

Available online at www.sciencedirect.com



Physics



Physics Procedia 58 (2014) 26 - 29

# 26th International Symposium on Superconductivity, ISS 2013

# Material-parameter dependence of superconductivity in hightemperature cuprates

<sup>a</sup>\*Takashi Yanagisawa, <sup>b</sup>Mitake Miyazaki, <sup>a</sup>Kunihiko Yamaji

<sup>a</sup>Electronics and Photonics Research Institute, National Institute of Advanced Industrial Science and Technology (AIST), Tsukuba Central 2, 1-1-1 Umezono, Tsukuba, Ibaraki 305-8568, Japan

<sup>b</sup>Hakodate National College of Technology, 14-1 Tokura-cho, Hakodate, Hokkaido 042-8501, Japan

### Abstract

We show that there is an interesting correlation between material parameters and critical temperature  $T_c$  in cuprate high temperature superconductors. Our analysis is based on the d-p model, that is, the three-band Hubbard model including d and p orbitals explicitly. This model contains many parameters; the transfer integrals  $t_{dp}$  and  $t_{pp}$ , the energy levels  $\varepsilon_p$  and  $\varepsilon_d$ , and the Coulomb interaction parameters  $U_d$  and  $U_p$ . Our main results are the following: (a)  $T_c$  increases as  $\varepsilon_p - \varepsilon_d$  is increased for  $U_p = 0$ , (2)  $T_c$  is lowered with increase of  $U_p$  when  $\varepsilon_p - \varepsilon_d > 0$ , (3)  $T_c$  is increased with increase of  $U_p$  when  $\varepsilon_p - \varepsilon_d > 0$ , (4)  $T_c$  has a minimum at near  $\varepsilon_p - \varepsilon_d = 0$  as a function of  $\varepsilon_p - \varepsilon_d$  when  $U_d$  and  $U_p$  are comparable, (5)  $U_d$  induces  $d_{x2-y2}$  pairing while  $U_p$  induces  $d_{xy}$  pairing, (6)  $T_c$  has a peak as a function of  $t_{pp}$ . The results imply that  $T_c$  will increase if we can suppress  $U_p$ . The role of  $U_p$  is consistent with the experimental tendency that  $T_c$  increases as the relative ratio of the hole density at oxygen site to that at copper site is increased, which means that when  $U_p$  increases, the number of p holes is decreased and  $T_c$  is also decreased.

© 2014 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/3.0/). Peer-review under responsibility of the ISS 2013 Program Committee

Keywords: high-temperature superconductivity; band parameter; interaction parameter; gap equation; material dependence

# 1. Introduction

The study of high-temperature superconductivity has been addressed extensively since the discovery of cuprate superconductors [1]. It is primarily important to clarify electronic states in the  $CuO_2$  plane [2-5]. Electron

\* Corresponding author. Tel.: +81-29-861-5374; fax: +81-29-861-5569. *E-mail address:* t-yanagisawa@aist.go.jp correlation is important in cuprates [6-13] and relationship between material parameters and critical temperature  $T_c$  is also significant in the study of cuprate high temperature superconductors. There are two kinds of material parameters. The first one is related to the band structure and the Fermi surface; they are transfer integrals  $t_{dp}$ ,  $t_{pp}$  and the levels of d and p electrons. The  $t_{dp}$  is the transfer integral between nearest d and p orbitals in the CuO<sub>2</sub> plane, and  $t_{pp}$  is that between nearest p orbitals. The other is concerning with the strength of interactions such as the Coulomb interactions,  $U_d$  and  $U_p$ , and the electron-phonon interaction. In our previous studies, the transfer integrals play an important role to stabilize superconductivity and striped states [7-13] as well as to obtain a finite bulk limit of the superconducting condensation energy [14,15].

There are experimentally known correlation between material parameters and  $T_c$ . For example,  $T_c$  increases as the relative ratio of hole density at O site against that at Cu site is increased [16]. This indicates that doping hole carrier at O site is more favorable for higher  $T_c$ . The hole carrier density is determined by the relative level difference  $\varepsilon_p$ - $\varepsilon_d$  and Coulomb interactions  $U_d$  and  $U_p$ . Thus,  $T_c$  would crucially depend on these parameters.

The purpose of this paper is to show relationships between  $T_c$  and material parameters on the basis of the d-p model in the CuO<sub>2</sub> plane. We investigate the gap equation that is derived for the effective interaction in terms of  $U_d$  and  $U_p$ . We use a weak-coupling formulation to solve the gap equation and show correlation between  $T_c$  and material parameters.

# 2. Hamiltonian and gap equation

The model is the Hamiltonian that contains d and p electrons [7,8,17]:

$$\begin{split} H &= \varepsilon_{d} \sum_{i\sigma} d_{i\sigma}^{+} d_{i\sigma} + \varepsilon_{p} \sum_{i\sigma} \left( p_{i+\bar{x}/2,\sigma}^{+} p_{i+\bar{x}/2,\sigma} + p_{i+\bar{y}/2,\sigma}^{+} p_{i+\bar{y}/2,\sigma} \right) \\ &+ t_{dp} \sum_{i\sigma} \left[ d_{i\sigma}^{+} \left( p_{i+\bar{x}/2,\sigma} + p_{i+\bar{y}/2,\sigma} - p_{i-\bar{x}/2,\sigma} - p_{i-\bar{y}/2,\sigma} \right) + h.c. \right] \\ &+ t_{pp} \sum_{i\sigma} \left( p_{i+\bar{y}/2,\sigma}^{+} p_{i+\bar{x}/2,\sigma} - p_{i+\bar{y}/2,\sigma}^{+} p_{i-\bar{x}/2,\sigma} - p_{i-\bar{y}/2,\sigma}^{+} p_{i+\bar{x}/2,\sigma} + p_{i-\bar{y}/2,\sigma}^{+} p_{i-\bar{x}/2,\sigma} + h.c. \right)$$
(1)  
$$&+ U_{d} \sum_{i} n_{i\uparrow}^{d} n_{i\downarrow}^{d} + U_{p} \sum_{i} \left( n_{i+\bar{x}/2\uparrow}^{p} n_{i+\bar{x}/2\downarrow}^{p} + n_{i+\bar{y}/2\uparrow}^{p} n_{i+\bar{y}/2\downarrow}^{p} \right) \end{split}$$

where  $n_{i\sigma}^d$  and  $n_{i+\hat{\mu}/2\sigma}^p$  ( $\mu=x, y$ ) are number operators for d and p electrons, respectively.  $U_d$  and  $U_p$  indicate the Coulomb interaction for d and p electrons, respectively. We examine the doped case within the hole picture where the lowest band is occupied up to the Fermi energy  $\mu$ . The non-interacting part is written as

$$H_{0} = \sum_{k\sigma} \left( d_{k\sigma}^{+} p_{xk\sigma}^{+} p_{yk\sigma}^{+} \right) \begin{pmatrix} \varepsilon_{d} - \mu & \varepsilon_{xk} & \varepsilon_{yk} \\ -\varepsilon_{xk} & \varepsilon_{p} - \mu & \varepsilon_{pk} \\ -\varepsilon_{yk} & \varepsilon_{pk} & \varepsilon_{p} - \mu \end{pmatrix} \begin{pmatrix} d_{k\sigma} \\ p_{xk\sigma} \\ p_{yk\sigma} \end{pmatrix},$$
(2)

where  $\varepsilon_{xk} = 2it_{dp}\sin(k_x/2)$ ,  $\varepsilon_{yk} = 2it_{dp}\sin(k_y/2)$  and  $\varepsilon_{pk} = -4t_{pp}\sin(k_x/2)\sin(k_y/2)$ .  $p_{\mu k\sigma}$  and  $d_{k\sigma}$  are Fourier transforms of  $p_{i+\mu/2,\sigma}$  and  $d_{i\sigma}$ , respectively. The eigenvectors of this matrix give the corresponding weights of d and p electrons.

The gap equation was derived by means of the perturbation theory in terms of  $U_d$  [18,19,20]. The inclusion of  $U_p$  is recently achieved to give the effective interaction [21]:

$$V_{kk'} = \frac{1}{N} \sum_{p\alpha\beta} \frac{f_{k+k'+p}^{\alpha} - f_{p}^{\beta}}{\varepsilon_{\beta}(p) - \varepsilon_{\alpha}(k+k'+p)} \left| \sum_{i=d, p_{x}, p_{y}} U_{i} z_{i}^{\alpha}(p) z_{i}^{\beta}(k+k'+p) z_{i}^{0}(k) z_{i}^{0}(k') \right|^{2},$$
(3)

where  $\varepsilon_{\alpha}(\mathbf{k})$  is the dispersion relation of the  $\alpha$ -th band ( $\alpha = 0, 1, 2$ ) and  $f_{\mathbf{k}}^{\alpha}$  is the Fermi distribution function. We adopt that  $\alpha = 0$  indicates the lowest band which gives a dominant contribution. The subscript *i* mean *d*,  $p_x$  and  $p_y$  components where we set  $U_p = U_{px} = U_{py}$ .  $z^{\alpha} \equiv (z_d^{\alpha}, z_{px}^{\alpha}, z_{py}^{\alpha})$  indicates the eigenvector of the non-interacting Hamiltonian matrix shown above. Our gap equation is

$$\Delta_{k} = -\frac{1}{N} \sum_{k'} V_{kk'} \frac{\Delta_{k'}}{2E_{k'}},\tag{4}$$

where  $\Delta_k$  is the gap function and  $E_k = \sqrt{\xi_k^2 + \Delta_k^2}$  for  $\xi_k = \varepsilon_k^0 - \mu$ .

#### 3. Results and Discussion

The magnitude of the gap function is obtained as  $\Delta = \exp\left(-2t_{dp}^2 / xU_d^2\right)$  in the weak-coupling formulation [22, 23] where we take  $t_{dp}$  as an energy unit. The exponent x indicates the strength of superconductivity. We first show the result for  $U_p = 0$ . With increase of  $\Delta_{dp} = \varepsilon_p - \varepsilon_d$ , the exponent x, namely  $T_c$ , increases quickly as seen in Fig.1 where x is shown as a function of  $\Delta_{dp}$  using  $t_{pp} = 0$  and  $U_p = 0$  at the hole carrier density  $n_h = 0.13$ .



Fig.1. x as a function of  $\Delta_{dp}$  with  $t_{pp} = 0$  and  $U_p = 0$ . Fig.2. x as a function of  $t_{pp}$  with  $\Delta_{dp} = 3$  and  $U_p = 0$ .

The gap functions are specified by one of irreducible representations of the square lattice. We have five irreducible representations  $A_1$ ,  $A_2$ ,  $B_1$ ,  $B_2$  and E [22]. The  $d_{x2-y2}$  symmetry is in  $B_1$  and  $d_{xy}$  symmetry is in  $B_2$ . The Fig.1 implies that  $d_{x2-y2}$  pairing symmetry is realized for all the  $\Delta_{dp}$  when  $U_p = 0$ . The Fig.2 shows x as a function of  $t_{pp}$  where a sharp peak indicates a peak of the density of states due to the van Hove singularity.

Next we examine the effect of  $U_p$ . The Fig.3 shows  $x vs U_p/U_d$  for  $\Delta_{dp} = \varepsilon_p - \varepsilon_d = 3$  and  $t_{pp} = 0$  at the doped hole density  $n_h = 0.13$ . This obviously indicates that  $T_c$  decreases as  $U_p$  is increased, that is,  $U_p$  suppresses  $T_c$ . This is because the pairing symmetries induced by  $U_d$  and  $U_p$  are different from each other. This means that  $T_c$  will increase if we can suppress  $U_p$ . Hence, when  $U_p > 0$ ,  $U_p$  induces  $d_{xy}$ -symmetry paired state. In fact, when  $U_p = U_d$ ,  $d_{x^2-y^2}$ pairing state changes into  $d_{xy}$  state as  $\Delta_{dp}$  is reduced from positive to negative values as shown in Fig.4. Here we mention that in this figure, x of  $A_1$  pairing state is very close to that of  $d_{xy}$ . We also found that  $T_c$  is increased with increase of  $U_p$  when  $\varepsilon_p - \varepsilon_d < 0$ . Lastly, we note that the role of  $U_p$  is consistent with the experiment concerning relationship between  $T_c$  and the relative ratio of hole density at O site to that at Cu site [16, 21]. This is because we have lower  $T_c$  and lower p-hole density when  $U_p$  grows large.



Fig.3. x as a function of  $U_p/U_d$  for  $\Delta_{dp} = 3$  and  $t_{pp} = 0$ . Fig.4. x as a function of  $\Delta_{dp}$  for  $U_p = U_d$  and  $t_{pp} = 0$ .

As a summary, we have investigated material-parameter dependence of  $T_c$ . We summarize as follows: (a)  $T_c$  increases as  $\varepsilon_p - \varepsilon_d$  is increased for  $U_p = 0$ , (2)  $T_c$  is lowered with increase of  $U_p$  when  $\varepsilon_p - \varepsilon_d > 0$ , (3)  $T_c$  is increased with increase of  $U_p$  when  $\varepsilon_p - \varepsilon_d < 0$ , (4)  $T_c$  has a minimum at near  $\varepsilon_p - \varepsilon_d = 0$  as a function of  $\varepsilon_p - \varepsilon_d$  when  $U_d$  and  $U_p$  are comparable, (5)  $U_d$  induces  $d_{x2-y2}$  pairing while  $U_p$  induces  $d_{xy}$  pairing, (6)  $T_c$  has a peak as a function of  $t_{pp}$ .

## Acknowledgements

We thank J. Kondo for useful discussions.

#### References

- [1] The Physics of Superconductors Vol.I and Vol. II edited by Bennemann KH, Ketterson JB. Springer-Verlag, Berlin (2003).
- [2] Hirsch JE, Loh D, Scalapino DJ, Tang S. Phys. Rev. B39 243 (1989).
- [3] Scalettar RT, Scalapino DJ, Sugar RL, White SR. Phys. Rev. B39 243 (1989).
- [4] Weber C, Lauchi A, Mila F, Giamarchi T. Phys. Rev. Lett. 102 017005 (2009).
- [5] Lau B, Berciu M, Sawatzky GA. Phys. Rev. Lett. 106 036401 (2011).
- [6] Yanagisawa T, Koike S, Yamaji K, J. Phys. Soc. Jpn. 67 3867 (1998).
- [7] Yanagisawa T, Koike S, Yamaji K. Phys. Rev. B64 184509 (2001).
- [8] Yanagisawa T, Koike S, Yamaji K, Phys. Rev. B67 132408 (2003).
- [9] Koike S, Yamaji K, Yanagisawa T, J. Phys. Soc. Jpn. 68 1657 (1999).
- [10] Yanagisawa T, Miyazaki M, Yamaji K. J. Phys. Soc. Jpn. 78 013706 (2009).
- [11] Miyazaki M, Yanagisawa T, Yamaji K. J. Phys. Soc. Jpn. 73 1643 (2004).
- [12] Miyazaki M, Yamaji K, Yanagisawa T, Kadono R. J. Phys. Soc. Jpn. 78 043706 (2009).
- [13] Yanagisawa T, Miyazaki M, Yamaji K. J. Mod. Phys. 4 No.6A 33 (2013).
- [14] Yamaji K, Yanagisawa T, Koike S. Physica B284-288 415 (2000).
- [15] Yamaji K, Yanagisawa T, Miyazaki M, Kadono R. J. Phys. Soc. Jpn. 80 083702 (2011).
- [16] Zheng G-q, Kitaoka Y, Ishida K, Asayama K. J. Phys. Soc. Jpn. 64 2524 (1995).
- [17] Koikegami S, Yamada K. J. Phys. Soc. Jpn. 69 768 (2000).
- [18] Koikegami S, Yanagisawa T. J. Phys. Soc. Jpn. 70 3499 (2001); 71 671 (2001) (E).
- [19] Koikegami S, Yanagisawa T. Phys. Rev. B67 134517 (2003).
- [20] Yanagisawa T. New J. Phys. 10 023014 (2008).
- [21] Yamaji K, Yanagisawa T. Physica C497 93 (2013).
- [22] Kondo J. J. Phys. Soc. Jpn. 70 808 (2001).
- [23] Kondo J. Bull. Electrotech. Lab. 64 67 (2001).