Dalitz analyses with $B \to Dh(h')$ decays at LHCb

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Abstract

We present studies performed by the LHCb experiment with beauty to open charmed meson decays $B \to Dh(h')$, $h' = \pi, K$. Using the 1 fb$^{-1}$ results of $B^+ \to (hh')(p)h^+$, 1 fb$^{-1}$ results of $B^+ \to (K^+\pi^+\pi^-)p h^+$ and 1 fb$^{-1} + 2$ fb$^{-1}$ results of model-independent Dalitz plot analysis of $B^+ \to (K_S hh)D K^+$ modes, $\gamma$ is constrained to be $(67 \pm 12)^\circ$. Further LHCb measurements with multi-body $B$ or $D$ decays are presented in the proceeding. The results include the model-dependent measurements of $B^+ \to (K_S hh)D K^+$ and the $CP$ violation measurements of $B^+ \to (K_S K\pi)D h^+$ and $B \to (hh')_D K^0$.

Keywords: $CP$ violation, CKM angle $\gamma$, Dalitz plot analysis

1. Introduction

The Cabibbo-Kobayashi-Maskawa (CKM) [1, 2] angle $\gamma = \arg[-V_{ud}V_{ub}^*/(V_{cd}V_{cb}^*)]$ is one of the least known parameters of the CKM Unitarity Triangle (UT). Precise measurement of $\gamma$ from tree-level interference is crucial to overconstrain the UT for the Standard Model test. The comparison between $\gamma$ measured from tree-level and loop-level decay amplitudes may shed light on new physics.

The angle $\gamma$ can be accessed by many channels with interference between tree-level $b \to c$ decay amplitude $A(b \to c)$ and $b \to u$ decay amplitude $A(b \to u)$ where

$$A(b \to u) / A(b \to c) = r_B e^{i\delta_B - \gamma}, \quad A(b \to \bar{u}) / A(b \to \bar{c}) = r_B e^{i\delta_B + \gamma}.$$  

$r_B$ and $\delta_B$ are the amplitude ratio and strong phase difference between the two transitions.

Several time-integrated methods to measure $\gamma$ using $B \to Dh (h = \pi, K)$ are established, where $D$ indicates the coherent mixture of $D^0$ and $\bar{D}^0$ states and $B$ is a $B^+(B^0)$ or $B^-(\bar{B}^0)$. These methods are categorised according to the decay of $D$:

1. Gronau-London-Wyler (GLW) method [3, 4]: $D$ decays to 2-body CP eigenstates, i.e. $K^+K^-$, $\pi^+\pi^-$.  

2. Atwood-Dunietz-Soni (ADS) method [5]: $D$ decays to $K\pi$ where the favoured $b \to c$ transition are with the doubly-Cabibbo-suppressed $D^0 \to K^+\pi^-$ decay, and the suppressed $b \to u$ transition are with the Cabibbo-favoured $\bar{D}^0 \to K^+\pi^-$ decay.


4. Grossman-Ligeti-Soffer (GLS) method [7]: the method is similar as ADS method, but with $D$ decays to singly-Cabibbo-suppressed three-body final states, i.e. $K_S K\pi$.

All these decay channels add sensitivity to $\gamma$ measurement and it is important to combine them for better precision. Measurements from Babar experiment and Belle experiment give their combinations of $\gamma = (69^{\pm 11}_{-17}^+) \circ$ [8] and $\gamma = (68^{+15}_{-14}) \circ$ [9] respectively. A combination of results from both experiments gives $\gamma = (67 \pm 11) \circ$ [10].

Using its 1 fb$^{-1}$ GLW and ADS measurements of $B^+ \to (hh')(p)h^+$ [11], 1 fb$^{-1}$ ADS measurement of $B^+ \to (K^+\pi^+\pi^-)p h^+$ [12] and 1 fb$^{-1}$ [13] + 2 fb$^{-1}$ [14] model-independent GGSZ results of $B^+ \to (K_S hh)D K^+$, the LHCb experiment performs the com-
bination on $\gamma$ [15]. The CLEO inputs on $D$ system parameters [16] are used to further constrain the system. Other external constraints include the $D^0$ mixing parameters [17] from the LHCb experiment and direct $CP$ violation of $D^0 \rightarrow K^+K^-$ and $D^0 \rightarrow \pi^+\pi^-$ from HFAG [18]. The 1-CL plot of $\gamma$ combination is shown in Fig. 1. The $\gamma$ is constrained to be $(67 \pm 12)^\circ$ with $\delta_B^K$

and $r_B^K$ for $B \rightarrow DK$ to be $(114.3_{-67}^{+12})^\circ$ and $0.0923^{+0.0078}_{-0.0080}$ respectively.

In the following sections, new measurements from the LHCb experiment which are not yet included in the $\gamma$ combinations are presented.

2. Model Dependent GGSZ Measurement

Besides the model-independent measurement of GGSZ analysis using external inputs from CLEO with binned Dalitz plot [16], a model-dependent analysis [19] is performed using models from previous measurements by the Babar experiment [20, 21]. Each resonant component and their interferences are explicitly described in the frame work of Dalitz plot technique. The Dalitz plot distributions of $K_S\pi^+\pi^-$ from $B^+ \rightarrow DK^\pm$ are shown in Fig. 2.

We have around 640 $B^+ \rightarrow DK^\pm$ and 8870 $B^\pm \rightarrow D\pi^\pm$ signals reconstructed based on 1 fb$^{-1}$ data. A simultaneous fit with $B$ mass and Dalitz plot distributions of $B^\pm \rightarrow DK^\pm$ and $B^\pm \rightarrow D\pi^\pm$ are performed to extract the Cartesian $CP$ violation variables defined as:

$$x_2 = \text{Re}[r_B^K e^{i\delta_B^K}], \quad y_2 = \text{Im}[r_B^K e^{i\delta_B^K}],$$

and the following results are obtained:

$$x_2 = (+2.7 \pm 4.4^{+0.8}_{-0.8} \pm 0.1)\%,$$
$$x_4 = (-8.4 \pm 4.5 \pm 0.9 \pm 0.3)\%,$$
$$y_2 = (+1.3 \pm 4.8^{+0.8}_{-0.8} \pm 0.3)\%,$$
$$y_4 = (-3.2 \pm 4.8 \pm 0.9 \pm 0.7)\%,$$

where the first uncertainty is statistical, the second is systematic and the third is systematic uncertainties due to Dalitz modelling. It results in a $\gamma$ value of $(84_{-49}^{+49})^\circ$. The measurement is in agreement with the 1 fb$^{-1}$ model-independent results [13].

3. $CP$ violation measurement with $B \rightarrow (K_SK\pi)Dh$

The first GLS analysis using singly-Cabibbo-suppressed $D$ decays $D \rightarrow K_SK\pi$ is performed using 3 fb$^{-1}$ data [22]. The decays are divided into eight categories according to the charge of $B$ meson, charge of kaon from $D$ decays and type of bachelor particle $h$ ($\pi$ or $K$). Decays with same charge for $B$ meson and kaon from $D$ decays are called Same Sign (SS) and those with opposite charge are named as Opposite Sign (OS).

The invariant mass distributions are shown in Fig. 3 ($B^+$) and Fig. 4 ($B^-$) for $B^\pm \rightarrow (K_SK\pi)_D K^\pm$ decays.
We have reconstructed around 216 signals for $B^+ \rightarrow (K_SK\pi)pK^+$ and 3108 signals for $B^+ \rightarrow (K_SK\pi)p\pi^+$. The decay widths of the eight categories are expressed as:

\[
\Gamma_{SS}^B = z[1 + r_\beta^0 r_\beta^0 + 2r_\beta r_\mu r_\kappa \cos(\delta_B \pm \gamma - \delta)],
\]

\[
\Gamma_{OS}^B = z[r_\beta^2 + r_\mu^2 + 2r_\beta r_\mu \cos(\delta_B \pm \gamma + \delta)],
\]

where $r_\beta$ value is around 0.1 for $B \rightarrow DK$ decays and 0.015 for $B \rightarrow D\pi$ decays; $r_D$ is the amplitude difference between $D^0 \rightarrow K_SK^+\pi^-$ and $D^0 \rightarrow K_S K^-\pi^+$. $\kappa$ and $\delta$ are the coherence factor and average phase over certain Dalitz plot region and are taken from CLEO [23]. The value of $r_D$, $\kappa$ and $\delta$ depend on the Dalitz plot region chosen. Two Dalitz plot regions are studied and their values are summarised in Table 1.

Table 1: $r_D$, $\delta$ and $\kappa$ parameters from CLEO measurements [23].

<table>
<thead>
<tr>
<th>$D$ Parameters</th>
<th>$r_D$ (%)</th>
<th>$\delta$</th>
<th>$\kappa$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole Dalitz Region</td>
<td>59.2 ± 4.4</td>
<td>8.3 ± 15.2</td>
<td>0.73 ± 0.08</td>
</tr>
<tr>
<td>$K^*$ region</td>
<td>35.6 ± 3.4</td>
<td>26.5 ± 15.8</td>
<td>1.00 ± 0.16</td>
</tr>
</tbody>
</table>

Seven ratios are built from the eight yields and their values are shown in Table 2 where $A_i$ indicates the charge asymmetry defined as $(N_i^- - N_i^+)/N_i^+$ and $R_i$ gives the ratios of flavour-averaged yields between two processes listed in the subscript.

Using ratios listed in Table 2 and inputs from CLEO listed in Table 1, scans of the $\chi^2$ probabilities over the $\gamma - r_D$ plane are performed for the whole Dalitz region and for the $K^*(892)^+$ region shown in Fig. 5. The results are consistent with previous LHCb combination and will be valuable for the future global fits of the CKM angle $\gamma$.

4. **CP violation studies with $B^0 \rightarrow (hh^{(*)})D\bar{K}$**

Results shown in previous sections are with the decay channels of $B \rightarrow DK$, similar analysis is also performed on self-tagged neutral $B$ decays of $B^0 \rightarrow D\bar{K}^{(*)}$. Due to the broad width of $K^{*0}(892)$, a dilution factor of $(0.95 \pm \ldots$
0.03) is needed when extracting angle $\gamma$ according to toy studies based on the Dalitz plot structure of $B^0 \to D K^0$.

Using the 3 fb$^{-1}$ data, the invariant mass distributions of the reconstructed $B^0(B_s)$ and $\bar{B}^0(B_s)$ candidates are shown in Fig. 6 and Fig. 7 respectively for different $D$ decay channels [24]. The yields of $B^0$ and $B_s^0$ are obtained through simultaneous fit to the invariant mass distributions. We have 8.6$\sigma$ and 5.8$\sigma$ observation for the GLW mode of $D \to K^+ K^-$ and $D \to \pi^+ \pi^-$. The significance of ADS mode is 2.9$\sigma$.

Twelve ratios between different yields are calculated. In the GLW modes, the relative partial decay-rate asymmetry $A_{d(s)}^{B}$ and ratio of flavour-averaged partial widths of the $B \to DK^*$ with the $D$ decaying to a $CP$-even eigenstate $R^{d(h)}$ are given:

\[
A_d^{B} = \frac{\Gamma(B \to (h^+)D\overline{K}^0) - \Gamma(B \to (h^-)D\overline{K}^0)}{\Gamma(B \to (h^+)D\overline{K}^0) + \Gamma(B \to (h^-)D\overline{K}^0)},
\]

\[
A_s^{B} = \frac{\Gamma(B \to (h^+)D\overline{K}^0) - \Gamma(B \to (h^-)D\overline{K}^0)}{\Gamma(B \to (h^+)D\overline{K}^0) + \Gamma(B \to (h^-)D\overline{K}^0)},
\]

\[
R_d^{B} = \frac{2 \times \Gamma(B \to D^{(s)}\overline{K}^0) \Gamma(B \to D_{CP}^{(s)}\overline{K}^0)}{\Gamma(B \to D^{(s)}\overline{K}^0) + \Gamma(B \to D_{CP}^{(s)}\overline{K}^0)}.
\]

The measured results for the six ratios are:

\[
A_d^{B} = -0.198^{+0.144+0.019}_{-0.145-0.020}, \quad A_s^{B} = -0.092^{+0.217+0.019}_{-0.217-0.019},
\]

\[
A_d^{B} = -0.044^{+0.073+0.019}_{-0.073-0.020}, \quad A_s^{B} = 0.064^{+0.131+0.019}_{-0.131-0.019},
\]

\[
R_d^{B} = 1.054^{+0.165+0.044}_{-0.153-0.044}, \quad R_s^{B} = 1.214^{+0.283+0.053}_{-0.252-0.053}.
\]

The first uncertainty is statistical and the second is systematic. Ratios between the flavour-averaged partial widths of $B^0$ and $B^0_s$ decays with $D \to h^+ h^-$ is defined as

\[
R_{ds}^{B} = \frac{\Gamma(B \to (h^+K^-)D\overline{K}^0) + \Gamma(B \to (h^-K^+)D\overline{K}^0)}{\Gamma(B \to (h^+K^-)D\overline{K}^0) + \Gamma(B \to (h^-K^+)D\overline{K}^0)}.
\]

The measured results are

\[
R_{ds}^{B} = 0.103^{+0.018+0.009}_{-0.016-0.009}, \quad R_{ds}^{B} = 0.147^{+0.040+0.012}_{-0.036-0.012}.
\]

For $D \to K\pi$ decay modes, the ratio of suppressed $B^0 \to D(\pi^+K^-)K^{0*}$ to favoured $B^0 \to D(K^+\pi^-)K^{0*}$ decay widths are measured separately for $B^0$ and $B^0$:

\[
R_d^{B} = \frac{\Gamma(B \to (\pi^+K^-)D\overline{K}^0)}{\Gamma(B \to (\pi^-K^+)D\overline{K}^0)} = 0.057^{+0.029+0.009}_{-0.027-0.012},
\]

\[
R_d^{B} = \frac{\Gamma(B \to (\pi^-K^+)D\overline{K}^0)}{\Gamma(B \to (\pi^+K^-)D\overline{K}^0)} = 0.056^{+0.022+0.009}_{-0.030-0.012}.
\]

The $B^0 - B^0_s$ asymmetry $A_{\gamma}^{K^*}$ with two kaons from the $D$ and the $K^{0*}$ decays having same charge is defined as:

\[
A_{\gamma}^{K^*} = \frac{\Gamma(B \to (K^-\pi^+)D\overline{K}^0) - \Gamma(B \to (K^+\pi^-)D\overline{K}^0)}{\Gamma(B \to (K^-\pi^+)D\overline{K}^0) + \Gamma(B \to (K^+\pi^-)D\overline{K}^0)}.
\]

Similar asymmetry $A_{\gamma}^{K^*}$ between $B^0_s - B^0$ with two kaons having opposite charge is written as:

\[
A_{\gamma}^{K^*} = \frac{\Gamma(B \to (K^-\pi^+)D\overline{K}^0) - \Gamma(B \to (K^+\pi^-)D\overline{K}^0)}{\Gamma(B \to (K^-\pi^+)D\overline{K}^0) + \Gamma(B \to (K^+\pi^-)D\overline{K}^0)}.
\]

The measured results are

\[
A_{\gamma}^{K^*} = -0.032^{+0.041+0.019}_{-0.041-0.020}, \quad A_{\gamma}^{K^*} = -0.014^{+0.025+0.019}_{-0.025-0.019}.
\]

The measured ratios are used to extract information for $\gamma$ measurement. The $p$-value from the profile likelihood projected over $r_B(DK^{0*})$ and $\gamma$ plane is shown in Fig. 8. The results are compatible with the LHCb combination of $\gamma$ shown in Sec. 1. The ratio of the amplitudes of the decay $B^0 \to DK^{0*}$ with $b \to u$ and $b \to c$ transitions is found to be $0.240^{+0.055}_{-0.048}$. It is compatible and more accurate than previous results from Babar [25].
Figure 6: Invariant mass distributions of \((K^+ \pi^-) p K^{*0}\), \((\pi^+ K^-) p K^{*0}\), \((K^+ K^-) p K^{*0}\) and \((\pi^+ \pi^-) p K^{*0}\) (from top to bottom). The data (black points) and the fitted model (black line) are shown. The fitted components are indicated in the legend.

Figure 7: Invariant mass distributions of \((K^+ \pi^-) p K^{*0}\), \((\pi^+ K^-) p K^{*0}\), \((K^+ K^-) p K^{*0}\) and \((\pi^+ \pi^-) p K^{*0}\) (from top to bottom). The data (black points) and the fitted model (black line) are shown. The fitted components are indicated in the legend.
5. Conclusion

The LHCb experiment has performed abundant measurements to extract CKM angle $\gamma$ using beauty to open charm decays. Using the 1 fb$^{-1}$ results of $B^+ \rightarrow (h h')_D^+ h^+$, 1 fb$^{-1}$ results of $B^+ \rightarrow (K^+\pi^+\pi^-\pi^-)_D h^+$ and 1 fb$^{-1} + 2$ fb$^{-1}$ results of model-independent Dalitz plot analysis of $B^+ \rightarrow (K_S h h)_D K^+$ modes, $\gamma$ is constrained to be $(67 \pm 12)^\circ$.

Further measurements with multi-body final states in $B$ or $D$ decays are presented, these decays include the 1 fb$^{-1}$ model-dependent GGSZ analysis, the first GLS measurement using 3 fb$^{-1}$ data and the GLW/ADS measurements using $B^0 \rightarrow DK^{*0}$ decays with 3 fb$^{-1}$ data. The $\gamma$ information extracted from these decays are compatible with the LHCb $\gamma$ combination results and will be used to further constrain angle $\gamma$.

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Figure 8: Two-dimensional projections of the $p$-value in $r_2(DK^{*0})-\gamma$ plane. The contours show the 1σ (black), 2σ (medium grey) and 3σ (light grey) likelihood. The vertical line and hatched band represent the best-fit value of $\gamma$ and the 68.3% confidence level interval shown in Sec. 1.