Then the current accelerating phase might be followed by a subsequent recontraction and then again an expansion, and life could begin all over again. Of course the new burst of life might not have any memory of our current cycle, so that this does not provide an altogether satisfactory solution to the problem of enabling life to continue indefinitely [2].

However, while it is not given to us to choose what kind of universe we live in now, we do have the freedom to improve our strategy for continued existence. It is of course hubris to believe that humans can at this point foresee all the ideas that all future life forms will come up with to save themselves. In the future, there may be many ways to work around

the basic premises we have here assumed. We list here a few of the ones one can imagine. Perhaps Dyson’s scaling hypothesis could be violated under the correct conditions. Perhaps someday one can find a way to create and use wormholes. Then we could either bring in energy from far distant points in the universe for our use, or we could travel to some other more congenial place in the universe where there are still sufficient resources for our consumption. Another alternative would be to create a universe in a lab, along the lines of suggestions made by Guth and Farhi [28], and then move into it. Future beings are likely to apply new technology and sophistication, far beyond anything we can anticipate, towards these questions of survival.

Acknowledgements

K.F. acknowledges support from the Department of Energy via the University of Michigan as well as the Institute for Particles, Strings, and Astroparticle Physics at Columbia University. W.H.K. is supported by ISCAP and the Columbia University Academic Quality Fund. ISCAP gratefully acknowledges the generous support of the Ohrstrom Foundation. We thank Glenn Starkman and Edward Baltz for helpful conversations.

References

- [1] J.D. Barrow, F.J. Tipler, *Nature* 276 (1978) 453.
- [2] F.J. Dyson, *Rev. Mod. Phys.* 51 (1979) 447.
- [3] A.G. Reiss, et al., *Astron. J.* 116 (1998) 1009.
- [4] S. Perlmutter, et al., *Astrophys. J.* 517 (1999) 565.
- [5] J.D. Barrow, F. Tipler, *The Anthropic Cosmological Principle*, Oxford Univ. Press, Oxford, 1986.
- [6] L.M. Krauss, G.D. Starkman, *Astrophys. J.* 531 (2000) 22, astro-ph/9902189.
- [7] K. Freese, F.C. Adams, J.A. Frieman, E. Mottola, *Nucl. Phys. B* 287 (1987) 797.
- [8] J. Frieman, C. Hill, A. Stebbins, I. Waga, *Phys. Rev. Lett.* 75 (1995) 2077.
- [9] P.J.E. Peebles, B. Ratra, *Astrophys. J. Lett.* 325 (1988) L17; B. Ratra, P.J.E. Peebles, *Phys. Rev. D* 37 (1988) 3406.
- [10] C. Wetterich, *Nucl. Phys. B* 302 (1988) 668.
- [11] D. Wands, E.J. Copeland, A.R. Liddle, *Ann. New York Acad. Sci.* 688 (1993) 647.
- [12] P.G. Ferreira, M. Joyce, *Phys. Rev. Lett.* 79 (1997) 4740.
- [13] R.R. Caldwell, R. Dave, P.J. Steinhardt, *Phys. Rev. Lett.* 80 (1998) 1582.

- [14] E.J. Copeland, A.R. Liddle, D. Wands, *Phys. Rev. D* 57 (1998) 4686.
- [15] I. Zlatev, L. Wang, P.J. Steinhardt, *Phys. Rev. Lett.* 82 (1999) 896;
P.J. Steinhardt, L. Wang, I. Zlatev, *Phys. Rev. D* 59 (1999) 123504.
- [16] P. Brax, J. Martin, *Phys. Rev. D* 61 (2000) 103502;
P. Brax, J. Martin, A. Riazuelo, *Phys. Rev. D* 62 (2000) 103505.
- [17] T. Barreiro, E.J. Copeland, N.J. Nunes, *Phys. Rev. D* 61 (2000) 127301.
- [18] L. Armendola, *Phys. Rev. D* 62 (2000) 043511.
- [19] J. Frieman, C. Hill, A. Stebbins, I. Waga, *Phys. Rev. Lett.* 75 (1995) 2077, astro-ph/9505060.
- [20] K. Freese, M. Lewis, astro-ph/0201229.
- [21] W. Fischler, A. Kashani-Poor, R. McNees, S. Paban, hep-th/0104181.
- [22] C. Deffayet, G. Dvali, G. Gabadadze, *Phys. Rev. D* 65 (2002) 044023.
- [23] R.C. Tolman, *Relativity, Thermodynamics, and Cosmology*, Oxford Univ. Press, Clarendon Press, Oxford, 1934.
- [24] J.D. Barrow, M.P. Dabrowski, *Mon. Not. R. Astron. Soc.* 275 (1995) 850.
- [25] L. Smolin, *The Life of the Cosmos*, Oxford Univ. Press, Oxford, 1999.
- [26] K. Freese, W.H. Kinney, M. Lewis, G. Starkman, in preparation.
- [27] P.J. Steinhardt, N. Turok, hep-th/0111030.
- [28] E. Farhi, A.H. Guth, J. Guven, *Nucl. Phys. B* 339 (1990) 417.