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Universality of the Collins–Soper–Sterman nonperturbative function in vector boson production

Anton V. Konychev^a, Pavel M. Nadolsky^{b,*}^a Department of Physics, Indiana University, Bloomington, IN 47405-7105, USA^b High Energy Physics Division, Argonne National Laboratory, Argonne, IL 60439-4815, USA

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Abstract

We revise the b_* model for the Collins–Soper–Sterman resummed form factor to improve description of the leading-power contribution at nearly nonperturbative impact parameters. This revision leads to excellent agreement of the transverse momentum resummation with the data in a global analysis of Drell–Yan lepton pair and Z boson production. The nonperturbative contributions are found to follow universal quasi-linear dependence on the logarithm of the heavy boson invariant mass, which closely agrees with an estimate from the infrared renormalon analysis.

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Transverse momentum distributions of heavy Drell–Yan lepton pairs, W , or Z bosons produced in hadron–hadron collisions present an interesting example of factorization for multi-scale observables. If the transverse momentum q_T of the electroweak boson is much smaller than its invariant mass Q , $d\sigma/dq_T$ at an n th order of perturbation theory includes large contributions of the type $\alpha_s^n \ln^m(q_T^2/Q^2)/q_T^2$ ($m = 0, 1, \dots, 2n - 1$), which must be summed through all orders of α_s to reliably predict the cross section [1]. The feasibility of all-order resummation is proved by a factorization theorem, first formulated for e^+e^- hadroproduction [2,3], stated by Collins, Soper, and Sterman (CSS) for the Drell–Yan process [4], and recently proved by detailed investigation of gauge transformations of k_T -dependent parton densities [5,6].

The heavy bosons acquire non-zero q_T mostly by recoiling against QCD radiation. The CSS formalism accounts for both the short- and long-wavelength QCD radiation by means of a Fourier–Bessel transform of a resummed form factor $\tilde{W}(b)$ introduced in impact parameter (b) space. The perturbative contribution, characterized by $b \lesssim 0.5 \text{ GeV}^{-1}$, dominates in W and Z boson production at all values of q_T . The nonpertur-

bative contribution from $b \gtrsim 0.5 \text{ GeV}^{-1}$ is not negligible at $q_T < 20 \text{ GeV}$ in the precision measurements of the W boson mass M_W at the Tevatron and LHC [7]. The model for the nonperturbative recoil is the major source of theoretical uncertainty in the extraction of M_W from the experimental data. This uncertainty must be reduced in order to measure M_W with accuracy of about 30 MeV in the Tevatron Run-2 and 15 MeV at the LHC. The nonperturbative model presented below approaches the level of accuracy desired in these measurements.

The nonperturbative component [described by the function $\mathcal{F}_{\text{NP}}(b, Q)$ given in Eq. (4)] can be constrained in a few experiments by exploiting process-independence, or universality, of $\mathcal{F}_{\text{NP}}(b, Q)$, just as the universal k_T -integrated parton densities are constrained with the help of inclusive scattering data. The universality of $\mathcal{F}_{\text{NP}}(b, Q)$ in unpolarized Drell–Yan-like processes and semi-inclusive deep-inelastic scattering (SIDIS) follows from the CSS factorization theorem [5]. In the study presented here, we carefully investigate agreement of the universality assumption with the data in a global analysis of fixed-target Drell–Yan pair and Tevatron Z boson production. We revise the nonperturbative model used in the previous studies [8,9] and improve agreement with the data without introducing additional free parameters. Renormalization-group invariance requires $\mathcal{F}_{\text{NP}}(b, Q)$ to depend linearly on $\ln Q$ [3,4]. With our latest revisions put in place, the global q_T fit clearly prefers

* Corresponding author.

E-mail address: nadolsky@hep.anl.gov (P.M. Nadolsky).

a simple function $\mathcal{F}_{\text{NP}}(b, Q)$ with universal $\ln Q$ dependence. The new $\mathcal{F}_{\text{NP}}(b, Q)$ has reduced dependence on the collision energy \sqrt{S} comparatively to the earlier fits. The slope of the $\ln Q$ dependence found in the new fit agrees numerically with its estimate made with methods of infrared renormalon analysis [10,11].

The function $\mathcal{F}_{\text{NP}}(b, Q)$ primarily parametrizes the “power-suppressed” terms, i.e., terms proportional to positive powers of b . When assessed in a fit, $\mathcal{F}_{\text{NP}}(b, Q)$ also contains admixture of the leading-power terms (logarithmic in b terms), which were not properly included in the approximate leading-power function $\tilde{W}_{\text{LP}}(b)$ [cf. Eq. (4)]. In contrast, estimates of $\mathcal{F}_{\text{NP}}(b, Q)$ made in the infrared renormalon analysis explicitly remove all leading-power contributions from $\mathcal{F}_{\text{NP}}(b, Q)$ [11]. While the recent studies [9–13] point to an approximately Gaussian form of $\mathcal{F}_{\text{NP}}(b, Q)$ [$\mathcal{F}_{\text{NP}}(b, Q) \propto b^2$], they disagree on the magnitude of $\mathcal{F}_{\text{NP}}(b, Q)$ and its Q dependence. The source of these differences can be traced to the varying assumptions about the form of the leading-power function $\tilde{W}_{\text{LP}}(b)$ at $b < 2 \text{ GeV}^{-1}$, which is correlated in the fit with $\mathcal{F}_{\text{NP}}(b, Q)$. The exact behavior of $\tilde{W}(b)$ at $b > 2 \text{ GeV}^{-1}$ is of reduced importance, as $\tilde{W}(b)$ is strongly suppressed at such b . The new improvements described here (excellent agreement of $\mathcal{F}_{\text{NP}}(b, Q)$ with the data and renormalon analysis) result from modifications in the model for $\tilde{W}_{\text{LP}}(b)$ at $b < 2 \text{ GeV}^{-1}$. The improvements are preserved under variations of the large- b form of $\tilde{W}_{\text{LP}}(b)$ in a significant range of the model parameters.

Our Letter follows the notations in Ref. [9]. The form factor $\tilde{W}(b)$ factorizes at all b as [2–4]

$$\tilde{W}(b) = \sum_{j=q,\bar{q}} \frac{\sigma_j^{(0)}}{S} e^{-S(b,Q)} \mathcal{P}_j(x_1, b) \mathcal{P}_{\bar{j}}(x_2, b), \quad (1)$$

where $\sigma_j^{(0)}/S$ is a constant prefactor [4], and $x_{1,2} \equiv e^{\pm y} Q/\sqrt{S}$ are the Born-level momentum fractions, with y being the rapidity of the vector boson. The b -dependent parton densities $\mathcal{P}_j(x, b)$ and Sudakov function

$$\mathcal{S}(b, Q) \equiv \int_{b_0^2/b^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[\mathcal{A}(\alpha_s(\bar{\mu})) \ln\left(\frac{Q^2}{\bar{\mu}^2}\right) + \mathcal{B}(\alpha_s(\bar{\mu})) \right] \quad (2)$$

are universal in Drell–Yan-like processes and SIDIS [5]. When the momentum scales Q and b_0/b (where $b_0 \equiv 2e^{-\gamma_E} \approx 1.123$ is a dimensionless constant) are much larger than 1 GeV , $\tilde{W}(b)$ reduces to its perturbative part $\tilde{W}_{\text{pert}}(b)$, i.e., its leading-power (logarithmic in b) part evaluated at a finite order of α_s :

$$\begin{aligned} \tilde{W}(b) \Big|_{Q, b_0/b \gg 1 \text{ GeV}} &\approx \tilde{W}_{\text{pert}}(b) \\ &\equiv \sum_{j=q,\bar{q}} \frac{\sigma_j^{(0)}}{S} e^{-\mathcal{S}_P(b,Q)} [\mathcal{C} \otimes f]_j(x_1, b; \mu_F) [\mathcal{C} \otimes f]_{\bar{j}}(x_2, b; \mu_F). \end{aligned} \quad (3)$$

Here $\mathcal{S}_P(b, Q)$ and $[\mathcal{C} \otimes f]_j(x, b; \mu_F) \equiv \sum_a \int_x^1 d\xi/\xi \times \mathcal{C}_{ja}(x/\xi, \mu_F b) f_a(\xi, \mu_F)$ are the finite-order approximations to the leading-power parts of $\mathcal{S}(b, Q)$ and $\mathcal{P}_j(x, b)$. $f_a(x, \mu_F)$

is the k_T -integrated parton density, computed in our study by using the CTEQ6M parameterization [14]. $\mathcal{C}_{ja}(x, \mu_F b)$ is the Wilson coefficient function. We compute $\mathcal{S}_P(b, Q)$ up to $O(\alpha_s^2)$ and \mathcal{C}_{ja} up to $O(\alpha_s)$.

In Z boson production, the maximum of $b\tilde{W}(b)$ is located at $b \approx 0.25 \text{ GeV}^{-1}$, and $\tilde{W}_{\text{pert}}(b)$ dominates the Fourier–Bessel integral. In the examined low- Q region, the maximum of $b\tilde{W}(b)$ is located at $b \approx 1 \text{ GeV}^{-1}$, where higher-order corrections in powers of α_s and b must be considered. We reorganize Eq. (1) to separate the leading-power (LP) term $\tilde{W}_{\text{LP}}(b)$, given by the model-dependent continuation of $\tilde{W}_{\text{pert}}(b)$ to $b \gtrsim 1 \text{ GeV}^{-1}$, and the nonperturbative exponent $e^{-\mathcal{F}_{\text{NP}}(b,Q)}$, which absorbs the power-suppressed terms:

$$\tilde{W}(b) = \tilde{W}_{\text{LP}}(b) e^{-\mathcal{F}_{\text{NP}}(b,Q)}. \quad (4)$$

At $b \rightarrow 0$, the perturbative approximation for $\tilde{W}(b)$ is restored: $\tilde{W}_{\text{LP}} \rightarrow \tilde{W}_{\text{pert}}$, $\mathcal{F}_{\text{NP}} \rightarrow 0$. The power-suppressed contributions are proportional to even powers of b [10]. Detailed expressions for some power-suppressed terms are given in Ref. [11]. At impact parameters of order 1 GeV^{-1} , we keep only the first power-suppressed contribution proportional to b^2 :

$$\mathcal{F}_{\text{NP}} \approx b^2 (a_1 + a_2 \ln(Q/Q_0) + a_3 \phi(x_1) + a_3 \phi(x_2)) + \dots, \quad (5)$$

where a_1, a_2 , and a_3 are coefficients of magnitude less than 1 GeV^2 , and $\phi(x)$ is a dimensionless function. The terms $a_2 \ln(Q/Q_0)$ and $a_3 \phi(x_j)$ arise from $\mathcal{S}(b, Q)$ and $\ln[\mathcal{P}_j(x_j, b)]$ in $\ln[\tilde{W}(b)]$, respectively. We neglect the flavor dependence of $\phi(x)$ in the analyzed region dominated by scattering of light u and d quarks. \mathcal{F}_{NP} is consequently a universal function within this region. The dependence of \mathcal{F}_{NP} on $\ln Q$ follows from renormalization-group invariance of the soft-gluon radiation [3]. The coefficient a_2 of the $\ln Q$ term has been related to the vacuum average of the Wilson loop operator and estimated within lattice QCD as $0.19_{-0.09}^{+0.12} \text{ GeV}^2$ [11].

The preferred \mathcal{F}_{NP} is correlated in the fit with the assumed large- b behavior of \tilde{W}_{LP} . We examine this correlation in a modified version of the b_* model [3,4]. The shape of \tilde{W}_{LP} is varied in the b_* model by adjusting a single parameter b_{max} . Continuity of \tilde{W} and its derivatives, needed for the numerical stability of the Fourier transform, is always preserved. We set $\tilde{W}_{\text{LP}}(b) \equiv \tilde{W}_{\text{pert}}(b_*)$, with $b_*(b, b_{\text{max}}) \equiv b(1 + b^2/b_{\text{max}}^2)^{-1/2}$. $\tilde{W}_{\text{LP}}(b)$ reduces to $\tilde{W}_{\text{pert}}(b)$ as $b \rightarrow 0$ and asymptotically approaches $\tilde{W}_{\text{pert}}(b_{\text{max}})$ as $b \rightarrow \infty$. The b_* model with a relatively low $b_{\text{max}} = 0.5 \text{ GeV}^{-1}$ was a choice of the previous q_T fits [8,9]. However, it is natural to consider b_{max} above 1 GeV^{-1} in order to avoid ad hoc modifications of $\tilde{W}_{\text{pert}}(b)$ in the region where perturbation theory is still applicable. To implement $\tilde{W}_{\text{pert}}(b_*)$ for $b_{\text{max}} > 1 \text{ GeV}^{-1}$, we must choose the factorization scale μ_F such that it stays, at any b and b_{max} , above the initial scale $Q_{\text{ini}} = 1.3 \text{ GeV}$ of the DGLAP evolution for the CTEQ6 PDF's $f_a(x, \mu_F)$. We keep the usual choice $\mu_F = C_3/b_*(b, b_{\text{max}})$, where $C_3 \sim b_0$, for $b_{\text{max}} \leq b_0/Q_{\text{ini}} \approx 0.86 \text{ GeV}^{-1}$. Such choice is not acceptable at $b_{\text{max}} > b_0/Q_{\text{ini}}$, as it would allow $\mu_F < Q_{\text{ini}}$. Instead, we choose $\mu_F = C_3/b_*(b, b_0/Q_{\text{ini}})$ for $b_{\text{max}} > b_0/Q_{\text{ini}}$, i.e., we

substitute b_0/Q_{ini} for b_{max} in μ_F to satisfy $\mu_F \geq Q_{\text{ini}}$ at any b . Aside from $f_a(x, \mu_F)$, all terms in $\tilde{W}_{\text{pert}}(b)$ are known, at least formally, as explicit functions of $\alpha_s(1/b)$ at all $b < 1/\Lambda_{\text{QCD}}$. We show in Ref. [15] that this prescription preserves correct re-summation of the large logarithms and is numerically stable up to $b_{\text{max}} \sim 3 \text{ GeV}^{-1}$.

We perform a series of fits for several choices of b_{max} by closely following the previous global q_T analysis [9]. We consider a total of 98 data points from production of Drell–Yan pairs in E288, E605, and R209 fixed-target experiments, as well as from observation of Z bosons with $q_T < 10 \text{ GeV}$ by CDF and DØ detectors in the Tevatron Run-1. See Ref. [9] for the experimental references. Overall normalizations for the experimental cross sections are varied as free parameters. Our best-fit normalizations agree with the published values within the systematical errors provided by the experiments, with the

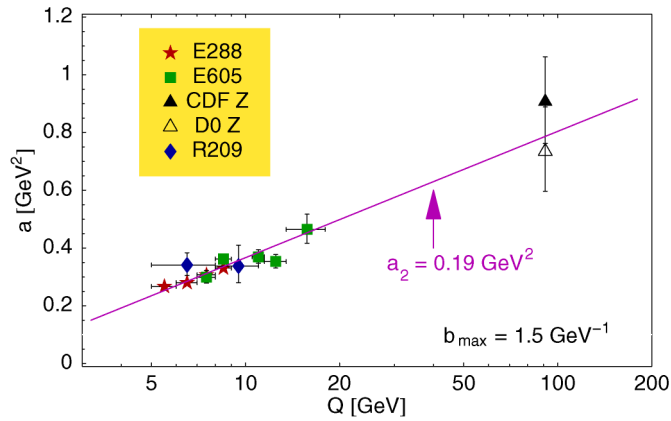


Fig. 1. The best-fit values of $a(Q)$ obtained in independent scans of χ^2 for the contributing experiments. The vertical error bars correspond to the increase of χ^2 by unity above its minimum in each Q bin. The slope of the line is equal to the central-value prediction from the renormalon analysis [11].

exception of the CDF Run-1 normalization (rescaled down by 7%).

To test the universality of \mathcal{F}_{NP} , we individually examine each bin of Q . We choose $\mathcal{F}_{\text{NP}} = a(Q)b^2$ and independently fit it to each of the 5 experimental data sets to determine the most plausible normalization in each experiment. We then set the normalizations equal to their best-fit values and examine χ^2 at each Q as a function of $a(Q)$. For $b_{\text{max}} = 1 - 2 \text{ GeV}^{-1}$, the best-fit values of $a(Q)$ follow a nearly linear dependence on $\ln Q$ [cf. Fig. 1]. The slope $a_2 \equiv da(Q)/d(\ln Q)$ is close to the renormalon analysis expectation of 0.19 GeV^2 [11]. The agreement with the universal linear $\ln Q$ dependence worsens if b_{max} is chosen outside the region $1 - 2 \text{ GeV}^{-1}$. Since the best-fit $a(Q)$ are found independently in each Q bin, we conclude that the data support the universality of \mathcal{F}_{NP} , when b_{max} lies in the range $1 - 2 \text{ GeV}^{-1}$. In another test, we find that each experimental data set individually prefers a nearly quadratic dependence on b , $\mathcal{F}_{\text{NP}} = a(Q)b^{2-\beta}$, with $|\beta| < 0.5$ in all five experiments.

To further explore the issue, we simultaneously fit our model to all the data. We parametrize $a(Q)$ as $a(Q) \equiv a_1 + a_2 \ln[Q/(3.2 \text{ GeV})] + a_3 \ln[100x_1x_2]$. This parametrization coincides with the BLNY form [9], if the parameters are renamed as $\{g_1, g_2, g_1g_3\}(\text{BLNY}) \rightarrow \{a_1, a_2, a_3\}(\text{here})$. It agrees with the generic form of $\mathcal{F}_{\text{NP}}(b, Q)$ in Eq. (5), if one identifies $\phi(x) = \ln(x/0.1)$. We carry out two sequences of fits for $C_3 = b_0$ and $C_3 = 2b_0$ to investigate the stability of our prescription for μ_F and sensitivity to $\mathcal{O}(\alpha_s^2)$ corrections. The dependence on C_3 is relatively uniform across the whole range of b_{max} , indicating that our choice of μ_F for $b_{\text{max}} > b_0/Q_{\text{ini}}$ is numerically stable.

Fig. 2 shows the dependence of the best-fit χ^2 , a_1 , a_2 , and a_3 on b_{max} . As b_{max} is increased above 0.5 GeV^{-1} assumed in the BLNY study, χ^2 rapidly decreases, becomes relatively flat at $b_{\text{max}} = 1 - 2 \text{ GeV}^{-1}$, and grows again at $b_{\text{max}} > 2 \text{ GeV}^{-1}$.

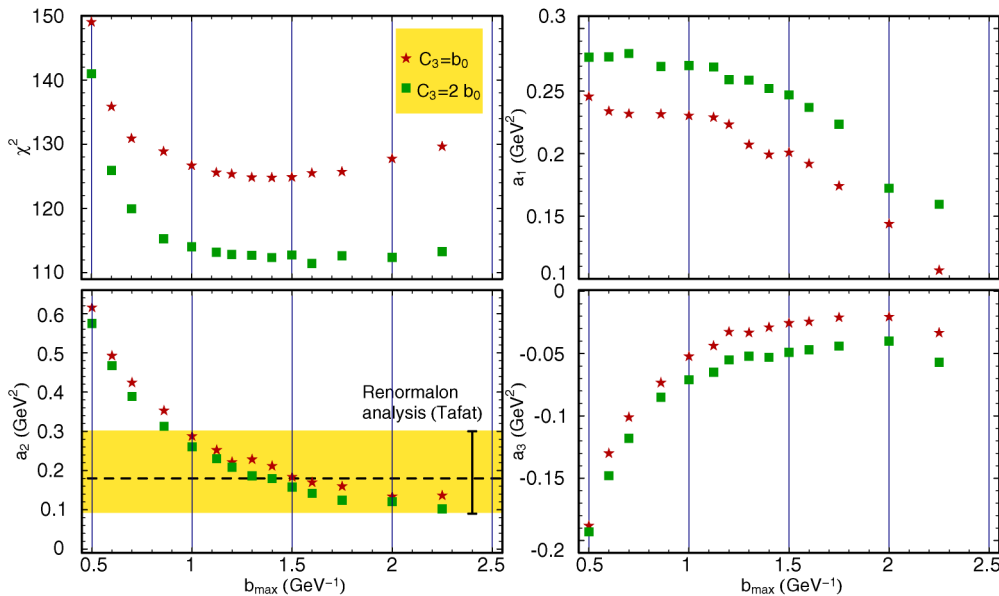


Fig. 2. The best-fit χ^2 and coefficients a_1 , a_2 , and a_3 in $\mathcal{F}_{\text{NP}}(b, Q)$ for different values of b_{max} , $C_3 = b_0$ (stars) and $C_3 = 2b_0$ (squares). The size of the symbols approximately corresponds to 1σ errors for the shown parameters.

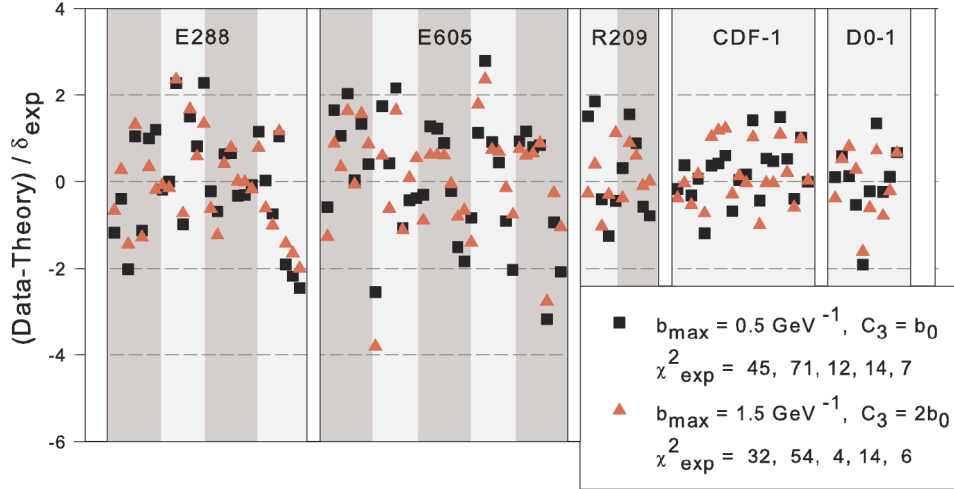


Fig. 3. Differences between the measured (data) and theoretical (theory) cross sections, divided by the experimental error δ_{exp} in each (Q, q_T) bin. The values of χ^2 for each experiment in the two fits are listed in the legend in the same order.

The global minimum of $\chi^2 = 125(111)$ is reached at $b_{\max} \approx 1.5 \text{ GeV}^{-1}$, where all data sets are described equally well, without major tensions among the five experiments. The improvement in χ^2 mainly ensues from better agreement with the low- Q experiments (E288, E605, and R209), while the quality of all fits to the Z data is about the same. This is illustrated by Fig. 3, which shows the differences between the measured and theoretical cross sections, divided by the experimental errors δ_{exp} , as well as the values of χ^2 in each experiment, in two representative fits for $b_{\max} = 0.5 \text{ GeV}^{-1}$, $C_3 = b_0$ (squares) and $b_{\max} = 1.5 \text{ GeV}^{-1}$, $C_3 = 2b_0$ (triangles). The data are arranged in bins of Q (shown by gray background stripes) and q_T , with both variables increasing from left to right. For $b_{\max} = 1.5 \text{ GeV}^{-1}$, the (data – theory) differences are reduced on average in the entire low- Q sample, resulting in lower χ^2 in three low- Q experiments. Two outlier points in the E605 sample (the first point in the second Q bin and fifth point in the fifth Q bin) disagree with the other E288 and E605 data in the same Q and x region and contribute 15–25 units to χ^2 at any b_{\max} . If the two outliers were removed, one would find $\chi^2/\text{d.o.f.} \approx 1$ at the global minimum.

The magnitudes of a_1 , a_2 , and a_3 are reduced when b_{\max} increases from 0.5 to 1.5 GeV^{-1} . In the whole range $1 \leq b_{\max} \leq 2 \text{ GeV}^{-1}$, a_2 agrees with the renormalon analysis estimate. The coefficient a_3 , which parametrizes deviations from the linear $\ln Q$ dependence, is considerably smaller (< 0.05) than both a_1 and a_2 (~ 0.2). A reasonable quality of the fit is retained if a_3 is set to zero by hand: χ^2 increases by ≈ 5 in such a fit above its minimum in the fit with a free a_3 . In contrast, χ^2 increases by > 200 units if $a_3 = g_1 g_3$ is set to zero at $b_{\max} = 0.5 \text{ GeV}^{-1}$, as it was noticed in the BLNY study.

The preference for the values of b_{\max} between 1 and 2 GeV^{-1} indicates, first, that the data do favor the extension of the b range where $\tilde{W}_{\text{LP}}(b)$ is approximated by the exact $\tilde{W}_{\text{pert}}(b)$. In Z boson production, this region extends up to $3\text{--}4 \text{ GeV}^{-1}$ as a consequence of the strong suppression of the large- b tail by the Sudakov exponent. The fit to the Z data is

actually independent of b_{\max} within the experimental uncertainties for $b_{\max} > 1 \text{ GeV}^{-1}$. The best-fit form factors $b\tilde{W}(b)$ for $b_{\max} = 0.5$ and 1.5 GeV^{-1} in Z boson production are shown in Fig. 4(a).

In the low- Q Drell–Yan process, continuation of $b\tilde{W}_{\text{pert}}(b)$ far beyond $b \approx 1 \text{ GeV}^{-1}$ raises objections, since $b\tilde{W}_{\text{pert}}(b)$ has a maximum and is unstable with respect to higher-order corrections at $b \approx 1.2\text{--}1.5 \text{ GeV}^{-1}$. The dubious large contributions to $\tilde{W}_{\text{pert}}(b)$ in this b region would deteriorate the quality of the fit. The b_* prescription with $b_{\max} < 2 \text{ GeV}^{-1}$ reduces the impact of the dubious terms on $\tilde{W}(b)$: for b_{\max} small enough, the maximum of $\tilde{W}_{\text{pert}}(b_*)$ is only reached at $b \gg 1.2 \text{ GeV}^{-1}$, where it is suppressed by $e^{-\mathcal{F}_{\text{NP}}(b, Q)}$. The best-fit form factors for the E605 kinematics, divided by the best-fit normalizations of the E605 data N_{fit} , are shown in Fig. 4(b).

If a very large b_{\max} comparable to $1/\Lambda_{\text{QCD}}$ is taken, $\tilde{W}_{\text{LP}}(b)$ essentially coincides with $\tilde{W}_{\text{pert}}(b)$, extrapolated to large b by using the known, although not always reliable, dependence of $\tilde{W}_{\text{pert}}(b)$ on $\ln b$. Similar, but not identical, extrapolations of $\tilde{W}_{\text{pert}}(b)$ to large b are realized in the models [12,13], which describe the Z data well, in accord with our own findings. In Z boson production, our best-fit $a(M_Z) = 0.85 \pm 0.10 \text{ GeV}^2$ agrees with 0.8 GeV^2 found in the extrapolation-based models, and it is about a third of 2.7 GeV^2 predicted by the BLNY parametrization. Our results support the conjecture in [12] that a_3 is small if the exact form of $\tilde{W}_{\text{pert}}(b)$ is maximally preserved. To describe the low- Q data, the model [12] allowed a large discontinuity in the first derivative of $\tilde{W}(b)$ at b equal to the separation parameter $b_{\max}^{QZ} = 0.3\text{--}0.5 \text{ GeV}^{-1}$, where switching from the exact $\tilde{W}_{\text{pert}}(b)$ to its extrapolated form occurs [cf. Fig. 4(b)]. In the revised b_* model, such discontinuity does not happen, and $\tilde{W}_{\text{LP}}(b)$ is closer to the exact $\tilde{W}_{\text{pert}}(b)$ in a wider b range at low Q than in the model [12]. The two models differ substantially at $b \approx 1 \text{ GeV}^{-1}$, as seen in Fig. 4(b).

To summarize, the extrapolation of $\tilde{W}_{\text{pert}}(b)$ to $b > 1.5 \text{ GeV}^{-1}$ is disfavored by the low- Q data sets, if a purely Gaussian form of \mathcal{F}_{NP} is assumed. The Gaussian approxima-

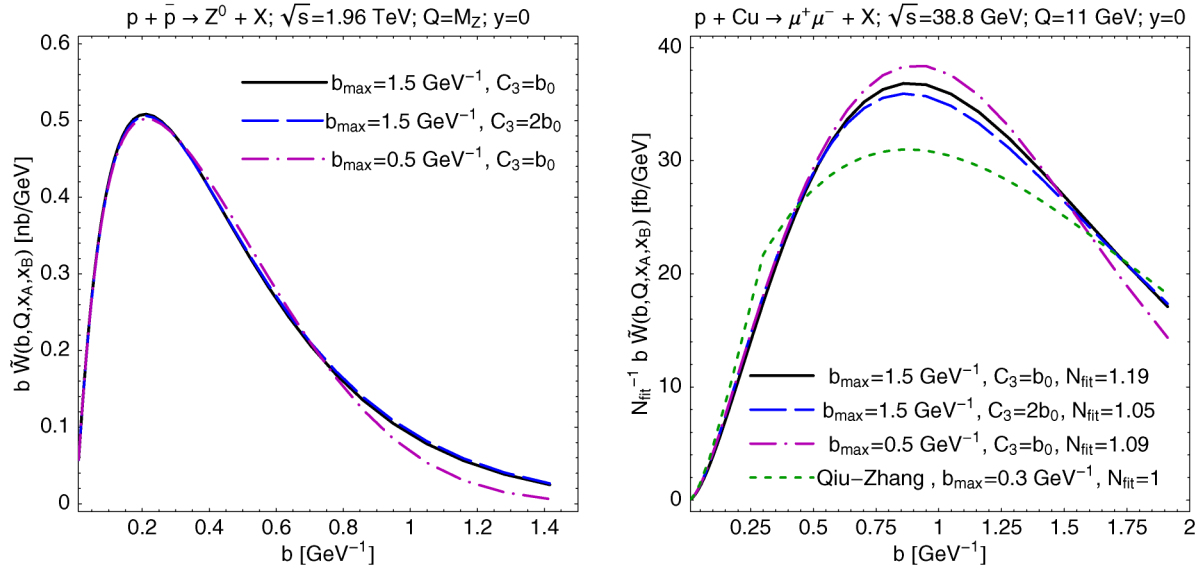


Fig. 4. The best-fit form factors $b\tilde{W}(b)$ in (a) Tevatron Run-2 Z boson production; (b) E605 experiment. In the E605 case, $b\tilde{W}(b)$ are divided by the best-fit normalizations N_{fit} for the E605 data, and the form factor in the Qiu–Zhang parametrization [12] for $b_{\text{max}}^{QZ} = 0.3 \text{ GeV}^{-1}$ is also shown.

tion is adequate, on the other hand, in the b_* model with b_{max} in the range $1\text{--}2 \text{ GeV}^{-1}$. Here variations in b_{max} are compensated well by adjustments in a_1 , a_2 , and a_3 , and the full form factor $b\tilde{W}(b)$ stays approximately independent of b_{max} . The best-fit parameters in \mathcal{F}_{NP} are quoted for $b_{\text{max}} = 1.5 \text{ GeV}^{-1}$ as $\{a_1, a_2, a_3\} = \{0.201 \pm 0.011, 0.184 \pm 0.018, -0.026 \pm 0.007\} \text{ GeV}^2$ for $C_3 = b_0$, and $\{0.247 \pm 0.016, 0.158 \pm 0.023, -0.049 \pm 0.012\} \text{ GeV}^2$ for $C_3 = 2b_0$. In Ref. [15], the experimental errors are propagated into various theory predictions with the help of the Lagrange multiplier and Hessian matrix methods, discussed, e.g., in Ref. [14]. We find that the global fit places stricter constraints on \mathcal{F}_{NP} at $Q = M_Z$ than the Tevatron Run-1 Z data alone. Theoretical uncertainties from a variety of sources are harder to quantify, and they may be substantial in the low- Q Drell–Yan process. In particular, χ^2 for the low- Q data improves by 14 units when the scale parameter C_3 in μ_F is increased from b_0 to $2b_0$, reducing the size of the finite-order $\tilde{W}_{\text{pert}}(b)$ at low Q . The best-fit normalizations N_{fit} also vary with C_3 . The dependence of the quality of the fit on the arbitrary factorization scale μ_F indicates importance of $\mathcal{O}(\alpha_s^2)$ corrections at low Q , but does not substantially increase uncertainties at the electroweak scale. Indeed, the $\mathcal{O}(\alpha_s^2)$ corrections and scale dependence are smaller in W and Z production. In addition, the term $a_2 \ln Q$, which arises from the soft factor $S(b, Q)$ and dominates \mathcal{F}_{NP} at $Q = M_Z$, shows little variation with C_3 [cf. Fig. 2(c)]. Consequently, the revised b_* model with $b_{\text{max}} \approx 1.5 \text{ GeV}^{-1}$ reinforces our confidence in transverse momentum resummation at electroweak scales by exposing the soft-gluon origin and universality of the dominant nonperturbative terms at collider energies.

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