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Information Fusion Identification Method for the Multi-dimension ARMA Signal with Sensor Bias and Common Disturbance Noise

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Abstract

For the multisensor multi-dimension autoregressive moving average(ARMA) signal system with a common disturbance measurement noise and sensor bias, when the model parameters, sensor bias and noise variances are all unknown, their consistent estimates are obtained by the multistage information fusion identification method. Firstly, by multi-dimension recursive extended least squares (RELS) algorithm, the estimates of the autoregressive parameters and sensor bias are obtained. Secondly, applying the correlation method, the estimates of the measurement noise variances are obtained. Finally, the fused estimates of the moving average(MA) parameters and the process noise variances are obtained by the Gevers-Wouters algorithm with a dead band. A simulation example verifies the consistency of unknown parameters estimates.

Keywords: Inforamtion fusion; Multi-dimension ARMA signal; Sensor bias; Common disturbance noise; Model parameter identification

1. Introduction

The estimate problem of the multi-channel autoregressive moving average (ARMA) signal has received increasing attention. The classical system identification methods for the signal system have the limitations that they are only suitable the system without measurement noise or with only a single sensor

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having measurement noise. The multisensor information fusion identification is a new field, whose aim is to combine or to weight the local estimators or local measurements from each sensor, and to obtain the fused estimators whose performance is better than that of the local estimators. Especially, in order to improve the robustness and reliability of the fused estimation, we take the average of local estimates as the fused estimate. The presented method[1] solved the estimation problem of unknown parameters for the single-channel ARMA signal, applying the RELS algorithm and the correlation method. For the single channel ARMA signal with a common disturbance noise, the paper[2] gave the estimation result by the RIV algorithm, the correlation method and the Gever-Wouters algorithm, when the model parameters and these noise variance are unknown.

The multi-dimension and the multiple identification method[3] for the multi-channel signal is presented. But it only suit for ARMA signal with white noise. When the measurement of the signal has been affected by a common disturbance noise, it is that the measurement noises are correlated. The reference [4] presented the multi-stage identification method to estimate unknown ARMA model parameters and the measurement noise variances. However, when measurements have a sensor bias, the RIV algorithm used in [3] can't estimate the unknown sensor bias. Recently, the reference [5] presented a multistage identification method for multisensor single channel ARMA signal with a common disturbance white noise and sensor bias. Therefore, this paper extends the results in [5] to the multisensor multichannel case.

2. System model

Consider the multisensor multi-dimension ARMA signal with common disturbance noises and sensor bias

$$A(q^{-1})s(t)=C(q^{-1})w(t)\,, \tag{1}$$

$$y_j(t)=s(t)+b+v_j(t),\,, \tag{2}$$

$$v_j(t)=\xi(t)+\eta_j(t),\, j=1,\cdots,L, \tag{3}$$

where t is the discrete time, $s(t)\in R^m$ is the signal, $w(t)\in R^m$ is the process noise, $y_j(t)$ and $v_j(t)$ are the measurement and measurement noise of the j th sensor subsystem, b is the sensor bias, but $v_j(t)$ contain a common disturbance noise $\xi(t)\in R^m$, q^{-1} is the backward shift operator with $q^{-1}s(t)=s(t-1)$, $A(q^{-1})$ and $C(q^{-1})$ are the polynomial matrices having the form as $X(q^{-1})=I_m+X_1q^{-1}+\cdots+X_{n_x}q^{-n_x}$, I_m is the $m\times m$ identity matrix, n_x is the order, X_k is the $m\times m$ coefficient matrix.

Assumption 1. $w(t)$, $\xi(t)$ and $\eta_j(t)$ are uncorrelated white noises with zero means and variances Q_w, Q_ξ and $Q_{\eta j}$.

Assumption 2. $(A(q^{-1}),C(q^{-1}))$ is left coprime, and $A(q^{-1})$ and $C(q^{-1})$ are stable.

Assumption 3. The sensor bias b , noise variances Q_w, Q_ξ and Q_{η_j} , and coefficient matrices in $A(q^{-1})$ and $C(q^{-1})$ are unknown, but the order n_a and n_c are known with $n_a>n_c$.

Assumption 4. The measurement data $y_j(t)(j=1,\cdots,L)$ are bounded for t with probability one.

3. Information Fusion Estimates of Unknown Model Parameters and Noise Variances

When the ARMA signal model parameters, the sensor bias and noise variances are unknown, we need to estimate them and to obtain their consistent estimates using the multistage identification method.

3.1. Fused estimates of model parameters for $A(q^{-1}), b$

Substituting (1) and (3) into (2) yields

$$A(q^{-1})y_j(t) = C(q^{-1})w(t) + A(q^{-1})b + A(q^{-1})(\xi(t) + \eta_j(t)). \tag{4}$$

Setting

$$D_j(q^{-1})\varepsilon_j(t) = C(q^{-1})w(t) + A(q^{-1})(\xi(t) + \eta_j(t)), \tag{5}$$

$$\rho = A(q^{-1})b = (I_m + A_1 + \cdots + A_{n_a})b\,, \tag{6}$$

we have the innovation model

$$A(q^{-1})y_j(t) = \rho + D_j(q^{-1})\varepsilon_j(t)\ . \tag{7}$$

Hence for the jth sensor, we have the corresponding least squares (LS) structure of (7) as

$$y_j(t) = \Theta_j\varphi_j(t) + \varepsilon_j(t), j = 1, \cdots, L, \tag{8}$$

$$\varphi_j(t) = [-y_j^{\mathrm{T}}(t-1), \cdots, -y_j^{\mathrm{T}}(t-n_a), 1, \varepsilon_j^{\mathrm{T}}(t-1), \cdots, \varepsilon_j^{\mathrm{T}}(t-n_d)]^{\mathrm{T}}, \tag{9}$$

$$\Theta_j = [A_1, \cdots, A_{n_a}, \rho, D_{j1}, \cdots, D_{jn_d}] \in R^{m\times(mn_a+1+mn_d)}. \tag{10}$$

Theorem 1. For the jth sensor subsystem with multi-dimension stationary ARMAX model(7), the multi-dimension recursive extended least squares (RELS) estimate $\hat{\Theta}_j(t)$ of Θ is

$$\hat{\Theta}_j(t) = \hat{\Theta}_j(t-1) + \frac{[y_j(t) - \hat{\Theta}_j(t-1)\varphi_j(t)]\varphi_j^{\mathrm{T}}(t)P_j(t-1)}{1+\varphi_j^{\mathrm{T}}(t)P_j(t-1)\varphi_j(t)}, \tag{11}$$

$$P_j(t) = P_j(t-1) - \frac{P_j(t-1)\varphi_j(t)\varphi_j^{\mathrm{T}}(t)P_j(t-1)}{1+\varphi_j^{\mathrm{T}}(t)P_j(t-1)\varphi_j(t)}, \tag{12}$$

with initial value $\hat{\Theta}_j(t_0) = \Theta_{j0}, P_j(t_0) = P_0, y_j(t) = 0 (t \le 0)$,where

$$\hat{\Theta}_j(t) = [\hat{A}_{1j}(t), \cdots, \hat{A}_{n_a j}(t), \hat{\rho}_j(t), \hat{D}_{j1}(t), \cdots, \hat{D}_{jn_d}(t)]\,, \tag{13}$$

$$\hat{\varphi}_j(t) = [-y_j^{\mathrm{T}}(t-1), \cdots, -y_j^{\mathrm{T}}(t-n_a), 1, \hat{\varepsilon}_j(t-1), \cdots, \hat{\varepsilon}_j(t-n_d)]^{\mathrm{T}}\ , \tag{14}$$

and $\hat{\varepsilon}_j(k)$ in $\hat{\varphi}_j(t)$ can be obtained by

$$\hat{\varepsilon}_j(k) = y_j(k) - \hat{\Theta}_j(k)\hat{\varphi}_j(k)(k = t-1, t-2, \cdots, t-n_d)\ . \tag{15}$$

And if $D_j(q^{-1})$ satisfies the positive real condition[6], then the multi-dimension RELS estimate $\hat{\Theta}_j(t)$ for different sensor converges to the true value Θ with probability 1, as

$$\hat{\Theta}_j(t) \to \Theta, j = 1, \cdots, L, \text{as} \quad t \to \infty, w.p.1. \tag{16}$$

Theorem 2. Information fusion estimates $\hat{A}_{kf}(t)$ of A_k and $\hat{b}_f(t)$ of b for the multi-dimension stationary ARAX model (1)—(3) are consistent, i.e.

$$\hat{A}_{kf}(t) \to A_k, \hat{b}_f(t) \to b, \quad k = 1, \cdots, n_a, \text{as} \quad t \to \infty, w.p.1. \tag{17}$$

Proof. From Theorem 1, we have local parameter estimates $\hat{A}_{kj}(t), \hat{\rho}_j(t)$ based on the jth sensor. The fused estimates $\hat{A}_{kf}(t)$ and $\hat{\rho}_f(t)$ are defined as

$$\hat{A}_{kf}(t) = \frac{1}{L}\sum_{j=1}^{L}\hat{A}_{kj}(t), \quad \hat{\rho}_f(t) = \frac{1}{L}\sum_{j=1}^{L}\hat{\rho}_j(t)\,. \tag{18}$$

From the consistency of $\hat{A}_{kj}(t), \hat{\rho}_j(t)$, we can easily obtain the consistency of $\hat{A}_{kf}(t), \hat{\rho}_f(t)$. From (6), we define the fused estimate of b as

$$\hat{b}_f(t) = (I_m + \hat{A}_{1f}(t) + \cdots + \hat{A}_{n_a f}(t))^{-1}\hat{\rho}_f(t). \tag{19}$$

Hence, from the consistency of $\hat{A}_{kf}(t), \hat{\rho}_f(t)$, we have that (17) holds. The proof is completed.

3.2. Fused estimates of noise variances Q_ξ and $Q_{\eta j}$

Defining

$$z_j(t) = A(q^{-1})y_j(t) - \rho, \tag{20}$$

we have

$$z_j(t) = C(q^{-1})w(t) + A(q^{-1})(\xi(t) + \eta_j(t)). \tag{21}$$

Defining the correlation function $R_{zij}(k) = \mathrm{E}[z_i(t)z_j^{\mathrm{T}}(t-k)], k = 0,1,\cdots,n_a$ of $z_i(t)$ and $z_j(t)(i,j = 1,\cdots,L)$, and computing the correlation function for $k = n_c+1,\cdots,n_a$ yields

$$R_{zij}(k) = \begin{cases} \sum_{u=k}^{n_a} A_u Q_\xi A_{u-k}^{\mathrm{T}}, & i \neq j, \\ \sum_{u=k}^{n_a} A_u [Q_\xi + Q_{\eta_j}] A_{u-k}^{\mathrm{T}}, & i = j. \end{cases} \tag{22}$$

Defining the sampled correlation function $R_{zij}^t(k)$ of $R_{zij}(k)$ at time t as

$$R_{zij}^t(k) = \frac{1}{t}\sum_{u=1}^{t} z_i(u) z_j^{\mathrm{T}}(u-k), \tag{23}$$

from the ergodicity[6] of stationary stochastic process, we have

$$R_{zij}^t(k) \to R_{zij}(k), \quad \text{as} \quad t \to \infty, w.p.1. \tag{24}$$

When noise variances Q_w, Q_ξ and $Q_{\eta j}$ are unknown, substituting these estimates $\hat{A}_{kf}$, $\hat{\rho}_f(t)$ into(20), we have $\hat{z}_j(t)$. And the sampled correlation function estimate $\hat{R}_{zij}^t(k)$ is

$$\hat{R}_{zij}^t(k) = \frac{1}{t}\sum_{u=1}^{t} \hat{z}_i(u)\hat{z}_j^{\mathrm{T}}(u-k). \tag{25}$$

From (23)-(25) and Assumption 4, we can prove

$$\hat{R}_{zij}^t(k) \to R_{zij}(k), \quad \text{as} \quad t \to \infty, w.p.1. \quad . \tag{26}$$

For different $k(k = n_c+1,\cdots,n_a)$, substituting $\hat{A}_{kf}(t), \hat{R}_{zij}^t(k)$ into the first equality of (22) yields local estimates $\hat{Q}_{\xi ijk}(t)$. Then, substituting $\hat{Q}_{\xi ijk}(t)$, $\hat{A}_{kf}(t)$ and $\hat{R}_{zij}^t(k)$ into the second equality of (22) yields the local estimates $\hat{Q}_{\eta jk}(t)$ [7] .

Taking the average value of local estimates $\hat{Q}_{\xi ijk}(t)$, the fused estimate $\hat{Q}_\xi(t)$ of Q_ξ is defined as

$$\hat{Q}_\xi(t) = \frac{2}{L(L-1)(n_a - n_c)}\sum_{i=1}^{L-1}\sum_{j=i+1}^{L}\sum_{k=n_c+1}^{n_a}\hat{Q}_{\xi ijk}(t), \tag{27}$$

and taking the average value of local estimates $\hat{Q}_{\eta jk}(t)$, fused estimates $\hat{Q}_{\eta j}(t)$ of $Q_{\eta j}$ are defined as

$$\hat{Q}_{\eta j}(t)=\frac{1}{n_a-n_c}\sum_{k=n_c+1}^{n_a}\hat{Q}_{\eta jk}(t). \tag{28}$$

3.3. Fused estimates of model parameters C_k and noise variance Q_w

Defining

$$m(t)=C(q^{-1})w(t), \tag{29}$$

and $m(t)$ is a stationary stochastic process, whose correlation function $R_m(k)$ is defined as

$$R_m(k)=\mathrm{E}[m(t)m^{\mathrm{T}}(t-k)],\ k=0,1,\cdots,n_c. \tag{30}$$

which is

$$R_m(k)=\sum_{j=k}^{n_c}C_jQ_wC_{j-k}^{\mathrm{T}},\ k=0,\cdots n_c \tag{31}$$

From (21), we have

$$z_j(t)=m(t)+A(q^{-1})(\xi(t)+\eta_j(t)). \tag{32}$$

Computing the correlation function of two sides of (32), and for different $k(k=0,\cdots,n_c)$, substituting $\hat{R}_{zij}^t(k)$, $\hat{A}_{kf}(t)$, $\hat{Q}_\xi(t)$ and $\hat{Q}_{\eta_j}(t)$ into it, we can obtain the correlation function estimate of $m(t)$

$$\hat{R}_{mij}^t(k)=\hat{R}_{zij}^t(k)-\sum_{u=k}^{n_a}\hat{A}_{uf}(t)[\hat{Q}_\xi(t)+\hat{Q}_{\eta_j}(t)\delta_{ij}]\hat{A}_{u-k,f}^{\mathrm{T}}(t),k=0,\cdots,n_a. \tag{33}$$

For (31), based on $\hat{R}_{mij}^t(k)$, applying the Gevers-Wouters algorithm with a dead band[8], we can obtain local estimates $\hat{C}_{kij}(t)(k=1,\cdots,n_c,i,j=1,\cdots,L)$ and $\hat{Q}_{wij}(t)$.

Taking the average value of these local estimates $\hat{C}_{kij}(t)$ and $\hat{Q}_{wij}(t)$, fused estimates $\hat{C}_{kf}(t)$ of C_k , and $\hat{Q}_w(t)$ of Q_w are defined by

$$\hat{C}_{kf}(t)=\frac{1}{L^2}\sum_{i=1}^{L}\sum_{j=1}^{L}\hat{C}_{kij}(t),\quad \hat{Q}_w(t)=\frac{1}{L^2}\sum_{i=1}^{L}\sum_{j=1}^{L}\hat{Q}_{wij}(t). \tag{34}$$

Theorem 3. For the multisensor multi-dimension ARMA signal system (1)--(3) with Assumptions 1--3, local and fused white noise estimates and MA parameters are consistent, i.e.

$$\hat{Q}_\xi(t)\to Q_\xi,\ \hat{Q}_{\eta j}(t)\to Q_{\eta j},\hat{Q}_w(t)\to Q_w,\hat{C}_{kf}(t)\to C_k,\ j=1,\cdots,L,\ k=1,\cdots,n_c,\ \text{as}\ t\to\infty, w.p.1. \tag{35}$$

Proof. Form the first equality of (22), for fixed *i*,*j*,*k*, the each element of Q_ξ is a continuous of the element of $R_{zij}(k),A_1,\cdots A_{n_a}$, i.e. $Q_\xi=f_{i,j,k}(R_{zij}(k),A_1,\cdots A_{n_a})$. Substituting $\hat{R}_{zij}^t(k)$ and $\hat{A}_{kf}(t)$ into it yields the local estimators

$$\hat{Q}_{\xi ijk}(t)=f_{i,j,k}(\hat{R}_{zij}^t(k),\hat{A}_{1f},\cdots\hat{A}_{n_af}) \tag{36}$$

From (17) and (26), and the continuity of $f_{i,j,k}$, we have $\hat{Q}_{\xi ijk}(t)\to Q_\xi$. Then $\hat{Q}_\xi(t)\to Q_\xi$ holds. Similarly, we have $\hat{Q}_{\eta jk}(t)\to Q_{\eta j}$.

Similarly, each element of Q_w and $C_l(l=1,\cdots n_c)$ is a continuous of the element of $R_m(k),k=0,1,\cdots n_c$, i.e.

$$Q_w=f_w(R_m(0),\cdots R_m(n_c)),\quad C_k=f_k(R_m(0),\cdots R_m(n_c)) \tag{37}$$

$$\hat{Q}_w(t)=f_w(\hat{R}_{mij}^t(0),\cdots\hat{R}_{mij}^t(n_c)),\quad \hat{C}_{kij}(t)=f_k(\hat{R}_{mij}^t(0),\cdots\hat{R}_{mij}^t(n_c)) \tag{38}$$

From (33), we have $\hat{R}^t_{mij}(k) \to R_m(k)$. According to the continuity of f_w and f_k, we have $\hat{Q}_{wijk} \to Q_w, \hat{C}_{kij} \to C_k$. Then, from (34) we have $\hat{Q}_w \to Q_w, \hat{C}_l \to C_l$. The proof is completed.

4. Simulation Example

Consider the multi-dimension ARMA(2,1) signal $s(t)$ as (1)--(3) with 3-sensors, and their parameters $A_1, A_2, C_1, b, Q_w, Q_\xi, Q_{\eta j}$ are all unknown. The problem is to obtain their consistent estimates of these unknown parameters. In the simulation we take

$$A_1 = \begin{bmatrix} 0.8 & 0.3 \\ -0.5 & -0.4 \end{bmatrix}, A_2 = \begin{bmatrix} 0.6 & -0.3 \\ 0.2 & 0.9 \end{bmatrix}, C_1 = \begin{bmatrix} 0.2 & -0.1 \\ 0.5 & -0.4 \end{bmatrix}, b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix},$$

$$Q_w = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}, Q_\xi = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}, Q_{\eta 1} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, Q_{\eta 2} = \begin{bmatrix} 5 & 0 \\ 0 & 3 \end{bmatrix}, Q_{\eta 3} = \begin{bmatrix} 4 & 0 \\ 0 & 7 \end{bmatrix}.$$

Applying the multi-dimension RELS algorithm as Theorem 1 and Theorem 2, the fusion estimates of the AR parameters and the sensor bias are obtained, where curves of information fusion estimates $\hat{A}_{1f}(t)$ and $\hat{b}_f(t)$ are shown in Fig 1-Fig 2. Applying the correlation method, the fused estimates of Q_ξ and $Q_{\eta j}$ are obtained, where curves of $\hat{Q}_\xi(t)$ $\hat{Q}_{\eta 3}(t)$ are shown in Fig 3 and Fig 4. The curves of the information fusion estimates $\hat{C}_{1f}(t)$, $\hat{Q}_w(t)$ are shown in Fig 5--Fig 6, by the Gevers-Wouters algorithm with a dead band $T_d = 200$. Fig 1—Fig 6 verify the consistency of parameters and noise variances estimates, where $M(k,r)$ denotes the (k,r)th element of matrix M, straight lines denote the true values, solid curves denote fused estimates, and dot curves denote local estimates.

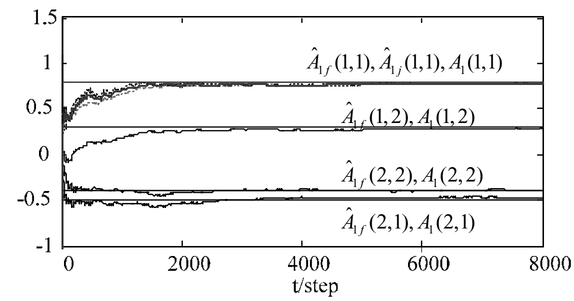

Fig 1. Curves of local and fused estimates of A_1

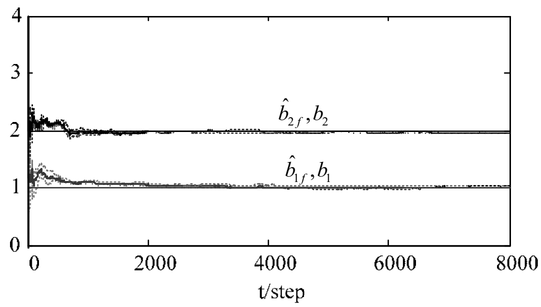

Fig 2. Curves of local and fused estimates of b

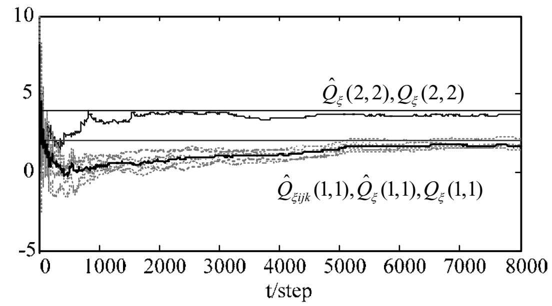

Fig 3. Curves of local and fused noise variances estimates of Q_ξ

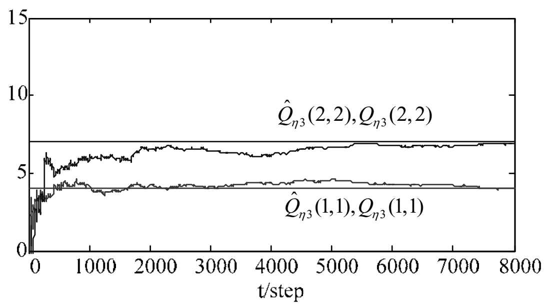

Fig 4. Curves of fused noise variances estimates of $Q_{\eta 3}$

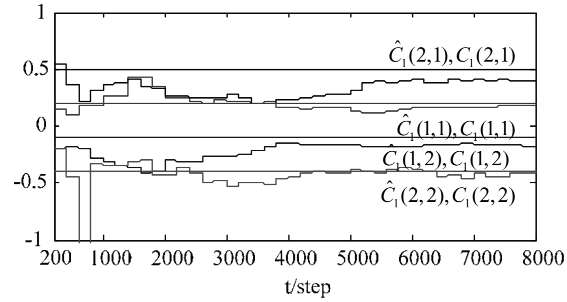

Fig 5. Curves of fused estimates of MA parameter C_1

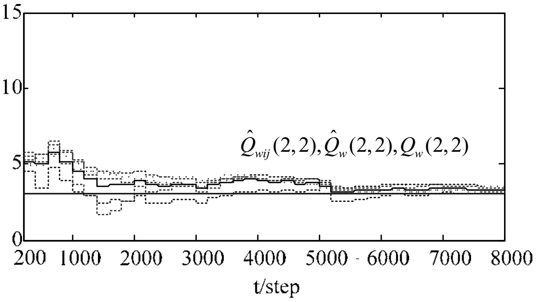

Fig 6. Curves of local and fused estimates of Q_w *(2,2)*

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