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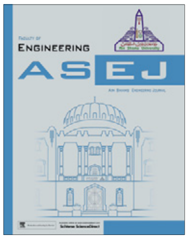

**CIVIL ENGINEERING**

# Macro-mechanical strut and tie model for analysis of fibrous high-strength concrete corbels

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**Abstract** Reinforced concrete (RC) corbels becoming a frequent attribute in the building construction with the increasing use of pre-cast high-strength concrete (HSC). The use of fibrous high-strength concrete (FHSC), increases corbel ductility and thus defines the mode of failure of the corbels, depending on the fiber parameters. In this paper a macro-mechanical strut and tie model is proposed for analysis of fibrous high-strength concrete corbels. In this model the fibers can be used as a replacement of horizontal stirrups, due to increasing of shear friction of (FHSC). The analytical macro-mechanical model takes into consideration the effect of fiber volume, fiber length, and fiber diameter, random distribution of fibers, fiber HSC interface and shear span-to-depth ratio, respectively. This model is compared with available experimental results found in literature and a good agreement is obtained. The parametric study is performed to examine for different parameters affecting the analysis of (FHSC) corbels using the proposed macro-mechanical strut and tie model.

## 1. Introduction

The use of high-strength concrete is widespread in modern structures. Many researchers focused on the shear friction behavior of high-strength concrete [1–4] and use this behavior in corbels as a structural application. Corbels are designed mainly to provide for vertical reaction and sometimes for horizontal forces. The reinforcement of the corbels is primary tension steel and horizontal bars. From the test results, it was found that corbels display several modes of failure [2]. The most common are the yielding of tension reinforcement, crushing or splitting of compression strut and localized bearing or shearing failure under the loading plate, respectively. The cracking pattern near failure load of high-strength reinforced concrete corbel is shown in Fig. 1. The current codes philosophy [5,6] of the design of corbels is based on shear–friction theory. In the code provision of design of corbels all the tensile stresses are carried by main reinforcement and horizontal hoops while the concrete tensile strength is totally neglected.

The uneven crack faces slide past one another, the projections on the crack faces over one another and force the crack apart, stretching any reinforcement crossing the crack to yield. The tensile force developed in the reinforcement is assumed to

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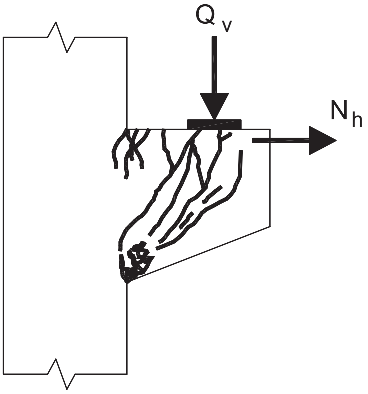

**Figure 1** Cracking pattern near failure load for high-strength reinforced concrete corbels.

compress the crack faces together which results frictional resistance across crack plan [1]. Many test results [7–12] are performed to predict the ultimate shear strength of fiber high-strength concrete (FHSC). Most of these tests are performed on the corbels and the other is localized on puss-off specimens. These test results reflect the influence of FHSC on shear friction strength. This means there is a need for design equation to evaluate the contribution of (FHSC) on the shear friction. The available experimental results [7–11] found in literature are performed on the corbels without horizontal ties and using fibers as partial or full replacement of horizontal ties. In the present paper, the evaluation of the contribution of fiber high-strength concrete (FHSC) corbels is presented through proposed simplified macro-mechanical model. The analytical macro-mechanical model takes into consideration the effect of fiber volume, fiber length, and fiber diameter, random distribution of fibers, fiber HSC interface and shear span to depth ratio. The analytical model compared with experimental results found in literature shows acceptable agreement. Different studies are preformed on the FHSC corbels include effect of fiber volume fraction, fiber length over fiber diameter, shear span-to-depth ration and FHSC strength, respectively.

## 2. Code provisions for design of corbels

The code previsions for design of RC corbels [5,6] approved that the shear force is transmitted through a defined plan by shear friction mechanism. The philosophy of the Egyptian Code (ECP 203-2007) [5] for shear friction provided that the shear force is transmitted by the reinforcement and neglecting the concrete shear strength. The reinforcement required to resist shear friction ($A_{sf}$) is given by the following relationship as:

$$A_{sf} = \frac{Q}{\mu_f(f_y/\gamma_s)} + \frac{N_h}{(f_y/\gamma_s)} \tag{1}$$

where ($Q_v$) and ($N_h$) are respectively, the vertical force and horizontal force transmitted through shear plan, as shown in Fig. 2. The value of ($f_y/\gamma_s$) is the yield stress of the reinforcement over the strength reduction factor of the reinforcing steel, on condition that the yield stress of the reinforcement is not greater than (400 MPa). The friction coefficient ($\mu_f$) is taken as (1.20) for monolithic concrete, (0.80) for construction jointed concrete with rough surface greater than (5 mm) and (0.50) for construction jointed concrete with rough surface less than (5 mm), respectively. The shear friction stress ($Q_v/bd$), as shown on Fig. 2, should not exceed the value ($0.225 f_{cu}/\gamma_c$); with maximum limit (5 MPa) as shown in Fig. 2, where ($b$) is the corbel width, ($d$) is the corbel depth, ($f_{cu}$) is the characteristic cube strength of concrete and ($\gamma_c$) is the strength reduction factor of concrete, respectively [5].

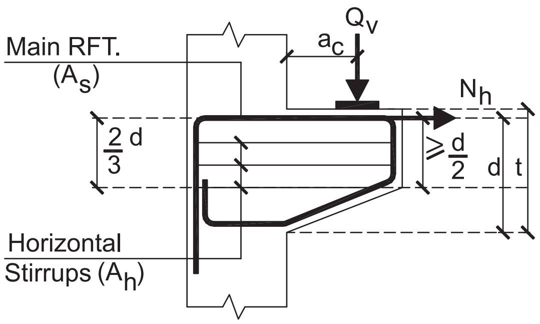

**Figure 2** Details of steel reinforcement arrangement and applied forces on the reinforced concrete corbels.

The reinforcement configuration of the corbels according to codes requirements [5,6] is shown in Fig. 2. The Egyptian Code [5] defines the corbels that have the value of ($a_c/d$) not greater than unity. The main reinforcement of the corbel ($A_s$) is shown in Fig. 2 and can be calculated according to (ECP 203-2007) [5], as the greater value of the following two equations as:

$$A_s = A_n + A_f \tag{2a}$$

or

$$A_s = A_n + (2/3)A_{sf} \tag{2b}$$

where the value of the main steel area ($A_f$) is the reinforcement required to resist flexure at the face of support and ($A_n$) is the value of main steel area required to oppose horizontal tension force ($N_h$), and can be calculated as:

$$A_n = N_h/(f_y/\gamma_s) \tag{3}$$

The main steel reinforcement ratio ($A_s/bd$) should be taken not less than ($0.03 f_{cu}/f_y$). The horizontal stirrups ($A_h$) are parallel to the main reinforcement ($A_s$) and distributed over (2/3) of the section depth nearby main reinforcement, can be calculated as:

$$A_h = 0.50(A_s - A_n) \tag{4}$$

## 3. Mechanical properties of fibrous concrete

Due to the fiber improvement of the mechanical properties of (HSC), it is considered suitable material to use specially in corbels. The compressive and tensile behavior of fibrous concrete is highly dependent on fiber properties, mixing, curing and concrete matrix [13–15]. The stress is transmitted from the matrix to fiber by shear deformation at the fiber–matrix interface as a result of the different mechanical properties of the fibers and the matrix. This explains why fibers can be used as a full replacement of horizontal stirrups in FHSC corbels.

Under uniaxial compressive behavior, many experimental studies and numerical models are carried out to study the effect of fiber addition to normal and high-strength concrete

[13]. It was found that the increase of fiber volume fraction and/or fiber aspect ratio increases the ductility of the stress–strain curves [13,14]. The compressive strength of the fibrous concrete slightly increased up to 9% greater than non-fibrous concrete. Thus, the equivalent stress block recommended by (ECP 203-2007) [5], could be used for fibrous concrete. The typical normalized stress–strain curves of fibrous concrete with different volume fractions are shown in Fig. 3. The fiber increases the peak strain of the compressive stress–strain curve ($\varepsilon_{cu}$). To determine the peak compressive strain of fibrous concrete ($\varepsilon_{cf}$), a trend linear relationship between ($\varepsilon_{cf}$) and fiber parameter ($I$) is proposed [16] as:

$$\varepsilon_{cf} = 0.0008I + \varepsilon_{cu} \tag{5}$$

where ($I$) is the fiber parameter and equal to ($V_f L/\phi$) and ($\varepsilon_{cu}$) is the peak compressive strain of non-fibrous concrete, while ($V_f$) is the fiber volume fraction and ($L/\phi$) is the fiber length to diameter or fiber aspect ratio, respectively.

In the post peak behavior the area under the stress–strain curve is highly increased by increasing the fiber volume fraction and/or fiber aspect ratio.

The defeat of ductility problem of non-fibrous high-strength concrete in tension can be effectively treated by adding fibers to the normal concrete [14]. The stress–strain relationship is nearly linear up to cracking strain of the concrete matrix. Practically, the fibrous concrete has nearly, the same values of cracking strain and cracking strength as non-fibrous concrete. This is due to the crack initiation process is considered highly dependent on the matrix not on the fiber. The analytical model in tension is adopted here uses also the equivalent stress block. The maximum value of the equivalent tensile stress block is given as ($2/3f_{cf}$), where $f_{cf}$ is given as follows:

$$f_{cf} = \alpha\tau V_f L/\phi \tag{6}$$

The orientation efficiency factor is given as ($\alpha$) while, the interfacial bond strength is specified as ($\tau$). The value of ($\alpha\tau$) is obtained from a regression analysis [14] as (4.55 MN/m$^2$). The cracking tensile strength of non-fibrous concrete ($f_{ctr}$) is given as a function of characteristic cube compressive strength ($f_{cu}$) and calculated according to (ECP 203-2007) [5], as follows:

$$f_{ctr} = 0.6\sqrt{f_{cu}} \tag{7}$$

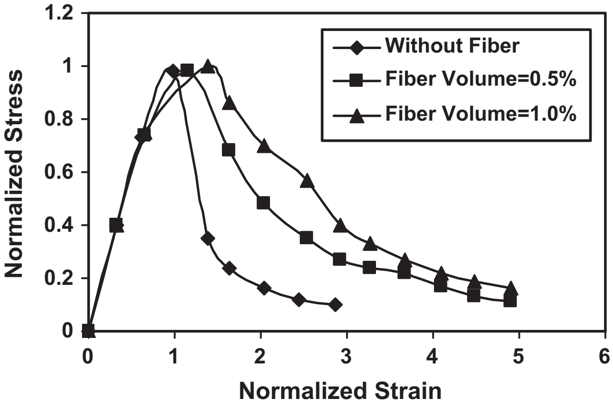

**Figure 3** Typical normalized stress–strain curves of fibrous concrete with different fiber volume fractions [13].

After the stress reach peak tensile strength sudden drop on the tensile strength due to the transformation of tensile stresses from matrix to the fibers. With increasing crack width, the fibers are pullout of the matrix resulting in decreasing tensile stress of the composite. A considerable improvement of ductility was found, on the post-peak tensile strength of the fibrous concrete.

## 4. Design of fibrous concrete corbel under bending/flexural

To determine the value of ($A_f$) in Eq. (2a), the following subsection explain how to calculate this flexural reinforcement value. It was also recommended to choose the partial safety factor for fibrous concrete in tension ($\gamma_{ct}$) equal to that of steel reinforcement ($\gamma_s$) because the effect of fiber takes place after the cracking of concrete occurs which represents the same effect of steel reinforcement [15]. The ultimate limit moment ($M_u$) for singly reinforced sections of rectangular beams, solid slabs and T-section having neutral axis inside its flange can be calculated [15] using the following equation in conjunction with Fig. 4 as follows:

$$M_u = (A_f f_y/\gamma_s)(d - 0.4c) + 2/3(f_{cf}/\gamma_{ct})(d - c)b(0.5d + 0.1c) \tag{8}$$

The equivalent stress block depth ($a$) is calculated from the using the following equation as:

$$a = [A_f f_y/\gamma_s + 2/3(f_{cf}/\gamma_{ct})bd]/[b(0.67f_{cu}/\gamma_c + 0.83f_{cf}/\gamma_{ct})] \tag{9}$$

The above value of the equivalent stress block depth ($a$) should satisfy the ECP conditions [5,15]. The maximum bending moment and reinforcement ratio can be calculated as:

$$M_u = R_{\max} f_{cu} b d^2/\gamma_c \tag{10}$$

where ($R_{\max}$) is the factor of maximum limit of bending moment. For double reinforced concrete section, the maximum moment of resistance of the section could be increased further than single reinforced one [5].

To control the cracking of RC corbels subjected to flexure and provide with tensile reinforcement only and to ensure enough section ductility, the steel reinforcement ratio should be greater than minimum reinforcement. Because of the cracking moment is governing by the concrete and the tensile strength for fibrous concrete is the same for non-fibrous concrete, the minimum reinforcement ratio for non-fibrous concrete can be applied for fibrous concrete. It was also, noted that the

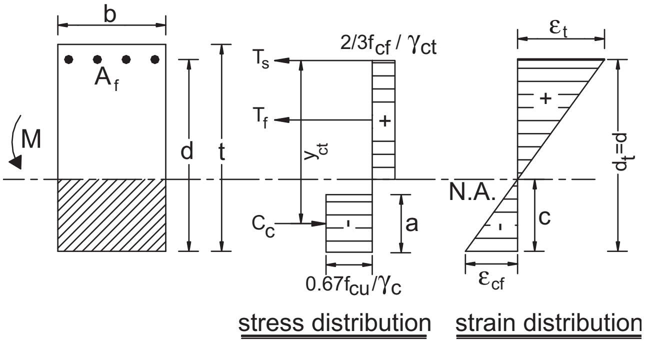

**Figure 4** Analytical model for design of fibrous concrete single and double reinforced section.

compressive strength of fibrous concrete is slightly increased than non-fibrous concrete. So, the value ($f_{cu}$) of non-fibrous concrete can be reasonably used for fibrous concrete. As shown from the following section the fibers increase the flexural capacity of the section in addition to use as full replacement of the stirrups to resist shear friction. This alters the corbel failure mode to tend as steady-mechanism of shear-flexural failure mode instead of flexural rather than splitting shear failure mechanism. This means that the fiber increase the term of ductility of (FHSC) corbels.

## 5. Evaluation of fiber contribution

The existence of fibers in concrete mixes is mainly to improve the post-cracking tensile performance of concrete and has a small effect on its compressive strength [13,14]. The effective area of fibers should be calculated and insert its effect in tension zone as mentioned in Section 4 of this paper. Every single fiber in the tension zone of the corbel may be considered as a small longitudinal rebar of area ($a_f$). The distribution of the fiber in the tension zone of the RC corbel is random and depends on fiber volume fraction, fiber (diameter/length) ratio, boundary conditions, mixing and placement method.

For these reasons, fibers are expected to orient randomly in the three dimensions but, the vibration of high strength concrete during mixing and placement process make fiber to reorient in the horizontal plan. The final form is intermediate between two and three dimensions. The orientation factor ($\beta$) has different values [17] according to geometric conditions. Thus, the number of effective fibers per unit cross section of the corbel ($S$) could be evaluated as:

$$S = \beta(V_f/a_f) \tag{11}$$

Therefore, the area of the fibers per unit area of the section ($S \cdot a_f$) is equal to ($\beta \cdot V_f$).

The proposed analytical model to calculate the effective fibrous section properties of (FHSC) is shown in Fig. 5. As shown from Fig. 5, the representation of flexural reinforcement and the fibers in tension zone is very clear. The depth of neutral axes ($z$) can be determined as follows:

$$bz^2/2 = nA_s(d - z) + K_f b(d - z) \tag{12}$$

The value of ($n$) is the modular ratio of reinforcing steel and equal to the modulus of elasticity of reinforcing steel to the modulus of elasticity of concrete ($E_s/E_c$), where ($K_f$) is the fiber parameter and can be derived in conjunction with Eq. (11) as:

$$K_f = m\beta V_f \tag{13}$$

The value of ($m$) is the modular ratio of fibers and equal to the modulus of elasticity of fibers to the modulus of elasticity of concrete ($E_f/E_c$). Eq. (12) can rewritten as follows:

$$z^2 + C_1 z - C_1 d = 0.0 \tag{14a}$$

where

$$C_1 = 2\left(\frac{nA_s}{b} + K_f\right) \tag{14b}$$

The value of ($z$) can be obtained by solving the Eq. (14a). For non-fibrous (HSC) section the value of ($K_f$) can be taken as zero.

## 6. The propose macro-mechanical strut- and tie model

The applied loads acting on the RC corbel and the corresponding internal forces are as shown in Fig. 6. The downward force ($Q_v$) is pulled by the tension force in the main steel and forms the diagonal compression strut. The horizontal force ($N_h$) is balanced with the tension force on the main corbel, with neglecting the shifting effect in this study. The internal horizontal shear force ($Q_h$) is the resultant of the horizontal force in concrete ($C_c$) and the tension force in the main reinforcing steel ($T_s$) and outer horizontal force ($N_h$) as follows:

$$Q_h = T_s - N_h = C_c \tag{15}$$

The distance ($H$) between the compression force in concrete and tension force in the reinforcing steel bars is calculated in conjunction with Eq. (14) as:

$$H = d - z/3 \tag{16}$$

The macro-mechanical strut- and tie model proposed in this paper (see Fig. 7) is a modified model proposed in [2]. The proposed macro-mechanical strut- and tie model is shown in Fig. 7. The downward vertical force ($Q_v$) is forming a compression strut with angle of inclination ($\theta$) equal to $\tan^{-1}$ ($H/a_c$). After development of cracks, the steel bars will be subjected to tension force and concrete acts as compression strut as shown in Fig. 7. Statically indeterminate load paths are proposed by [2] for macro-mechanical strut and tie model. The formation of compression strut between loading point and the support is failed by crushing of concrete, while the fibers carry the tension tie instead of horizontal stirrups. There are three paths for the load, as shown in Fig. 7 vise; the vertical, horizontal and diagonal directions. But, the vertical stirrups are not mainly detailed in the corbels, so that the vertical

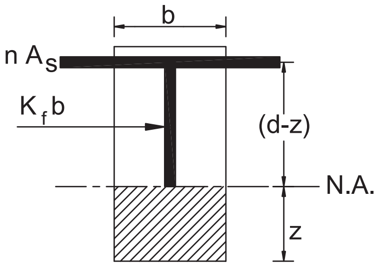

**Figure 5** Proposed analytical model to calculate the effective fibrous section properties.

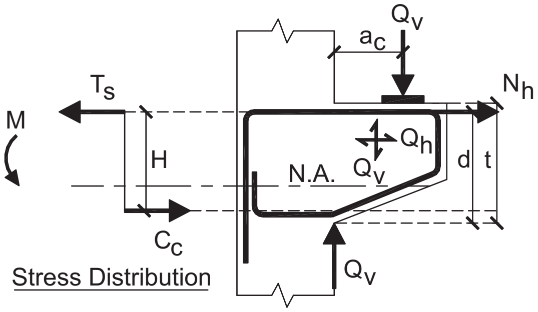

**Figure 6** Applied loads acting on the corresponding internal forces.

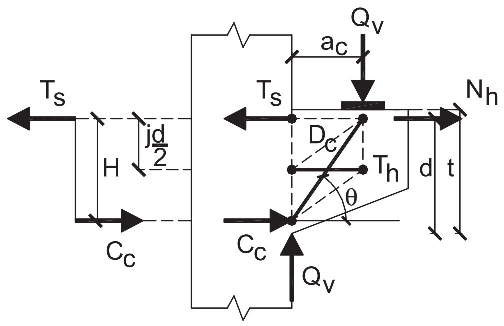

**Figure 7** Proposed macro-mechanical strut- and tie model for fibrous high-strength concrete corbels.

mechanism is not recognized on this study. The tension tie in the vertical direction will be developed first, but concrete will be cracked due to not detailing of vertical stirrups. In this case the corbels transmit the vertical load through diagonal and horizontal direction only. For this reason, an incomplete macro-mechanical strut and tie model is developed by Hwang et al. [2] for normal concrete and developed in this paper for (FHSC). The diagonal mechanism is a compression strut inclined by angle ($\theta$), while the tension tie is horizontal and carried by fibrous high-strength concrete in tension in place of steel stirrups.

The horizontal mechanism composed of two flat-struts and horizontal tie, as shown in Fig. 7. The horizontal tie is resisted by force ($T_h$) derived from fibrous high-strength concrete in tension and can be analyzed in this paper as tension force can be carried by fibers ($T_f$), as shown in Fig. 4 and can be calculated as follows:

$$T_h = [(2/3)(f_{cf}/\gamma_{ct})b(d - a/0.8)]J_t \tag{17}$$

The effective force of the diagonal compression strut ($D_c$) can be calculated as follows:

$$D_c = [f_{cu}ab]J_c \tag{18}$$

where ($J_t$) and ($J_c$) are the adjustment factors for tension tie and compression strut, respectively. The value of ($J_t$) and ($J_c$) are considered equal to (0.80) and (0.70) respectively, while the value of ($a$) can be derived using Eq. (9).

The equilibrium of the diagonal compression strut can be re-written in the form of vertical shear force ($Q_v$) as follows [2]:

$$Q = D_c \sin\theta + T_h \tan\theta \tag{19}$$

Also, the ratios of the contribution of diagonal compression strut and horizontal tension tie to carry the vertical shear force ($Q_v$) can be calculated as follows [2]:

$$D_c \sin\theta : T_h \tan\theta = R_c : R_t \tag{20}$$

where ($R_c$) and ($R_t$) are the ratio of the corbel shear resisted by diagonal compression and tension tie, respectively and it can be calculated as [3]:

$$R_{c=1-\delta_h} \tag{21a}$$

$$R_{t=\delta_h} \tag{21b}$$

where ($\delta_h$) is the friction coefficient of the horizontal shear transferred by horizontal tie and defined as follows [2,3]:

$$\delta_h = (2\tan\theta - 1)/3 \quad \text{for} \quad 0 \le \delta_h \le 1 \tag{22}$$

The value ($\delta_h$) indicates that the entire shear can be resisted by horizontal mechanism if $\theta \geqslant \tan^{-1}(2)$ and the entire shear is resisted by diagonal mechanism if $\theta \leqslant \tan^{-1}(1/2)$.

## 7. Validation and parametric studies of the proposed model

The proposed model is incorporated into computer program to calculate the strut and tie model components ($D_c$) and ($T_h$), respectively. The validation of the numerical model is carried out to compare the model with the experimental data found in literature [8–11]. A comparison between calculated ($Q_v$) using the adopted strut and tie model and experimentally predicted values, are shown in Table 1. The numerical predicted results, calculated from adopted model, show a good agreement, as shown in Fig. 8. The obtained standard deviation is equal to (0.071).

Parametric studies are performed in current paper to detect the most important parameters. The most important parameters include the effect of fiber volume percentage, fiber aspect ratio and concrete strength, respectively. The adopted corbel is of depth 125 mm and shear span ($a_c$) of 80 mm with 155 mm width. The steel reinforcement ratio is chosen to be 1.20% which represents acceptable intermediate design value used in the precast concrete corbels. The fiber volume changes from 0.5% up to 2.20%, is adopted to represent the effect of change of fiber volume fraction. During change of fiber volume fraction, the fiber aspect ratio ($L/\phi$) is constant and equal to 60. The characteristic cube concrete compressive strength is chosen to be 51.70 MPa which represents the most used value in the precast manufacturing.

The effect of fiber volume fraction on the maximum vertical load carrying capacity applied on corbels is shown in Fig. 9. It can be shown from this figure that the loading carrying capacity of the corbel is increased with increasing the fiber volume fraction. This increase up to percentage of volume fraction is equal to about 1.50% after this volume increase the effect of increasing of the maximum vertical load carrying capacity is no longer efficient. However, it is not recommended to use values of volume fraction higher than 1.50% due to fiber balling [14].

The effect of change of fiber aspect ratio is studied on different aspect ratio ($L/\phi$) is studied for different values starting from 30 up to 120 with constant most public use fiber volume fraction equal to (1%) and characteristic cube compressive strength equal to 51.70 MPa. The effect of increasing fiber aspect ratio on the maximum vertical shear force carried by the corbel is shown in Fig. 10. It was found that the optimum fiber aspect ratio is (55) up to (60) after these values the influence of increase of fiber aspect ratio is no more efficient above these values.

A parametric study is carried out to show the effect of increase of concrete compressive strength on the maximum shear capacity of the RC corbels. The concrete strength range was chosen to start from characteristic cube strength from 40 MPa up to 70 MPa.

The fiber volume fraction of 1% with fiber aspect ration of 60 was chosen during this study. As shown from Fig. 11, the increase in the concrete cube strength increases the maximum vertical capacity of shear force till certain limit of concrete cube strength equal to 50 MPa. After this limit the curve tends to be plateau with slight increase in maximum shear capacity

**Table 1** Comparison between experimental and numerical results.

| Refs. | $a_c$ (mm) | $b$ (mm) | $d$ (mm) | $V_f$ (%) | $L/\phi$ | $A_f$ (mm$^2$) | $f_y$ (MPa) | $f_{cu}$ (MPa) | $Q_v$ (kN) Exp. | $Q_v$ (kN) Num. | Num./Exp. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| [8] | 80 | 152.5 | 123 | 1.66 | 60 | 226 | 340 | 49.3 | 153 | 160.8 | 1.05 |
| [8] | 80 | 155 | 124 | 1.66 | 60 | 226 | 340 | 51.69 | 160 | 163.13 | 1.02 |
| [8] | 80 | 154 | 125 | 0.74 | 60 | 100 | 340 | 45.2 | 92 | 85.60 | 0.93 |
| [9] | 110 | 153 | 123.5 | 1.75 | 60 | 226 | 340 | 45.98 | 126 | 124.1 | 0.98 |
| [9] | 110 | 156 | 122 | 1.50 | 60 | 220 | 340 | 44.70 | 118 | 120.32 | 1.02 |
| [9] | 110 | 153 | 122.5 | 2.00 | 60 | 220 | 340 | 40.26 | 126.5 | 132.18 | 1.045 |
| [9] | 80 | 153.5 | 123.5 | 2.50 | 60 | 220 | 340 | 50.30 | 171.5 | 180.16 | 1.05 |
| [9] | 120 | 154 | 120.2 | 2.00 | 60 | 270 | 340 | 48.30 | 132.5 | 138.40 | 1.044 |
| [9] | 110 | 153.5 | 124 | 2.25 | 60 | 260 | 340 | 46.10 | 144.5 | 150.20 | 1.039 |
| [9] | 80 | 154.1 | 122.1 | 2.50 | 60 | 220 | 340 | 44.0 | 164.5 | 162.20 | 0.986 |
| [10] | 89 | 152 | 122 | 0.70 | 92 | 157 | 400 | 57.0 | 133 | 122.35 | 0.92 |
| [10] | 89 | 152 | 121 | 2.10 | 92 | 220 | 400 | 57.4 | 171.2 | 205.50 | 1.20 |
| [11] | 52.5 | 153 | 121 | 0.70 | 92 | 101 | 400 | 56.3 | 125.8 | 129.41 | 1.029 |
| [11] | 125 | 153 | 118 | 0.70 | 92 | 226 | 400 | 56.33 | 110.1 | 114.98 | 1.044 |
| [11] | 64.5 | 153 | 118 | 0.70 | 92 | 226 | 400 | 67.90 | 179 | 163.65 | 0.914 |
| [12] | 125 | 152 | 120 | 0.70 | 48 | 160 | 400 | 63.70 | 84.5 | 78.60 | 0.93 |

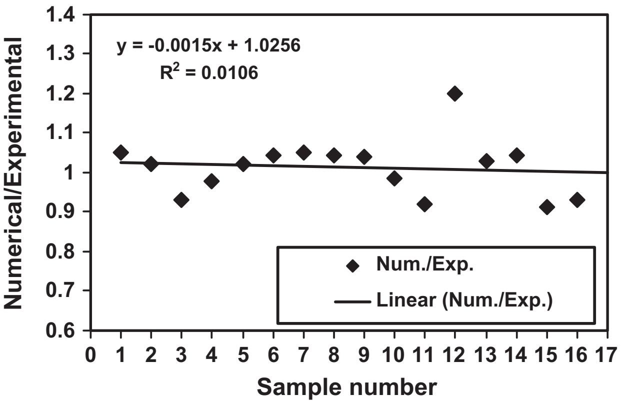

**Figure 8** Comparison between numerical and experimental results [8–12].

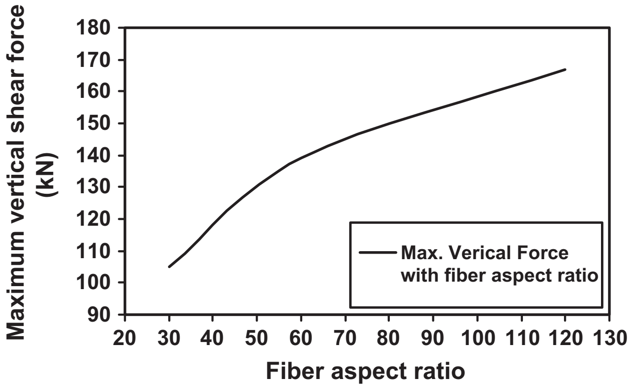

**Figure 10** Effect of increasing fiber aspect ratio on the maximum vertical load carrying capacity of corbels.

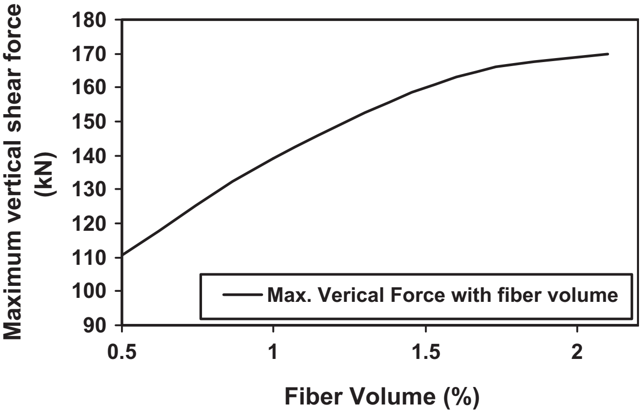

**Figure 9** Effect of increasing fiber volume fraction on the maximum vertical load carrying capacity of corbels.

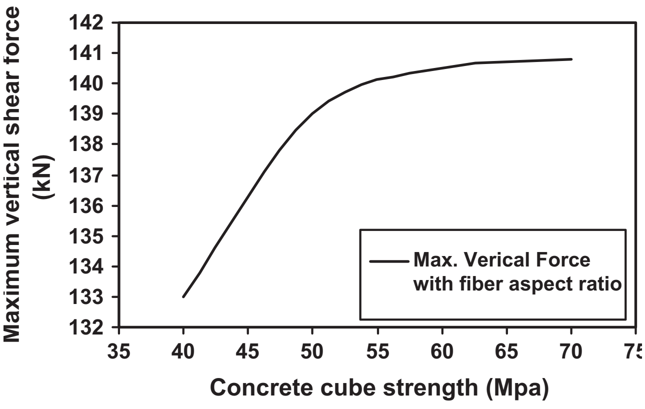

**Figure 11** Increasing of concrete cube compressive strength versus maximum vertical load carrying capacity.

with increasing the concrete compressive strength. This due to the failure mode is governing by the fiber fracture rather than fiber pullout. For the value engineering requirement, it is recommended to increase the fiber volume and/or fiber aspect ratio rather than increasing the concrete compressive strength greater than 50 MPa.

## 8. Conclusions

In this paper a macro-mechanical strut and tie model is proposed for analysis of fibrous high-strength concrete corbels. In this model the fibers can be used as a partial or full replacement of horizontal stirrups, due to increasing of shear friction of (FHSC). The proposed analytical macro-mechanical model is taken into consideration the effect of fiber volume, fiber length, and fiber diameter, random distribution of fibers, fiber HSC interface and shear span to depth ratio, respectively. The proposed model is incorporated into computer program to calculate the strut and tie model components. The validation of the numerical model is carried out to compare the model with the available experimental data founded in literature. A comparison between calculated maximum vertical shear force using the adopted strut and tie model and experimentally predicted values shows a good agreement with standard deviation is equal to (0.071).

Parametric studies are performed in the present paper to detect the most important parameters. The most important parameters include the effect of fiber volume percentage, fiber aspect ratio and concrete strength on the corbel behavior. It was found that the effect of fiber volume fraction on the maximum vertical load carrying capacity applied on corbels is increased with increasing the fiber volume fraction. This increase up to percentage of volume fraction equal to about 1.50% after this volume increase the effect of increasing of the maximum vertical load carrying capacity is no longer efficient. The effect of increasing fiber aspect ratio on the maximum vertical shear force carried by the corbel is match with the optimum fiber aspect ratio is (55) up to (60) after these values the influence of increasing of fiber aspect ratio is not more efficient than these values.

Also a parametric study is carried out on the effect of increasing of concrete strength on the maximum shear capacity of the RC corbels. As developed from parametric study, increasing of concrete compressive strength increases the maximum vertical capacity of shearing force till certain limit of concrete cube strength equal to 50 MPa. After this limit the curve tends to be plateau with slight increase in maximum shear capacity with increasing the concrete strength. This due to the failure mode which governed by the fiber fracture rather than fiber pullout. For the value engineering requirement, it is recommended to increase fiber volume and/or fiber aspect ratio rather than increase the concrete compressive strength greater than 50 MPa.

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