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Testing the Weisskopf–Wigner approximation by using neutral-meson–antimeson correlated states

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Abstract

We phenomenologically decompose the Weisskopf–Wigner approximation, as applied to the neutral flavoured meson (M^0) complexes, into three pieces and propose tests for these pieces. Our tests hold for general decay amplitudes and M^0–$\overline{M}^0$ mixing parameters. We concentrate on C-odd $M^0\overline{M}^0$ states and stress the importance of such tests in view of the variety of physics extracted from measurements on such complexes. Studying the feasibility of the tests confines one to the $K^0\overline{K}^0$ system at present. In particular, we show that the time dependence of the correlated decay $\phi \to K^0\overline{K}^0 \to 2(\pi^+\pi^-)$ is determined solely by the WWA and provides thus a clean test of it.

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The complex formed by a neutral flavoured meson M^0 (i.e., K^0, D^0, B_d^0, B_s^0) and its antiparticle $\overline{M}^0$ is an important environment for investigating (i) discrete symmetries CP, T and CPT, (ii) quantum-mechanical correlations and also (iii) new physics. The present phenomenology of this complex is commonly based on the Weisskopf–Wigner approximation (WWA) [1]. Since tiny effects are searched for in the (M^0, $\overline{M}^0$) complex, it is desirable to devise experimental tests of the WWA itself, independently of the physics searched for. One component of the WWA is the exponential decay law (for theoretical studies see, e.g., Ref. [2] and papers cited therein), which has been tested [3] in nuclear physics, and no deviations have been found, nor in particle physics. Recently, however, strong non-exponential decay features have been observed in a quantum-mechanical system [4]. Ever since the development of the WWA formalism for the (K^0, $\overline{K}^0$) complex [5,6], many studies have been performed to search for possible deviations from the WWA; these would be important in the evaluation of experimental data on the (M^0, $\overline{M}^0$) complex (for recent references see, e.g., [7]). In the present letter we study the evolution of the correlated C-odd $M^0\overline{M}^0$ state for testing the WWA. We will phenomenologically split the WWA into three basic components and investigate separately their consequences. We want to derive consequences of the WWA alone, independently of CP, T and CPT invariances (of mixing or of decay amplitudes) and the $\Delta Q = \Delta F$ rule, where F means flavour (i.e., F may be either S, C or B).

We first attempt a phenomenological anatomy of the WWA. If we denote the general probability amplitudes

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for the transitions $|M^0\rangle \to |M^0\rangle$, $|M^0\rangle \to |\overline{M}^0\rangle$, $|\overline{M}^0\rangle \to |M^0\rangle$ and $|\overline{M}^0\rangle \to |\overline{M}^0\rangle$, respectively, as $a(t), b(t), \bar{b}(t)$ and $\bar{a}(t)$, where t is the proper time, the WWA provides a model for these amplitudes. We shall propose tests for the pieces of that model. In order to define these pieces, we recall that the WWA introduces two independently propagating states

$$\begin{pmatrix} |M_H\rangle \\ |M_L\rangle \end{pmatrix} \xrightarrow{t} \begin{pmatrix} \Theta_H(t) & 0 \\ 0 & \Theta_L(t) \end{pmatrix} \begin{pmatrix} |M_H\rangle \\ |M_L\rangle \end{pmatrix}$$
$$\text{with} \quad \Theta_H(0) = \Theta_L(0) = 1. \tag{1}$$

The states [1] $|M_{H,L}\rangle$ are suitable linear combinations of the two flavour states $|M^0\rangle$ and $|\overline{M}^0\rangle$:

$$|M_H\rangle = p_H|M^0\rangle + q_H|\overline{M}^0\rangle, \quad |p_H|^2 + |q_H|^2 = 1,$$
$$|M_L\rangle = p_L|M^0\rangle - q_L|\overline{M}^0\rangle, \quad |p_L|^2 + |q_L|^2 = 1, \tag{2}$$

where $p_{H,L}$ and $q_{H,L}$ are complex constants. Because of the relevant approximation, the $[(|M^0\rangle, |\overline{M}^0\rangle) \leftrightarrow (|M_H\rangle, |M_L\rangle)]$ system is closed, the effect of the physical channels (like $\pi^+\pi^-$) of $\overset{(-)}{M}{}^0$ decay being taken into account by the details of the propagation functions $\Theta_{H,L}(t)$. Of course, the transformation (2) is invertible:

$$\begin{pmatrix} |M^0\rangle \\ |\overline{M}^0\rangle \end{pmatrix} = A^{-1} \begin{pmatrix} |M_H\rangle \\ |M_L\rangle \end{pmatrix}$$
$$\text{with} \quad A = \begin{pmatrix} p_H & q_H \\ p_L & -q_L \end{pmatrix}. \tag{3}$$

The "decoupled" propagation of $|M_{H,L}\rangle$ in Eq. (1) restricts the otherwise general coefficients a, b, $\bar{b}$, $\bar{a}$. Using (i) the closed nature of $[(|M^0\rangle, |\overline{M}^0\rangle) \leftrightarrow (|M_H\rangle, |M_L\rangle)]$, (ii) Eqs. (2) and (3) and (iii) the definitions for the coefficients a, b, $\bar{b}$, $\bar{a}$, one gets

$$\Omega \begin{pmatrix} |M_H\rangle \\ |M_L\rangle \end{pmatrix} = \Omega A \begin{pmatrix} |M^0\rangle \\ |\overline{M}^0\rangle \end{pmatrix} = A\Omega \begin{pmatrix} |M^0\rangle \\ |\overline{M}^0\rangle \end{pmatrix}$$
$$= A \begin{pmatrix} a & b \\ \bar{b} & \bar{a} \end{pmatrix} \begin{pmatrix} |M^0\rangle \\ |\overline{M}^0\rangle \end{pmatrix}$$
$$= A \begin{pmatrix} a & b \\ \bar{b} & \bar{a} \end{pmatrix} A^{-1} \begin{pmatrix} |M_H\rangle \\ |M_L\rangle \end{pmatrix}, \tag{4}$$

where Ω denotes the time-development operator. Using Eq. (1), this means

$$\begin{pmatrix} \Theta_H & 0 \\ 0 & \Theta_L \end{pmatrix} = A \begin{pmatrix} a & b \\ \bar{b} & \bar{a} \end{pmatrix} A^{-1}. \tag{5}$$

Comparing the diagonal elements, one obtains

$$\Theta_H = \frac{1}{2}(a + \bar{a}) + \gamma \quad \text{and} \quad \Theta_L = \frac{1}{2}(a + \bar{a}) - \gamma, \tag{6}$$

with

$$\gamma = \left\{ \frac{1}{2}(a - \bar{a})(p_H q_L - q_H p_L) + b p_H p_L + \bar{b} q_H q_L \right\} / D$$
$$\text{and} \quad D = p_H q_L + p_L q_H. \tag{7}$$

Computing the off-diagonal elements (12 and 21 elements) of Eq. (5), similarly gives

$$\Delta_{12} = \left\{ (a - \bar{a}) p_H q_H - b p_H^2 + \bar{b} q_H^2 \right\} / D,$$
$$\Delta_{21} = \left\{ (a - \bar{a}) p_L q_L + b p_L^2 - \bar{b} q_L^2 \right\} / D. \tag{8}$$

The vanishing of the off-diagonal elements Δ_{12} and Δ_{21} is the lack of "vacuum regeneration" (viz. the absence of $|M_{H,L}\rangle \to |M_{L,H}\rangle$ transitions) in the WWA. This directly gives the first piece of the WWA:

$$\text{WWA1:} \quad \bar{a}(t) - a(t) = \beta b(t), \quad \bar{b}(t) = \alpha b(t), \tag{9}$$

where

$$\alpha = \frac{p_H p_L}{q_H q_L} \quad \text{and} \quad \beta = \frac{p_L}{q_L} - \frac{p_H}{q_H}. \tag{10}$$

Of course, Eq. (9) holds for all t and any $\Theta_{H,L}(t)$. The second piece of the WWA arises from the diagonal elements of Eq. (5). Using Eq. (6), one obtains the t-dependence of the coefficients a, b, $\bar{b}$, $\bar{a}$ as

$$\text{WWA2:} \quad a + \bar{a} = \Theta_H + \Theta_L, \tag{11a}$$
$$b = q_H q_L (\Theta_H - \Theta_L)/D, \tag{11b}$$

wherein γ has been simplified with the help of Eq. (9). While Eq. (11b) is expressed in terms of b, one could have equivalently written $\bar{a} - a$ or $\bar{b}$ in terms of $\Theta_H - \Theta_L$, because of Eq. (9). Thus all the four coefficients a, b, $\bar{b}$, $\bar{a}$ are given in terms of the functions $\Theta_{H,L}$ which are so far not specified. One may note that Eq. (11a) is expected because it is merely the invariance of the trace under the similarity

[1] In the case of the neutral kaons, the long-lived variety K_L corresponds to the heavier state M_H, and the short-lived K_S corresponds to the lighter state M_L.

transformation expressed by Eq. (5). The third piece of the WWA is the specification of $\Theta_{H,L}$ in terms of the exponential law:

$$\text{WWA3:} \quad \Theta_{H,L}(t) = \exp(-it\lambda_{H,L}) \quad \text{with } \lambda_{H,L} = m_{H,L} - \frac{i}{2}\Gamma_{H,L}, \tag{12}$$

where, as usual, $m_{H,L}$ are the real masses and $\Gamma_{H,L}$ the real decay widths of $M_{H,L}$. With all the pieces of the WWA put in, one eventually arrives at (for a convenient summary, see, e.g., [8,9])

$$\begin{aligned} a(t) &= g_+(t) - \theta g_-(t), \\ b(t) &= \frac{q}{p}\sqrt{1-\theta^2}\, g_-(t), \\ \bar{a}(t) &= g_+(t) + \theta g_-(t), \\ \bar{b}(t) &= \frac{p}{q}\sqrt{1-\theta^2}\, g_-(t), \end{aligned} \tag{13}$$

where

$$g_\pm(t) = \frac{1}{2}\big\{\exp(-it\lambda_H) \pm \exp(-it\lambda_L)\big\}, \tag{14}$$

$$\frac{q}{p} = \sqrt{\frac{q_H q_L}{p_H p_L}}, \qquad \theta = \frac{q_H/p_H - q_L/p_L}{q_H/p_H + q_L/p_L}, \tag{15}$$

$$\alpha = \left(\frac{p}{q}\right)^2, \qquad \beta = 2\frac{p}{q}\frac{\theta}{\sqrt{1-\theta^2}}. \tag{16}$$

Note that θ, but not q/p, is rephasing-invariant. Thus both the real and the imaginary parts of θ are in principle measurable, and also $|q/p|$, but not the phase of q/p [8,10]. A non-zero θ signifies CPT and CP non-invariance in mixing; similarly, a non-zero $|q/p| - 1$ signifies T and CP non-invariance. In the usual explicit calculations based on the WWA, the full model of Eq. (13) is used. Our interest, in contrast, is in the three ingredients (WWA1, WWA2, WWA3) and testing them; the tests of WWA2 (without WWA3) are bound to be only qualitative because they can merely examine the differences between the unknown Θ_H and Θ_L.

We now consider decays of the correlated $M^0\overline{M}^0$ states

$$\begin{aligned} |\psi_\epsilon\rangle = \frac{1}{\sqrt{2}}\big[&\big|M^0(\vec{k})\big\rangle \otimes \big|\overline{M}^0(-\vec{k})\big\rangle \\ &+ \epsilon\big|\overline{M}^0(\vec{k})\big\rangle \otimes \big|M^0(-\vec{k})\big\rangle\big], \end{aligned} \tag{17}$$

where ϵ denotes the charge conjugation value; $\epsilon = +1$ for the C-even case, $\epsilon = -1$ for the C-odd case. If one detects the decay channel f at time t_ℓ and the channel g at time t_r, the decay rate of $|\psi_\epsilon\rangle$ is

$$\begin{aligned} &R_\epsilon(f, t_\ell; g, t_r) \\ &\quad = \frac{1}{2}\Big|(a_\ell\bar{b}_r + \epsilon\bar{b}_\ell a_r)A_f A_g + (b_\ell\bar{a}_r + \epsilon\bar{a}_\ell b_r)\bar{A}_f\bar{A}_g \\ &\qquad + \big(a_\ell\bar{a}_r + b_\ell\bar{b}_r + \epsilon(\bar{a}_\ell a_r + \bar{b}_\ell b_r)\big) \\ &\qquad\quad \times \frac{1}{2}(A_f\bar{A}_g + \bar{A}_f A_g) \\ &\qquad + \big(a_\ell\bar{a}_r - b_\ell\bar{b}_r - \epsilon(\bar{a}_\ell a_r - \bar{b}_\ell b_r)\big) \\ &\qquad\quad \times \frac{1}{2}(A_f\bar{A}_g - \bar{A}_f A_g)\Big|^2. \end{aligned} \tag{18}$$

In this expression the transition amplitudes are defined as

$$\begin{aligned} \langle f|T|M^0\rangle &= A_f, \qquad \langle f|T|\overline{M}^0\rangle = \bar{A}_f, \\ \langle g|T|M^0\rangle &= A_g, \qquad \langle g|T|\overline{M}^0\rangle = \bar{A}_g, \end{aligned} \tag{19}$$

and

$$\begin{aligned} \overset{(-)}{a}_\ell &\equiv \overset{(-)}{a}(t_\ell), \qquad \overset{(-)}{a}_r \equiv \overset{(-)}{a}(t_r), \\ \overset{(-)}{b}_\ell &\equiv \overset{(-)}{b}(t_\ell), \qquad \overset{(-)}{b}_r \equiv \overset{(-)}{b}(t_r). \end{aligned} \tag{20}$$

The form of Eq. (18) assumes only that aspect of the WWA which was mentioned immediately after Eq. (2). We shall consider the consequences of WWA1, WWA2, WWA3, successively, for the coefficients a, b, $\bar{b}$, $\bar{a}$. The aim is a comparison with experiment.

For the following we concentrate on the decay of $|\psi_-\rangle$, i.e., the C-odd case. Use of WWA1 gives

$$\begin{aligned} &R_-(f, t_\ell; g, t_r) \\ &\quad = \frac{1}{2}\Big|(a_\ell b_r - b_\ell a_r) \\ &\qquad \times \Big(\alpha A_f A_g - \bar{A}_f\bar{A}_g + \beta\frac{1}{2}\big(A_f\bar{A}_g + \bar{A}_f A_g\big)\Big) \\ &\qquad + \big(2a_\ell a_r - 2\alpha b_\ell b_r + \beta(a_\ell b_r + b_\ell a_r)\big) \\ &\qquad\quad \times \frac{1}{2}\big(A_f\bar{A}_g - \bar{A}_f A_g\big)\Big|^2. \end{aligned} \tag{21}$$

This is the general form of the decay rate where Eq. (9) has been used.

The case that

$$A_f\bar{A}_g - \bar{A}_f A_g = 0 \tag{22}$$

deserves particular attention, because then the time dependence is "factored out":

$$R_-(f, t_\ell; g, t_r) = \frac{1}{2}|a_\ell b_r - b_\ell a_r|^2 \times \left|\alpha A_f A_g - \bar{A}_f \bar{A}_g + \beta A_f \bar{A}_g\right|^2. \quad (23)$$

This relation is a powerful test of the property of lack of vacuum regeneration in the WWA: for all decay channels satisfying Eq. (22), the $(t_\ell \leftrightarrow t_r)$-symmetric time dependence must be the same for a given choice of M^0. For Eq. (22) to hold, the decay amplitudes for the channels f and g must be completely specified, using not only the particle content of f and g, but also their configurations of spins and momenta. One is, therefore, led to consider spinless decay products with a situation wherein there is no variable Lorentz scalar. Thus spinless two-body channels (e.g., $\pi^+\pi^-$) and effective two-body channels seem interesting. (One possibility is the 3π mode where one pion, say π_1, moves back-to-back with the remaining two, with no relative momentum between π_2 and π_3; another possibility arises when π_1 is created at rest. [2]) With this in mind one may write $f = g$ in Eq. (23).

For considering WWA2, Eqs. (11a) and (11b), we use also the first relation of Eq. (9) to obtain, apart from an overall constant,

$$\begin{aligned} a_\ell b_r - b_\ell a_r &\longrightarrow (\Theta_H + \Theta_L)_\ell(\Theta_H - \Theta_L)_r \\ &\quad - (\Theta_H - \Theta_L)_\ell(\Theta_H + \Theta_L)_r \\ &= 2\big(\Theta_L(t_\ell)\Theta_H(t_r) - \Theta_H(t_\ell)\Theta_L(t_r)\big). \end{aligned} \quad (24)$$

Due to WWA2, therefore, the unknown time dependence of Eq. (23) is now determined by the characteristics $\Theta_{H,L}(t)$ of the WWA. The greater the difference between $\Theta_H(t)$ and $\Theta_L(t)$, the more pronounced this time dependence would be because $\Theta_H - \Theta_L$ occurs linearly in Eq. (24). Unfortunately, this "test" of the WWA cannot be quantified because the $\Theta_{H,L}$ are yet unknown.

[2] These configurations have non-zero amplitudes in the standard theory of $K \to 3\pi$; see, e.g., Ref. [11]. We thank H. Neufeld for discussions on this point.

If one introduces the exponential law, viz. WWA3, the above feature comes to the surface: [3]

$$|a_\ell b_r - b_\ell a_r|^2 \longrightarrow e^{-\Gamma t_+}\left\{\cosh\left(\tfrac{1}{2}\Delta\Gamma t_-\right) - \cos(\Delta m\, t_-)\right\}, \quad (25)$$

where we have used the definitions $\Gamma = (\Gamma_H + \Gamma_L)/2$, $\Delta\Gamma = \Gamma_H - \Gamma_L$, $\Delta m = m_H - m_L$ and $t_\pm = t_\ell \pm t_r$. It is worth remarking that the time dependences (23), (24) and (25) for $R_-(f, t_\ell; f, t_r)$ do not depend on any assumptions about the decay amplitudes $\overset{(-)}{A}_f$ – in particular, their behaviour under CP, T and CPT transformations and the $\Delta Q = \Delta F$ rule; similarly, the constants $p_{H,L}$, $q_{H,L}$ have been kept general; CP, T and CPT non-invariances ($\beta \neq 0$, $|\alpha| \neq 1$) have been allowed throughout. The time dependence (25) of $R_-(f, t_\ell; f, t_r)$ is, therefore, the test of the full WWA; it is a specific version of the general $(t_\ell \leftrightarrow t_r)$-symmetric form in Eq. (23). Note that so far the WWA has been used in its general form, the exact values of the constants α and β appearing in Eq. (10) have not been exploited. A side remark: the vanishing of Eqs. (23), (24) and (25) for $t_\ell = t_r$ is merely the quantum-mechanical expectation that $R_-(f, t; f, t)$ vanishes (see, e.g., Ref. [13]). This feature is already present in Eq. (18) with $f = g$, $t_\ell = t_r$ and $\epsilon = -1$; it does not require WWA1, WWA2 and WWA3.

Another test of the general WWA framework of Eq. (18) arises by choosing $\epsilon = -1$ and $t_\ell = t_r$ for any $f \neq g$. Now, in complete contrast to Eq. (22), only the amplitude combination $A_f \bar{A}_g - \bar{A}_f A_g$ contributes to the rate. Once again, one has a factorization of the time dependence:

$$R_-(f, t_\ell; g, t_\ell) = \frac{1}{2}\left|a_\ell \bar{a}_\ell - b_\ell \bar{b}_\ell\right|^2 \left|A_f \bar{A}_g - \bar{A}_f A_g\right|^2, \quad (26)$$

leading to channel independence of the time dependence of R_-. If one now uses WWA1 + WWA2, along with the values of α and β from Eq. (10), the time dependence becomes

$$R_-(f, t_\ell; g, t_\ell)$$

[3] This result can be shown to be contained in the explicit calculation of Ref. [12], wherein the relevant result, Eq. (4), was obtained by making the assumption of CPT invariance, viz. $\beta = 0$. Our derivation is based on simpler and more general considerations.

$$\longrightarrow \left|\Theta_H(t_\ell)\Theta_L(t_\ell)\right|^2 = \frac{1}{16}\Big[\big(|\Theta_H(t_\ell)|+|\Theta_L(t_\ell)|\big)^2 - \big(|\Theta_H(t_\ell)|-|\Theta_L(t_\ell)|\big)^2\Big]^2. \tag{27}$$

Here, the difference between Θ_H and Θ_L makes only an additive contribution; its role is therefore not as important as in Eq. (24). With WWA3, one gets [4]

$$R_-(f,t_\ell;g,t_\ell) \to e^{-2\Gamma t_\ell} \tag{28}$$

as the channel-independent time dependence for a given choice of M^0, which tests the full WWA.

We now consider the feasibility of our WWA tests. For the tests in Eqs. (23), (24), (25), one needs spinless two-body (or, effectively two-body) channels. For $M^0 = B^0_{d,s}$, the relevant branching ratios are very small. For $M^0 = D^0$, the relevant $|\psi_-\rangle$ states have not yet been well-studied. This leads to $M^0 = K^0$. The obvious choice would then be to compare $\pi^+\pi^-$ with $\pi^0\pi^0$ for $f = g$ as a test of the channel independence of the correlated decay rate as a function of t_ℓ and t_r. In this case, the decay amplitudes $\overset{(-)}{A}_{f,g}$ are reasonably well studied. However, even without the WWA1, viz. already in Eq. (18), the time dependence is expected to be very nearly the same for these final states, as will be shown in the next paragraph. One is, therefore, led to compare the two choices $\pi\pi$ and special cases of $\pi\pi\pi$ for $f = g$ and $M^0 = K^0$. For further choices, one has to wait for future data.

We now give the detailed reason why the choices $\pi^+\pi^-$ and $\pi^0\pi^0$ for $f = g$ and $M^0 = K^0$ are not practically useful for testing the channel independence of the time dependence of the rate (23). Considering the rate (18), where WWA1 has *not* yet been used, we observe that the ratios $A_f A_f : \bar{A}_f \bar{A}_f : A_f \bar{A}_f$ for $f = g$ determine its time dependence. We will show that these ratios are the same for $f = \pi^+\pi^-$ and $f = \pi^0\pi^0$, except for quantities of second order of smallness, i.e., of the order of the CP-violating quantity ε' which denotes CP violation in the decay amplitudes [8]. In detail, let us write [14]

$$A_I = \langle I|T|K^0\rangle = a_I e^{i\delta_I}(1 + i\varphi_I + \beta_I + i\alpha_I), \tag{29a}$$
$$\bar{A}_I = \langle I|T|\overline{K}^0\rangle = a_I e^{i\delta_I}(1 - i\varphi_I - \beta_I + i\alpha_I), \tag{29b}$$

for the decay amplitudes to the two isospin states $I = 0,\ 2$ of the $\pi\pi$ system; $\varphi_I, \beta_I, \alpha_I$ are (supposedly small) real parameters expressing CP and T non-invariance, CP and CPT non-invariance, T and CPT non-invariance, respectively; the a_I are real; the δ_I are the $\pi\pi$ scattering phase shifts at the c.m. energy which equals the kaon mass. The isospin value I appears as subscript on various quantities. Then, with obvious meaning of the subscripts $+-$ and 00, one obtains

$$\begin{aligned} &A_{+-}A_{+-} : \bar{A}_{+-}\bar{A}_{+-} : A_{+-}\bar{A}_{+-} \\ &\quad = 1 : \big[1 - 2\sqrt{2}\,\omega\zeta - 4\sigma + 4\sigma(2\sigma + i\alpha_0)\big] \\ &\qquad : \big[1 - \sqrt{2}\,\omega\zeta - 2\sigma + 2\sigma(\sigma + i\alpha_0)\big], \end{aligned} \tag{30a}$$

$$\begin{aligned} &A_{00}A_{00} : \bar{A}_{00}\bar{A}_{00} : A_{00}\bar{A}_{00} \\ &\quad = 1 : \big[1 + 4\sqrt{2}\,\omega\zeta - 4\sigma + 4\sigma(2\sigma + i\alpha_0)\big] \\ &\qquad : \big[1 + 2\sqrt{2}\,\omega\zeta - 2\sigma + 2\sigma(\sigma + i\alpha_0)\big], \end{aligned} \tag{30b}$$

where we have used the notation

$$\begin{aligned} &\omega = \frac{a_2}{a_0} e^{i(\delta_2 - \delta_0)}, \qquad \sigma = \beta_0 + i\varphi_0, \\ &\zeta = i(\varphi_2 - \varphi_0) + (\beta_2 - \beta_0) + i(\alpha_2 - \alpha_0). \end{aligned} \tag{31}$$

Here, ω is small because of the $\Delta I = 1/2$ rule; we have retained small quantities up to only second order. Note that the only differences in the ratios in Eqs. (30a) and (30b) are due to the $\omega\zeta$ terms which are of the ε' type: they are proportional to a_2 and to a combination of φ_I, β_I and α_I. Usually, one takes $\beta_I = \alpha_I = 0$; then, $\omega\zeta$ is directly seen to be $\sqrt{2}\,\varepsilon'$. In the ratios in Eqs. (30a) and (30b), quantities of first (viz. σ) and zeroth (viz. 1) order of smallness are also present, apart from other quantities of second order (viz. σ^2, $\sigma\alpha_0$). Thus the ratios (30a) and (30b) are the same to a very good approximation. The observability of a difference in the time distributions for the $+-$ and 00 channels would require, therefore, very accurate data, in general, even without the WWA1.

Let us consider the feasibility of the test of WWA3 with $M^0 = K^0$ and $f = g = \pi^+\pi^-$, for which the corresponding rate is being measured at DAΦNE [15]. To get an estimate of the magnitude of the rate (23), we neglect the CPT-violation parameter β and the parameter ε'. Also, we retain small quantities to only the lowest contributing order. In this way we obtain

$$R_-(f,t_\ell;f,t_r) \simeq \big(\Gamma(K_S \to \pi^+\pi^-)\big)^2 |\eta_{+-}|^2 e^{-\Gamma t_+}$$

[4] Here, footnote 3 applies as well.

$$\times \left\{ \cosh \frac{1}{2} \Delta\Gamma t_- - \cos \Delta m t_- \right\}, \tag{32}$$

where η_{+-} denotes the ratio of the decay amplitudes of K_L and K_S into $\pi^+\pi^-$. DAΦNE will produce an adequate number of $K^0\overline{K}^0$ pairs so as to overcome the CP suppression in the rate (32) [15]; its *time dependence* is a clear consequence of the full WWA, irrespective of any T, CP and CPT violations or the validity of the $\Delta S = \Delta Q$ rule, as noted immediately following Eq. (25).

Now we come to the feasibility of the tests (26), (27), (28). The relevant time variable is only t_+ because $t_- = t_\ell - t_r$ vanishes now. But the variable t_+ is difficult to measure at the present asymmetric B factories [16] (see, e.g., also Ref. [17]). One is thus led to the choice $M^0 = K^0$ again. Since now $f \neq g$, one can consider many possible choices for f and g.

In summary, we have defined/derived the three pieces of the WWA from the point of view of phenomenological applications. We have then proposed tests for these pieces in a general way, without making any assumptions about the decay amplitudes $\overset{(-)}{A}_{f,g}$ of $\overset{(-)}{M}{}^0$ decay and about the constants $p_{H,L}$, $q_{H,L}$ of Eq. (2). Our overall framework of Eq. (18) assumes the WWA property of the closed nature of the $M^0\overline{M}^0$ system; our purpose was to see the successive consequences of the three pieces of the WWA. The tests (23), (24), (25) involve checking the channel independence of the observed rates, successively for WWA1, WWA2 and WWA3. The last of these, Eq. (25), tests also the exponential decay law for any choice of $f = g$ and M^0. The same holds for the three tests (26), (27), (28). All these, at present, are feasible for the choice $M^0 = K^0$. The first set, viz. (23), (24), (25), is further restricted to the comparison of the choices $\pi\pi$ and special cases of $\pi\pi\pi$ for $f = g$. Hopefully, such tests can be performed soon. For the other choices of M^0, one has to wait for the future.

References

[1] V. Weisskopf, E. Wigner, Z. Phys. 63 (1930) 54;
V. Weisskopf, E. Wigner, Z. Phys. 65 (1930) 18.

[2] C. Bernardini, L. Maiani, M. Testa, Phys. Rev. Lett. 71 (1993) 2687;
P. Facchi, S. Pascazio, Phys. Lett. A 241 (1998) 139.

[3] E.B. Norman, S.B. Gazes, S.G. Crane, D.A. Bennet, Phys. Rev. Lett. 60 (1988) 2246.

[4] M.C. Fischer, B. Gutiérrez-Medina, M.G. Raizen, Phys. Rev. Lett. 87 (2001) 040402.

[5] M. Gell-Mann, A. Pais, Phys. Rev. 97 (1955) 1387;
T.D. Lee, R. Oehme, C.N. Yang, Phys. Rev. 106 (1957) 340.

[6] P.K. Kabir, The CP Puzzle, Academic Press, London, 1968.

[7] C.B. Chiu, E.C.G. Sudarshan, B. Misra, Phys. Rev. D 16 (1977) 520;
C.B. Chiu, E.C.G. Sudarshan, Phys. Rev. D 42 (1990) 3712;
E.C.G. Sudarshan, C.B. Chiu, Phys. Rev. D 47 (1993) 2602;
L.A. Khalfin, Found. Phys. 27 (1997) 1549;
M. Nowakowski, Int. J. Mod. Phys. A 14 (1999) 589.

[8] G.C. Branco, L. Lavoura, J.P. Silva, CP Violation, Oxford University Press, Oxford, 1999.

[9] N.W. Tanner, R.H. Dalitz, Ann. Phys. (N.Y.) 171 (1986) 463;
M. Kobayashi, A.I. Sanda, Phys. Rev. Lett. 69 (1992) 3139;
M. Hayakawa, A.I. Sanda, Phys. Rev. D 48 (1993) 1150.

[10] L. Lavoura, J.P. Silva, Phys. Rev. D 60 (1999) 056003.

[11] L. Maiani, N. Paver, in: L. Maiani, G. Pancheri, N. Paver (Eds.), The Second DaΦne Physics Handbook, Vol. I, SIS-Pubblicazioni dei Laboratori di Frascati, Italy, 1995, p. 239;
G. D'Ambrosio, G. Ecker, G. Isidori, H. Neufeld, Z. Phys. C 76 (1997) 301.

[12] Z.-Z. Xing, Phys. Rev. D 53 (1996) 204.

[13] C.P. Enz, R.R. Lewis, Helv. Phys. Acta 38 (1965) 860.

[14] L. Wolfenstein, Nuovo Cimento A 63 (1969) 269.

[15] P.H. Eberhard, in: L. Maiani, G. Pancheri, N. Paver (Eds.), The Second DaΦne Physics Handbook, Vol. I, SIS-Pubblicazioni dei Laboratori di Frascati, Italy, 1995, p. 99.

[16] BELLE Collaboration, A. Abashian et al., Phys. Rev. Lett. 86 (2001) 2509;
BABAR Collaboration, B. Aubert et al., hep-ex/0107013.

[17] I. Dunietz, T. Nakada, Z. Phys. C 36 (1987) 503;
R. Aleksan et al., Phys. Rev. D 39 (1989) 1283.