Multi-state supernetworks: recent progress and prospects

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Abstract: Supernetworks have long been adopted to address multi-dimensional choice problems, which are thorny to solve for classic singular networks. Originated from combining transport mode and route choice into a multi-modal network, supernetworks have been extended into multi-state networks to include activity-travel scheduling, centered around activity-based models of travel demand. A key feature of the network extensions is that multiple choice facets pertaining to conducting a full activity program can be modeled in a consistent and integrative fashion. Thus, interdependencies and constraints between related choice facets can be readily captured. Given this advantage of integrity, the modeling of supernetwork has become an emerging topic in transportation research. This paper summarizes the recent progress in modeling multi-state supernetworks and discusses future prospects.

Key words: supernetwork; multi-modal; multi-state; activity-based models

1 Introduction

The last two decades have witnessed significant progress in developing and applying activity-based models of travel demand (Bowman and Ben-Akiva 2001; Hensen et al. 2009; Pinjari and Bhat 2011; Rasouli and Timmermans 2014). In this context, modeling daily activity patterns at a high degree of spatial and temporal resolution is deemed necessary to better predict traffic flows. In addition, the shift from the four-step model to activity-based travel demand forecasting involved the introduction of interdependencies in the various choice facets underlying activity-travel patterns, and a substitution of the trip as the unit of analysis to complex household activity-travel patterns as the focus of attention.

At the academic platform, activity-based analysis has become the norm. In contrast, the four-step model still dominates transportation planning across the world, although there is some evidence of diffusion and dissemination of activity-based models to planning practice, in particular in the United States and to some extent also in Europe. Yet, the models applied in practice seem to have addressed primarily long-term planning problems.
although their development has been triggered by the increasing relevance of transportation management. One of the reasons might be that the level of detail to address transportation management problems has remained insufficient. In addition, activity-based models have not fundamentally re-addressed the route choice and traffic flow problem. Rather, time-dependent O-D matrices predicted by activity-based models served as input to static or dynamic traffic assignment algorithms.

As a consequence, dedicated but less comprehensive models have been used to predict the impact of transportation management schemes primarily on departure time, route choice and in turn on traffic flows, travel times and congestion. A limitation of these models is their focus on a single aspect of travel behavior. Thus, these models are based on very strong assumptions about how travellers respond to management schemes. Comprehensive and complex models that predict with much detail how individuals and household organize their activities in time and space and respond to external changes are required.

At the core of activity-based models should be activity-travel scheduling that predicts how a set of activities planned for a given day are organized in time and space. Existing approaches fall short in fully representing activity-travel patterns at a high level of detail. In particular, parking choice and multi-modal trip chaining between private vehicles and public transport (PT) are often neglected. Moreover, since multiple choice dimensions are rarely modeled simultaneously, the feasibility of the activity-travel patterns are not checked in a global sense; meanwhile, synchronizations among individuals’ activity programs, transport networks, and network of facilities and services are not fully captured. Hence, it can be argued current models do not accurately capture all interdependencies.

The major reason is that an overall representation that allows fully representing the full activity-travel patterns and simultaneously modeling the choice facets is missing. Supernetworks, defined as networks of networks (Sheffi 1985) or networks beyond existing networks (Nagurney et al. 2002), may provide a solution to the stipulated limitations of existing models. Supernetworks thus allow systemically integrating different networks of service provisions, transport and activity-travel behavior in a single representation and they have the potential to simultaneously model multiple choice facets. In that sense, the combination of activity-based modeling and supernetwork approach would offer a promising way to address the aforementioned challenges.

Seminal work on this topic was conducted by Arentze and Timmermans (2004). They developed multi-state supernetworks, which allow synchronizing networks and modeling multi-faceted choices simultaneously in terms of the high choice dimensions involved. Over the last few years, their original multi-state supernetwork model has been systematically elaborated in Liao et al. (2010; 2011; 2012; 2013a; 2013b; 2014) to improve the representation efficiency and allow predicting activity-travel patterns with higher level of choice detail and dimensions. In this invited paper, these developments and identify prospects of future research are summarized.

2 Review of supernetwork model

Network extensions have a long history in addressing transportation problems. Dafermos (1972) was the first who demonstrated an abstract multiclass user traffic network by expansion of a road network. The importance of such abstract networks was accentuated by Sheffi and Daganzo (1978) for modeling mode and route choice in a so-called hypernetwork, which was later re-termed supernetwork (Sheffi 1985). The supernetwork was constructed by adding transfer links at locations in both sub-networks, i.e. car road network and transit network, where an individual can switch between transport modes. A path through this supernetwork expresses the choices of mode and routes.

Similar network extensions have been developed for modeling multi-modal trip chaining by Nguyen
and Pallottino (1989), Lozzano and Storchi (2002), and Carlier et al. (2003). Networks of all transport modes, i.e., car, bike, tram, pedestrian etc., are connected at every possible transfer locations.

The concept of supernetwork caught attention due to the intensive research and applications of Nagurney's group (2002, 2003, 2005). At a trip-based level, Nagurney et al. (2002) first introduced transaction links to model activity implementation. In their supernetwork representation, route choice and home/work location choice (commuting vs tele-commuting) can be modeled simultaneously. A path through the supernetwork represents choice of route that may involve virtual travel. However, multi-modal trip chaining was not taken into account. Therefore, it cannot be easily extended to model an activity program of multiple activities.

To model complete activity programs, Arentze and Timmermans (2004) suggested multi-state supernetworks, which integrate activity programs of individuals, multi-modal transport networks (Carlier et al. 2003) and locations of facilities/services. The multi-state supernetworks are constructed for each individual separately and are made up of as many copies of physical networks as different mode and activity states in an activity program execution. The mode state defines whether or not and which particular mode is used. In addition, to represent locations of picking-up a private vehicle and parking of a vehicle, the vehicle state defines where the private vehicle is (in use or parked somewhere). In each vehicle state, a link can be further mode-identified by its feature. The activity state defines which activities have already been conducted. Thus, different stages of conducting the activity programs can be associated with different activity-vehicle states (combinations of activity and vehicle states). In this representation, nodes represent real locations in space. In addition, the following links are distinguished:

1) Travel links connect different nodes of the same activity state representing the movement of the individual from one location to another; the modes can be walking, bike, car, or any PT modes.

2) Transition links connect the same nodes in the same activity states but different vehicle states (i.e., parking/picking-up a private vehicle or boarding/alighting).

3) Transaction links connect the same nodes in different activity states, representing the implementation of activities.

This so-called multi-state supernetwork provides a powerful framework for scheduling activity agendas. Fig. 1 schematically shows an example of supernetwork representation for an individual's activity program. A hexagon denotes a network unit that represent locations and transportation system, in which an angle denotes a location. The

![Fig. 1 Supernetwork representation of Arentze and Timmermans (2004).](image-url)
network units in different rows relate to different activity states, while those in different columns represent different vehicle states. State definitions will be described in detail in the next section. The links between the hexagons always lead to state changes. A derived feature is that any path from start to end represents a feasible activity-travel pattern including sequence of activities, mode, route, parking and activity location choice.

The path formed by the bold links (Fig. 1) denotes a full activity-travel pattern that the individual leaves home with car, park the car at the parking location and then goes to conduct \( A_2 \); afterwards, the individual drops the bags of \( A_2 \) in the car, conducts \( A_1 \), and goes back to the parking location to pick-up the car and finally returns home. The least-cost path through a multi-state supernetwork represents the preferred set of choices for the concerned individual to conduct the activity program. The costs of such a path provides a measure of accessibility that takes into account multi-modal and multi-activity trip chaining as well as the synchronization among the land use system and transport networks. The link costs on each component should be defined specifically since the time spent on them is perceived differently. If assigning static link costs, the optimal activity-travel pattern can be found by classic shortest path algorithms.

Ramadurai and Ukkusuri (2010) also proposed an activity-based supernetwork representation to model dynamic user-equilibrium (UE) for combined activity-travel choices. Since activity-travel patterns are represented as paths through the network, it is natural to adopt path-based assignment for UE. In their supernetwork representation, a potential activity location node is expanded with many copies to model choice of route and activity location simultaneously. However, choices of mode and parking are not considered in the representation. Another drawback is that extra nodes are needed for differentiating travel links and activity links to guarantee path feasibility.

The above supernetwork models to a large extent follow the logic of "path choice" (Nagurney et al. 2002) through the abstract networks. There are also other network-based models proposed to model multiple choice facets simultaneously. Most of them focused on modeling urban multi-modal trips, i.e. at the level of route choice and mode choice. Less attention has been paid to multi-modal freight supernetworks. Nevertheless, the supernetwork representation of these models can be generalized into the one proposed by Carlier et al. (2003). Furthermore, several supernetwork models have been proposed to analyze travel patterns in an activity-based context. Thus, activity-related choices such as activity location, start time, duration, and parking location are combined at some level with choice of route and mode. Specifically, ignoring parking location choice, several studies examined user equilibrium in activity-based time-space expanded transit networks (e.g., Li et al. 2010; Fu and Lam 2014). The network representations of these models can be generalized into multi-state supernetworks (Fig. 1). In other words, multi-state supernetworks represent the state-of-the-art for synchronizing networks and model multi-faceted choices simultaneously.

3 Recent progress

It is widely recognized that an integrated platform encompassing the networks of transportation and locations/facilities of services and as well as the activity programs of travellers is essential for any kind of mobility-related analysis (Waddell 2011). The multi-state supernetwork approach is suitable for integrating these elements to evaluate travellers' responses to a large spectrum of land-use and transport policies. This operational model was later extended to include additional facets of activity scheduling and optimized for large-scale micro-simulation. These extensions involved embedding new choice options such as modern modalities and joint travel arrangements, further increasing the relevance of the model for transport policy making. During the past few years, the original multi-state supernetwork model has been
systematically extended and enhanced. This section describes these elaborations and improvements. An individual’s activity program in general is defined as an activity-travel plan involving an individual leaving home with at most one private vehicle to conduct at least one out-of-home activity, and returning home with all activities conducted and all private vehicles at home.

3.1 An efficient supernetwork representation

Although multi-state supernetworks are advantageous to model interdependencies between different choice facets simultaneously, this merit comes with the expense of potential combination explosion of the network scale. As all the choice facets are interconnected and explicitly represented, a personalized supernetwork needs to incorporate as many copies of a network unit as there are possible states associated with the different stages of an activity program. Consequently, the derived supernetwork may become very large and possibly intractable even for a small activity program. One obvious cause of this problem is that the network unit (or base network) is the full transport network.

For that matter, Liao et al. (2010) proposed a more efficient multi-state supernetwork representation by allowing considerable reduction in network size, without the expense of representation power. The integrated network unit can be split, which can contribute to expressing the choice facets more clearly and reducing the scale. Transition or transaction links are used to interconnect network units of different states, and there is a large redundant part in each of them. For example, before a used private vehicle is parked, the individual cannot use networks of pedestrian and public transport; and on the other hand he/she cannot use car or bike network when it is parked. Therefore, the integrated network can be split into a set of PVNs (private vehicle networks) and a PTN (PT network). In the supernetwork, travel links are inside PVNs and PTNs; boarding/alighting PT links are inside PTNs only; parking/pick-uping and transaction links are used to interconnect PVNs and PTNs, and PTNs and PTNs respectively. More specifically, inside a PVN, there are only parking locations (including home location) and for each pair of locations there is a PVN connection, which involves only one mode. Inside a PTN, there are parking (if any) and activity locations connected by links which may include walking, waiting, boarding/alighting, and in-vehicle travel links. Using a pentagon and a hexagon to denote a PVN and PTN respectively (with the angles denoting locations), Fig. 2 shows an example of activity and vehicle state transfer. In Fig. 2(a), activity state 0 and 1 denote the activity is un-conducted and conducted respectively; in Fig. 2(b), there are three vehicle states, i.e., the vehicle being in use, or parking at \( P_1 \) or \( P_2 \).

Following the above logic, the action/choice space of conducting a whole activity program can be represented more efficiently. Fig. 3 is an example of a supernetwork representation for an individual’s AP, including one fixed activity (at \( A_1 \)) and one flexible activity (at \( A_2 \) assuming only one alternative location for the sake of simplic-
Fig. 3 Multi-state supernetwork representation

ity), and two private vehicles (car and bike). $P_1$, $P_2$ and $P_3$, $P_4$ are parking locations for car and bike respectively. $P_0$ and $P_5$ denote car and bike in use respectively. $s_1, s_2$ represents the activity states for $A_1$ and $A_2$. $H$ and $H'$ denote home at the start and end of the activity states respectively. It can be proven that any path from $H$ to $H'$ denotes a possible full daily activity-travel pattern (undirected links are bi-directed). The path denoted by the bold links shows that the individual leaves home by car to conduct the fixed activity at $A_1$ with parking at $P_2$, then returns home and switches to bike to conduct the flexible activity at $A_2$ with parking at $P_4$, and finally returns home.

3.2 Constructing personalized networks

As argued in their study (Arentze and Timmermans 2004), the multi-state supernetwork approach may still be feasible when personalized supernetworks are constructed for each individual. A personalized supernetwork does not just allow representing preferences and perceptions individual-specific, but also allows a reduction to the relevant subset of a transport network. Thus, personalized supernetworks are not only more accurate in the sense that they are tailored to the preferences and perceptions of an individual, but also reduce network size. The split between PTN and PVN is beneficial to the supernetwork representation. However, the size of a supernetwork still strongly depends on the size of location choice sets. To further reduce the required size of supernetwork representations, Liao et al. (2011; 2012) proposed a heuristic approach for constructing personalized PVNs and PTN, based on the notion that generally only a small set of locations are of interest to individuals.

The approach regards activity and parking location choice models. Given an activity program, depending on whether an activity has more than one alternative location or not, it can be classified as a fixed location or a flexible location. Consider for example the activity work. If the individual is required to be present at a specified work location, work is an activity with a fixed location. Similarly, home is regarded as a fixed location where the individual leaves from and returns to. In the model fixed locations are considered as given. By contrast, shopping often allows a location choice and, therefore, generally is an activity with flexible locations. For those with flexible locations, the individual may need to narrow down the choice set into a smaller consideration set. In this decision-making process, two key factors are
Fig. 4 Example of narrowing down the choice set

Drawn to scale

Parking) or $P + R$ facilities ($P + Rs$) to switch to PT for avoiding long distance riding or congestion and difficulty of parking in city centers. These three types of locations are potential options for parking. Heuristic rules are proposed to narrow down the potential parking locations (Liao et al. 2012).

For a private vehicle $v$, $v$ is taken as $c$ (car) or $b$ (bike), two distance circles with centers at home are set for $i$, acceptance distance $E_{tv}$ and limit distance $E_{tv}$ satisfying $E_{tv} < E_{tv}$ and $E_{tv}$. The rules are as follows:

1. With $v$, $i$ will not drive a distance over $E_{tv}$ away from home but may drive over a distance of $E_{tv}$.
2. If there is an activity location that lies out of circle $E_{tv}$, $i$ must find a parking location near a PT stop for $v$ inside circle $E_{tv}$.
3. If it lies between $E_{tv}$ and $E_{tv}$, $i$ may find a parking location near a PT stop inside circle $E_{tv}$.
4. Otherwise, $i$ will drive directly to the activity location.

If there are still too many feasible parking locations, those leading to shorter travel time are selected. Fig. 5 is an example, in which TH/1 is potential for bike parking, and TH/1, TH/2, $P + R/1$, $P + R/2$ and $A$ are potential for car parking.

Once related activity and parking locations are selected, PVN and PTN connections between those locations can be generated by simply extracting them by static route choice models. Hence, a
3.3 Incorporating time constraints and dependencies

A key parameter is the number of selected activity locations, on which the number of parking locations and the final scale of the supernetwork is contingent. The larger this parameter the more likely the optimal locations are covered by the choice model. Sensitivity analysis showed that the optimal locations could be selected by setting low values of this parameter.

As shown in Eq. (2), activity-travel components are only treated in a static way. However, time dependency is a common phenomenon in nearly all activity-travel components (links of the supernetworks). Moreover, location choice models aforementioned did not accommodate space-time constraints in the selection of locations. Without taking their time constraints and dependencies into account, the model tends to output inaccurate predictions in the temporal dimension and even wrong predictions in activity patterns and locations. For that consideration, the temporal dimension in multi-state supernetworks was substantially improved in Liao et al. (2013a), by (i) embedding space-time constraints into location selection models; and (ii) systematically incorporating time-dependency in the activity-travel components.

To embed space-time constraints, the definition of an activity program is modified that each activity and its corresponding locations have specific opening times. Lower bound time constraints can be built when considering the candidacy of a location for a flexible activity lying between two fixed activities, which comes down to the requirements: (i) the earliest finishing time of the first fixed activity plus the ideal minimum duration of the flexible activity should be less than the closing time of the candidate; (ii) the earliest finishing time of the flexible activity plus the ideal minimum duration of the second fixed activity should be less than the closing time of the second fixed location. These lower bounds cancel out impossible locations and can be handled in the selection process by adding control values on the ideal minimum duration. Similar logic can be applied to select parking location. Note again that the purpose of location choice models is to remove absolute inferior locations but not to pick out the best locations.

Furthermore, all "links" (considering a PVN or PTN as a link) except picking-up links are turned time dependent. In detail, PVN connections with car travel look up time-dependent travel speed profiles; PTN connections look up PT timetables (Pyrga et al. 2008); linear parking cost profiles are used to calculate duration-dependent parking cost; and activity durations have been recently proposed to look up time-expanded duration choice space (Liao et al. 2014). After adding these elements, the supernetwork topology (e.g. Fig. 3) remains the same, whereas most of the link costs are defined time-dependently as:

$$\text{dis} U_{ism} = \beta_{ism} X_{ism} + \varepsilon_{ism}$$

A side product is that the network structure may fail the FIFO (first-in-first-out) property (Dean 2004). Therefore, a bicriteria label-correcting scheduling algorithm was developed based on certain behavioural assumptions. As a result of
this fundamental elaboration, the multi-supernet-work model can more accurately represent highly detailed activity-travel patterns with multi-modal and multi-activity trip chaining. Moreover, to account for the generalized representation, refined behavioral assumptions and dominance relationships are proposed in a bi-criteria label-correcting algorithm to find the optimal activity-travel pattern. Analyses and formal proofs of the scheduling algorithm are also provided.

3.4 Representing new modalities

Several new modalities such as information and communication technologies (ICT), electric bike (E-bike) and public transport bike (PT-bike) have the potential to improve accessibility and mobility efficiency while reducing congestion and energy consumption. With the improved performance achieved by these new modality tools, individuals may adapt their activity-travel choice behavior. This notion has generated lots of interests from policy making. These new modalities have also been systematically represented in the multi-state supernetwork (Liao et al. 2010; 2012; 2013). The algorithm proposed for time-dependent multi-state supernetworks can still find the optimal pattern with these new modalities included in the representation.

3.4.1 ICT use

There is long established and developing field of research examining how ICT use and activity-travel interact. Although theoretical and empirical evidence shows that the interactions are highly complex, a notable point is that the short-term effects of ICT use on the implementation of daily activity programs can be identified as substitution, fragmentation and multi-tasking (Mokhtarian et al. 2005). This subsection discusses how these concepts can be represented in a multi-state supernetwork.

(1) Substitution

Travel is undertaken to access people, goods, services and opportunities. ICT is evidently enabling some people on some occasions to gain such access without travel. If ICT offers alternative means of conducting an activity, it may substitute going to a specific location to conduct the activity, and hence eliminate the travel to that location. The premise of substitution is that the location-based activity has the ICT-based counterpart and the individual has the access to this ICT service. Thus, this effect may occur when the premise is satisfied elsewhere other than at the physical activity locations. To capture this possibility, transaction links are decomposed to virtual and physical transaction links. While physical transaction links connect the actual activity locations of different activity states, virtual transaction links connect locations where the activities can be conducted via ICT use. An advantage over the supernetwork conceptualization of ICT substitution (Nagurney 2002), in which a virtual link connects the substitution and the physical activity location, is that this format allows the study of substitution embedded in an activity program potentially involving multiple activities and stops.

(2) Fragmentation

Couclelis (2004) has argued that the association between activity, place and time has weakened through ICT, thereby facilitating the decomposition of activities into multiple segments of sub-tasks that can be conducted at different times and/or locations, for example, part-day home-working. Such separation of activities into discrete pieces is commonly termed as fragmentation. The fragmentation of activities can occur on three levels: manner, space, and time (Couclelis 2004). To represent these, if an activity is likely to be decomposed into several subtasks, each subtask is regarded as a sub-activity in parallel with other activities. If all the states of these sub-activities turn from 0 to 1, this activity has been done. Substitution may also take effect in some of the sub-activities so that the manner of conducting the activity changes. If at least one sub-activity is substituted somewhere, the activity is fragmented spatially. Thus, substitution is a component of fragmentation.

(3) Multi-tasking

Multi-tasking is closely related to fragmentation
but differs in perspective. Multi-tasking is about whether several activities are conducted simultaneously during a time period (Kenyon and Lyons 2007). Multi-tasking can enable individuals to reconfigure their activity participation in an effective way thereby releasing time for additional activities. Two widely accepted types of multi-tasking are multi-tasking while traveling (emailing on a train) or at a fixed location (e.g., online shopping during work). For the time being, multi-tasking is considered for those pre-assigned in the activity program. For those outside the activity program, for instance, reading a book at trains or casual i-chatting during work, it is assumed that they do not have an influence in terms of change of activity states but merely on the components affecting the disutility of travel and transaction links. Similarly, the situation is also classified as this kind when a little part of an activity is multi-tasked during a period but not substantially enough to be regarded as a sub-activity (note above that a sub-activity is seen as a fragment and has its state of whether being done). For example, browsing a business report randomly on a train may not be seen as a fragmentation of work. Multi-tasking in these two ways will bring no change on the supernetwork structure but on the attributes of existing links.

Hence, multi-tasking can be represented in the supernetworks as; if the activity state changes while traveling, links of multi-tasking while traveling are added to connect different locations across different activity states; and if more than one (sub)activities’ states change at fixed locations simultaneously, links of multi-tasking are added to connect the same locations across multiple activity state changes.

(4) Over all supernetwork representation

Based on the elements described above, the effects of substitution, fragmentation and multi-tasking can be captured in extended multi-state supernetworks by adding extra activity states and transaction links. The steps for the supernetwork representation of an individual’s daily activity program that integrates transportation, land use and ICT can be described as follows:

Step 1: decompose activities into possible sub-tasks if the ICT counterparts exit, add the locations of ICT access to PTNs, and update PVNs if applicable.

Step 2: assign PTNs and PVNs to all the possible (sub)activity-vehicle states.

Step 3: connect PTN and PVNs with transition links and physical and virtual transaction links of substitution and multi-tasking.

Consider an activity program with two activities (A1 is working and A2 is shopping) and one private vehicle (car), and A1 prior to A2 due to opening time of A1 and A2. Suppose L1 and L2 are selected for A1 and A2 in PTN respectively, and P1 is selected for parking. Thus, the supernetwork representation with car as the departing mode can be depicted as Fig. 6(a). Suppose further that with ICT services, it is allowed to shop at home and work half day at home to avoid the traffic congestion in the morning or afternoon peak. Following the steps mentioned above, A1 is decomposed into two parts (A11 and A12), and A11, A12 and A2 can be substituted and possibly multi-tasked. Therefore, the supernetwork can be represented as Fig. 6(b) (some feasible links are removed for the sake of better illustration).

As shown, the space-time constraints can be relaxed by ICT use and the action space and thus solution space are enlarged considerably. Any path through the overall representation still represents a full activity-travel pattern that potentially involves the short-term effects of ICT.

3.4.2 E-bike

The promotion of fuel-efficient, space-saving and healthier transport modes brings bikes back into a focus of attention. Widespread usage of bikes has the potential to play an important role in addressing many notorious transport problems. However, the travel range of a normal bike is limited because of the physical capability of the bike (speed) and the rider (stamina). A range of factors are contributing to the emergence of E-bikes with a greater travel range and comfort (Rose 2012). The incorporation of E-bike in multi-state...
supernetworks is relatively straightforward (Liao et al. 2012). E-bike can be considered as a private vehicle with acceptance $E_A^+$ and limitation $E_A^-$ distance in-between those of bike and car respectively. Fig. 5 is then updated by Fig. 7, in which $TH/2$ is a potential parking for E-bike other than for a normal bike. With E-bike in an activity program, the vehicle states instead of activity states are expanded.

In most PT-bike systems, $i$ can rent one from a kiosk and must return it to a kiosk (not necessarily same to the previous one) within a period, otherwise there will be some punishment. To embed PT-bike in the multi-state supernetwork representation, the definition of a daily activity program is relaxed as: $i$ at a time leaves home with at most one private vehicle and returns home in the end with all the activities conducted, private vehicles parked at home, and PT-bike returned to a kiosk if any.

Based on the concepts of PVN and PTN, $i$ must be in a PTN when picking-up a PT-bike from a kiosk. When riding a PT-bike, $i$ is in a network different from PVN or PTN. On the one hand, $i$ can use the PT-bike as a private vehicle with the freedom to choose parking locations and routes; on the other hand, it must be returned at a kiosk after use. To capture the choice facet of using a PT-bike, suppose $i$ PPVN is a special case of a PVN where the individual is riding the PT-bike. A transfer link from a PTN to a PPVN denotes picking-up a PT-bike, and a link from a PPVN to a PTN represents returning a PT-bike. The usage of PT-bike for only a fraction of a trip can be expressed as Fig. 8(a), in which $K_1$ & $K_2$ are two bike kiosks and $L_1$ is the location for activity $A_1$. The sub-trip from $K_1$ to $K_2$ can be traveled by a
PT-bike. The situation that \( i \) use PT-bike to access or egress a PT stop belongs to this kind. Moreover, \( i \) can also use it to access destinations for conducting activities; then, \( i \) needs to park the PT-bike. The parking locations can be at the activity locations or elsewhere. Assume that once the PT-bike is parked, \( i \) goes into a PTN of PT-bike-PTTN. Fig. 8(b) shows an example of such, in which \( P_i \) is the parking location for PT-bike.

With the extended definition of activity-vehicle state (PPVN and PPTN), the multi-state super-networks can still be constructed as before by assigning basic networks to all the possible activity-vehicle states and interconnecting them. In the full representation, a path from \( H \) to \( H' \) still represents a feasible and consistent activity-travel pattern possibly containing the use of a PT-bike.

### 3.5 Joint travel and activity participation

Individuals undertake both independent and joint travel as a part of their daily activity-travel patterns. Travel surveys indicate that a significant portion (around 50%) of travel is implemented by joint travel (Vovsha et al. 2003). For example, individuals meet with other people at transport hubs or landmarks to travel jointly for business or leisure activities. In principle, organizing household travel is not fundamentally different. Accordingly, there is a growing interest in transportation research in studying inter-personal inter-dependencies in joint activity-travel patterns. However, in practice, modeling joint activity-travel decisions often turns out to be problematic due to the lack of data and model limitations. For one reason, there is always the involvement of higher choice dimensions than individual patterns; moreover, explicit representations of the joint patterns with other choice facets are needed (Carrasco et al. 2008). To implement joint activity-travel, individuals are subject to the coupling constraints, which define when and where individuals can join other individuals. This spatial and temporal coordination is also referred to as synchronization. Without synchronizing different individuals' joint travel patterns, inconsistent choices of mode and route tend to be produced.

Liao et al. (2013b) proposed a multi-state super-network framework to model the two-person joint travel problem (JTP), which was to find the optimal joint activity-travel pattern for two individuals. This process can be represented by introducing joint state, which defines the composition of the sub-group. Two types of transfer links are further introduced.

1. Meeting link: connecting the same nodes from networks of different joint states with more individuals involved in the end point.
2. Departing link: connecting the same nodes from networks of different joint states with fewer individuals involved in the end points.

Thus, a joint activity-travel trip starts with a meeting link and ends with a departing link. Fig. 9 shows an example of meeting links denoting that individual \( i \) and \( j \) may meet at \( A_1 \) and \( A_2 \), and then jointly travel to and conduct joint activity at \( B_1 \) and \( B_2 \). Meeting and activity location choices are consistently represented. A similar representation can be drawn for departing links. However, to capture the choice of where the joint group is split, individuals' networks are copied as many
times as the number of possible departing points. Unlike the meeting links with all meeting points that can converge to the same joint network, gathering departing links with different departing points in the same networks will cause inconsistent joint activity-travel patterns. Although only one episode of joint activity and the related joint travel routing algorithms were discussed in Liao et al. (2013b), the same logic can be extended for multiple episodes of joint activities.

4 Future prospects

In this article, the overview of the development of supernetworks application in the area of transportation research and, specifically, focus on some recent extensions of multi-state supernetworks are introduced. The main advantage of these multi-state supernetworks is the integrated representation of activities and travel, implying that well-known network algorithms can be applied to model scheduling decisions. The potential power of this representation has been shown, and recent progress have made these supernetworks appropriate for addressing time-dependent and constrained scheduling models. Hence, the multi-state supernetwork representation is not only a powerful candidate for activity-travel scheduling problems, but may also be of interest in the context of time geographic approaches that have received continuous attention (Neutens, et al., 2008; Kang and Scott, 2008; Yin, et al., 2011; Farber, et al., 2014). Despite the potential, at least two fundamental problems require attention in future research.

First, multi-state supernetworks need to be expanded to account for macro-micro relationships. The supernetwork models are primarily models of individual or two-person scheduling behavior. The accumulated choices of all travellers are manifested in aggregate traffic conditions on different links of a transportation network. These conditions may result in travel times that are inconsistent with the travel times that were used to optimally schedule the activities. Thus, future work should develop mechanisms that represent the feedback of macro states on individual scheduling behavior (user equilibrium) (Ramadurai and Ukkusuri, 2010). Of special importance is the question how individuals update their beliefs of travel times (and other features of the system) to understand learning and dynamics in activity-travel scheduling behavior.

Second, all current multi-state supernetworks involve deterministic representations of the transportation and the urban system, and are based on the behavioral principle of minimizing generalized costs or maximizing utility under conditions of full information. In particular in the context of short-term transportation management issues, these properties are too rigorous and highly limiting. Travel times show a substantial degree of inherent variability and are hard to predict by the traveler. Thus, future work should develop multi-state supernetwork representations that account for limited information and decision making under uncertainty in complex urban transportation systems. A consequence of the limited information is that individuals make decisions under uncertainty, questioning the validity of the currently adopted choice mechanisms (link costs functions and path finding algorithm). Hence, future research should also examine the effects of replacing the principle of minimizing generalized costs with more valid principles of choice and decision making under conditions of uncertainty.

References


