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A countable widely connected Hausdorff space

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Abstract

We construct a countable connected Hausdorff space in which every connected subset containing more than one point is dense. We prove that every regularly open-maximal topology of such a space also has this property, and in addition it admits no decomposition into two connected disjoint proper subsets containing more than one point.

Keywords: Countable connected; Widely connected; Biconnected; Regularly open-maximal topology

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In [1] Erdős poses the question of whether there exists a connected space such that the complement of every connected subset of it, containing more than one point, is nowhere dense in it. In order to solve this problem we first construct a countable connected Hausdorff space (S, τ) in which every connected subset containing more than one point, is dense. A connected space having this property is called widely connected (Swingle [9]). Then we consider a regularly open-maximal topology τ^* (Mioduszewski and Rudolf [7]) finer than τ and we prove that (S, τ^*) is also widely connected. Since in a regularly open-maximal topology every dense subset is open, it follows that in (S, τ^*) the complement of every connected subset containing more than one point, is closed and nowhere dense. It also follows that the intersection of every such a pair of connected subsets of (S, τ^*) is open dense. That is, the space (S, τ^*) admits no decomposition into two connected disjoint proper subsets containing more than one point. A connected space having this property is called biconnected (Kuratowski [5]). Thus the space (S, τ^*) being widely connected and biconnected answers a problem of Swingle [9] of whether a widely connected space can contain a biconnected subspace. An example of a biconnected widely connected space which is a subset of the plane has been constructed, under Continuum

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Hypothesis, by Miller in [6]. A stronger example of a biconnected space has been constructed, also under Continuum Hypothesis, by Rudin in [8]. In this example which also is a subset of the plane, the complement of every connected subset containing more than one point is at most countable. We note that in the above space (S, τ^*) the complement of every connected subset containing more than one point, is not necessarily a finite set. The problem of existence of such a space was posed by Watson in [11] and was answered by Gruenhage in [2], where he constructed a countable connected Hausdorff space under Martin's Axiom and a perfectly normal connected space under Continuum Hypothesis, in which the complement of every connected subspace containing more than one point is finite, hence nowhere dense. As far as I know, it is an open question of whether there exists a metrizable space in which the complement of every connected subset containing more than one point is nowhere dense.

1. The space S^2

For the construction of a countable widely connected space we basically use the same method as in [3] or [4]. The auxiliary space T which will be attached in every step of the construction is a countable Hausdorff space which is due to Urysohn [10]. This space contains two points a, b with the following two properties. The first is that $f(a) = f(b)$, for every continuous real valued function of T. The second is that the basic open neighbourhoods of the points a, b consist of double sequences. The way of attaching the space T is based on the transformation of a double sequence into a simple one. That is, in every step of the construction, the attaching will "follow" the function of this transformation. This way of attaching forces the final space to be widely connected.

We consider the set

$$T = \{a, b\} \cup \{a_{ij}, b_{ij}, c_i\colon\ i, j = 1, 2, \ldots\}$$

with the following topology. Every point a_{ij}, b_{ij}, is isolated. For every point c_i a basis of open neighbourhoods is the collection of sets

$$O_n(c_i) = \{c_i\} \cup \{a_{ij}, b_{ij}\colon\ j \geqslant n\}, \quad n = 1, 2, \ldots.$$

For the point a a basis of open neighbourhoods is the collection of sets

$$O_n(a) = \{a\} \cup \{a_{ij}\colon\ i \geqslant n,\ j = 1, 2, \ldots\}, \quad n = 1, 2, \ldots.$$

For the point b a basis of open neighbourhoods is the collection of sets

$$O_n(b) = \{b\} \cup \{b_{ij}\colon\ i \geqslant n,\ j = 1, 2, \ldots\}, \quad n = 1, 2, \ldots.$$

The space T is due to Urysohn [10]. It is a countable Hausdorff totally disconnected space having the property that $f(a) = f(b)$, for every continuous real valued function of T. The first deleted basic open neighbourhoods of the points a, b are the double sequences $\{a_{ij}\}$, $\{b_{ij}\}$ respectively.

Let $T^n(0)$, $n = 1, 2, \ldots$, be disjoint copies of T and let $x^n(0)$ be the copy of $x \in T$ in $T^n(0)$. We set $J^n(0) = T^n(0) \setminus \{a^n(0), b^n(0)\}$ and we attach the disjoint copies $J^n(0)$

to the set $\mathbb{N}$ of natural numbers identifying the point $a^n(0)$ with n and the point $b^n(0)$ with $n+1$. On the set

$$S^1 = \mathbb{N} \cup \bigcup_{n=1}^{\infty} J^n(0)$$

we define the following topology. Every point $a_{ij}^k(0)$, $b_{ij}^k(0)$, is isolated. For every point $c_i^k(0)$ a basis of open neighbourhoods is the collection of sets

$$O_n^1\left(c_i^k(0)\right) = \left\{c_i^k(0)\right\} \cup \left\{a_{ij}^k(0), b_{ij}^k(0)\colon\ j \geqslant n\right\}, \quad n = 1,2,\ldots.$$

For every $k \in \mathbb{N}$ a basis of open neighbourhoods is the collection of sets

$$O_n^1(k) = \{k\} \cup \left\{a_{ij}^1(0)\colon\ i \geqslant n,\ j = 1,2,\ldots\right\}, \quad n = 1,2,\ldots,$$

if $k = 1$ and

$$O_n^1(k) = \{k\} \cup \left\{b_{ij}^{k-1}(0), a_{ij}^k(0)\colon\ i \geqslant n,\ j = 1,2,\ldots\right\}, \quad n = 1,2,\ldots,$$

if $k > 1$.

Lemma 1. *The space S^1 has the following properties:*

(1) *It is countable Hausdorff totally disconnected.*

(2) *The set of isolated points is dense.*

(3) *Every continuous real valued function of S^1 restricted to $\mathbb{N}$ is constant.*

Proof. It is obvious. □

In order to construct the space S^2 we first observe that the first deleted basic open neighbourhood of every point of the subspace $\mathbb{N}$ of S^1, except for point 1, consists of a pair of double sequences. Thus if we set

$$x_{ij}^1(0) = a_{ij}^1(0),$$

and

$$x_{ij}^n(0) = \begin{cases} b_{\frac{i}{2}j}^{n-1}(0) & \text{if } i = 2,4,\ldots, \\ a_{\frac{i+1}{2}j}^{n}(0) & \text{if } i = 1,3,\ldots, \end{cases}$$

for $n = 2,3,\ldots$, it follows that the matrix $[x_{ij}^n(0)]$ represents the first deleted basic open neighbourhood of the point n. Each matrix $[x_{ij}^n(0)]$, $n = 1,2,\ldots$, whose rows are indexed by i and the columns by j, can be transformed into the simple sequence

$$x_{11}^n,\ x_{21}^n,\ x_{12}^n,\ x_{13}^n,\ldots$$

which can be considered as the nth row of a matrix $[y_k^n(0)]$ whose rows are indexed by n and the columns by k. Therefore the set

$$\bigcup_{n=1}^{\infty} \left(J^n(0) \setminus \left\{c_i^n(0)\colon\ i = 1,2,\ldots\right\}\right)$$

of isolated points of S^1 is represented by the matrix $[y_k^n(0)]$, whose nth row is now the first deleted basic open neighbourhood of the point n.

Let f_1 be the one-to-one function transforming the double sequence $[y_k^n(0)]$ into the simple sequence

$$y_1^1(0),\ y_1^2(0),\ y_2^1(0),\ y_3^1(0), \dots .$$

Let also, $T^n(1)$, $n = 1, 2, \dots$, be disjoint copies of the initial space T and let $x^n(1)$ be the copy of $x \in T$ in $T^n(1)$. We set $J^n(1) = T^n(1) \setminus \{a^n(1), b^n(1)\}$ and "following" the function f_1 we attach the copies $J^n(1)$ to S^1 identifying the point $a^n(1)$ with $f_1(n)$ and the point $b^n(1)$ with $f_1(n+1)$. Thus to every point $f_1(n)$, $n = 2, 3, \dots$, we attach the two copies $J^{n-1}(1)$ and $J^n(1)$, whereas to the point $f_1(1)$ we only attach the copy $J^1(1)$. Each copy is attached to different rows except for the copies

$$J^{a_{n+1}}(1), \quad a_{n+1} = a_n + 4n + 3,\ n = 1, 2, \dots,\ a_1 = 3$$

which are attached only to the first row.

On the set

$$S^2 = S^1 \cup \bigcup_{n=1}^{\infty} J^n(1)$$

we define the following topology. Every point $a_{ij}^k(1)$, $b_{ij}^k(1)$ is isolated. For every point $c_i^k(1)$ a basis of open neighbourhoods is the collection of sets

$$O_n^1\big(c_i^k(1)\big) = \big\{c_i^k(1)\big\} \cup \big\{a_{ij}^k(1), b_{ij}^k(1)\colon\ j \geqslant n\big\}, \quad n = 1, 2, \dots .$$

If x is an isolated point of S^1 then a basis of open neighbourhoods of x is the collection of sets

$$O_n^2(x) = \{x\} \cup \big\{a_{ij}^1(1)\colon\ i \geqslant n,\ j = 1, 2, \dots\big\}, \quad n = 1, 2, \dots,$$

if $f_1(1) = x$, and

$$O_n^2(x) = \{x\} \cup \big\{b_{ij}^{k-1}(1), a_{ij}^k(1)\colon\ i \geqslant n,\ j = 1, 2, \dots\big\}, \quad n = 1, 2, \dots,$$

if $f_1(k) = x$, $k > 1$. For every point $c_i^k(0)$ a basis of open neighbourhoods is the collection of sets

$$O_n^2\big(c_i^k(0)\big) = O_n^1\big(c_i^k(0)\big) \cup \bigcup_{f_1(t) \in O_n^1(c_i^k(0))} O_n^1\big(f_1(t)\big), \quad n = 1, 2, \dots .$$

For every $k \in \mathbb{N}$ a basis of open neighbourhoods is the collection of sets

$$O_n^2(k) = O_n^1(k) \cup \bigcup_{f_1(t) \in O_n^1(k)} O_n^1\big(f_1(t)\big).$$

Lemma 2. *The space S^2 has the following properties:*

(1) *It is countable Hausdorff totally disconnected.*

(2) *The set of isolated points is dense.*

(3) *Every continuous real valued function of S^2 restricted to S^1 is constant.*

(4) *If* $n-1$, n, $n+1$, $n=2,3,\ldots$, *are (successive) points of* $\mathbb{N}$ *then for every pair of basic open neighbourhoods* $O^1_{l_1}(n-1)$ *and* $O^1_{l_2}(n+1)$ *in* S^1, *of the points* $n-1$ *and* $n+1$, *respectively, there exists a basic open neighbourhood* $O^1_l(n)$ *in* S^1, *of the point* n *having the property that if* $f_1(t) \in O^1_l(n)$ *then*

$$f_1(t-1), f_1(t+1) \in O^1_{l_1}(n-1) \cup O^1_{l_2}(n+1).$$

Proof. Properties (1)–(3) are obvious. We prove property (4).

Let t be a natural number such that $f_1(t)$ belongs to some basic open neighbourhood of the point n. Hence $f_1(t)$ is a point of the nth row of the matrix $[y^n_k(0)]$, and hence $f_1(t) = y^n_k(0)$, for some n, k. Let $x^n_{ij}(0)$ be the corresponding position (entry) of $y^n_k(0)$ in the matrix $[x^n_{ij}(0)]$. The position of the points $f_1(t+1)$ and $f_1(t-1)$ in $[y^n_k(0)]$ depends on the direction of the function f_1 in $[y^n_k(0)]$. That is, if $f_1(t+1)$ belongs to $(n-1)$th row then the point $f_1(t+1) = y^{n-1}_{k+1}(0)$ corresponds to the point $x^{n-1}_{(i-1)(j+1)}(0)$.

If $f_1(t+1)$ belongs to $(n+1)$th row then the point $f_1(t+1) = y^{n+1}_{k-1}(0)$ corresponds to the point $x^{n+1}_{(i+1)(j-1)}(0)$.

If $f_1(t-1)$ belongs to $(n-1)$th row then the point $f_1(t-1) = y^{n-1}_{k+1}(0)$ corresponds to the point $x^{n-1}_{(i-1)(j+1)}(0)$.

If $f_1(t-1)$ belongs to $(n+1)$th row then the point $f_1(t-1) = y^{n+1}_{k-1}(0)$ corresponds to the point $x^{n+1}_{(i+1)(j-1)}(0)$.

Therefore in all cases, if $O_{l_1}(n-1)$ and $O_{l_2}(n+1)$ are basic open neighbourhoods of the points $n-1$ and $n+1$ respectively, then $O_l(n)$ for $l < \min\{l_1, l_2\} - 1$ is the required basic open neighbourhood of the point n. $\square$

2. The space S

In order to construct the final space S we first construct by induction the space S^{m+2}. For this, on the space $S^{m+1} = S^m \cup \bigcup_{n=1}^{\infty} J^n(m)$ we consider the subset $\bigcup_{n=1}^{\infty}(J^n(m) \setminus \{c^n_i(m)\colon i = 1,2,\ldots\})$ of isolated points. We transform this subset into the sequence of matrices $[x^{f_m(n)}_{ij}(m)]$, $n = 1,2,\ldots$, the nth term of which represents the first deleted basic open neighbourhood of the point $f_m(n)$ in S^{m+1}. We transform each nth term into the simple sequence

$$x^{f_m(n)}_{11}(m),\ x^{f_m(n)}_{21}(m),\ x^{f_m(n)}_{12}(m),\ x^{f_m(n)}_{13}(m),\ldots,$$

and we consider it as the nth row of a matrix $[y^{f_m(n)}_k(m)]$ whose rows are indexed by n and the columns by k, and whose nth row is now the first deleted basic open neighbourhood of the point $f_m(n)$ in S^{m+1}.

Let f_{m+1} be the one-to-one function transforming the double sequence $[y^{f_m(n)}_k(m)]$ into the simple sequence

$$y^{f_m(1)}_1(m),\ y^{f_m(2)}_1(m),\ y^{f_m(1)}_2(m),\ y^{f_m(1)}_3(m),\ldots.$$

Let also $T^n(m+1)$, $n = 1, 2 \ldots$ be disjoint copies of the initial space T and let $x^n(m+1)$ be the copy of $x \in T$ in $T^n(m+1)$. We set

$$J^n(m+1) = T^n(m+1) \setminus \{a^n(m+1), b^n(m+1)\}$$

and "following" the function f_{m+1} we attach the copies $J^n(m+1)$ to S^{m+1} identifying the point $a^n(m+1)$ with $f_{m+1}(n)$ and the point $b^n(m+1)$ with $f_{m+1}(n+1)$. Thus to every point $f_{m+1}(n)$, $n = 2, 3, \ldots$, we attach the two copies $J^{n-1}(m+1)$ and $J^n(m+1)$, whereas to the point $f_{m+1}(1)$ we only attach the copy $J^1(m+1)$. Each copy is attached to different rows except for the copies

$$J^{a_{n+1}}(m+1), \quad a_{n+1} = a_n + 4n + 3,\ n = 1, 2, \ldots,\ a_1 = 3,$$

which are attached only to the first row.

Finally we set $S^{m+2} = S^{m+1} \cup \bigcup_{n=1}^{\infty} J^n(m+1)$ and on the set S^{m+2} we define by induction the following topology. Every point $a^k_{ij}(m+1), b^k_{ij}(m+1)$, is isolated. For every point $c^k_i(m+1)$ a basis of open neighbourhoods is the collection of sets

$$O^1_n\big(c^k_i(m+1)\big) = \{c^k_i(m+1)\} \cup \{a^k_{ij}(m+1), b^k_{ij}(m+1)\colon\ j \geqslant n\},$$
$$n = 1, 2, \ldots .$$

If $x \in S^{m+1} \setminus S^n$ then a basis of open neighbourhoods of x is the collection of sets

$$O^2_n(x) = O^1_n(x) \cup \bigcup_{f_{m+1}(t) \in O^1_n(x)} O^1\big(f_{m+1}(t)\big),$$

$n = 1, 2, \ldots$, where $O^1_n(x)$ is a basis of open neighbourhoods of x in S^{m+1}. If $x \in S^m$ let k be the minimal natural number for which $x \in S^k$. Then a basis of open neighbourhoods of x is the collection of sets

$$O^{m+2-k}_n(x) = O^{m+1-k}_n(x) \cup \bigcup_{f_{m+1}(t) \in O^{m+1-k}_n(x)} O^1_n\big(f_{m+1}(t)\big), \quad n = 1, 2, \ldots .$$

The final space is the set $S = \bigcup_{m=1}^{\infty} S^m$ with the following topology. Let $x \in S$ and let m be the minimal natural number for which $x \in S^m$. If k is a natural number greater than m and if $O^{k-m}_n(x)$, $n = 1, 2, \ldots$, is a basis of open neighbourhoods of x in the space S^{m+k} then a basis of open neighbourhoods of x in S is the collection of sets

$$O_n(x) = \bigcup_{k=m+1}^{\infty} O^{k-m}_n(x), \quad n = 1, 2, \ldots .$$

Remark. The topology on S is a modification of the topology of $I(X)$ in [3]. Comparing the two topologies, it is not difficult to prove that the topology on S is strictly finer. By definition the topology on S is first countable while the topology in [4] is nowhere first countable [4, Theorem 1.2.5(c)].

Lemma 3. *The space S^{m+2} has the following properties:*

(1) *It is countable Hausdorff totally disconnected.*

(2) *The set of isolated points is dense.*

(3) *Every continuous real valued function of* S^{m+2} *restricted to* S^{m+1} *is constant.*

(4) *If* $f_m(n-1)$, $f_m(n)$, $f_m(n+1)$, $n = 2,3,\dots$, *are (successive) points of* S^m *then for every basic open neighbourhoods* $O^1_{l_1}(f_m(n-1))$ *and* $O^1_{l_2}(f_m(n+1))$ *in* S^{m+1}, *of the points* $f_m(n-1)$ *and* $f_m(n+1)$, *respectively, there exists a basic open neighbourhood* $O^1_l(f_m(n))$ *in* S^{m+1}, *of the point* $f_m(n)$, *having the property that if* t *is a natural number such that* $f_{m+1}(t) \in O^1_l(f_m(n))$ *then*

$$f_{m+1}(t-1), f_m(t+1) \in O^1_{l_1}(f_m(n-1)) \cup O^1_{l_2}(f_m(n+1)).$$

Proof. Properties (1)–(3) are obvious. The proof of (4) is similar to that of (4) of Lemma 2. □

Proposition 1. *The space* S *is countable Hausdorff widely connected.*

Proof. Obviously the space S is countable Hausdorff. That it is connected follows from property (3) of Lemma 3.

Let M be a connected subset of S containing more than one point. By the definition of topology on S it follows that the subspace $C = \{c^n_i(k)\colon i,\ n = 1,2,\dots,\ k = 0,1,\dots\}$ is discrete. Hence there exists $x \in M$ and $x \notin C$. Let m be the minimal natural number for which $x \in S^m$. Then x is an isolated point of S^m and therefore there exists a natural number $k > 1$ for which $f_m(k) = x$. Obviously if $k = 1$ for every $m = 1,2,\dots$, then M is discrete. We consider the points $y = f_m(k-1)$, $z = f_m(k+1)$ and we suppose that, $y, z \notin \mathrm{Cl}_S M$. Then there exist basic open neighbourhoods $O_{l_1}(y)$, $O_{l_2}(z)$ of the points y, z respectively, not intersecting $\mathrm{Cl}_S M$. The points y, x, z are successive and therefore by property (4) of Lemmas 2 and 3, and by the definition of topology on S, we can find for $n < \min\{l_1, l_2\}$, a basic open neighbourhood $O_n(x)$ of x having the property that if m, t is any pair of natural numbers such that $f_m(t) \in O_n(x)$ then $f_m(t-1)$, $f_m(t+1) \in O_{l_1}(y) \cup O_{l_2}(z)$. But then for the set

$$E(O_n(x)) = O_n(x) \cup \bigcup_{c \in \mathrm{Cl}\, O_n(x) \setminus O_n(x)} O_n(c)$$

it holds that $E(O_n(x)) \cap \mathrm{Cl}_S M$ is open and closed in $\mathrm{Cl}_S M$ for every $n < \min\{l_1, l_2\}$ which is impossible. Therefore either $y \in \mathrm{Cl}_S M$ or $z \in \mathrm{Cl}_S M$. Continuing in this manner first for the points $f_m(k+i), i = 2,3,\dots$, and then for the points $f_m(k-i)$, $i = 2,3,\dots,k-2$, we first conclude that all but finitely many isolated points of S^m belong to $\mathrm{Cl}_S M$, and then that every isolated point of S^m belongs to $\mathrm{Cl}_S M$. Hence every point of S^m belongs to $\mathrm{Cl}_S M$ because the set of isolated points is dense in S^m (Lemma 3). Since the space S^m is totally disconnected (Lemma 3) it follows that there exists a point of M belonging to $S \setminus S^m$ not belonging to C and not being of the form $f_l(1)$, $l > m$. This fact finally implies that $\mathrm{Cl}_S M = S$. □

Proposition 2. *Let* (S, τ) *be a Hausdorff widely connected space and* τ^* *be a regularly open-maximal topology finer than* τ. *Then the space* (S, τ^*) *is Hausdorff widely connected biconnected.*

Proof. Obviously the space (S,τ^*) is Hausdorff connected. We prove that if M is a connected subset containing more than one point then M is dense in τ^*. Suppose not. Then there exists an open set U in τ^* such that $U \cap \mathrm{Cl}_{\tau^*} M = \emptyset$. Hence $\mathrm{Cl}_{\tau^*} U \cap \mathrm{Int}_{\tau^*} \mathrm{Cl}_{\tau^*} M = \emptyset$. Observe that $\mathrm{Int}_{\tau^*} \mathrm{Cl}_{\tau^*} M \neq \emptyset$, for if it is empty then $\mathrm{Cl}_{\tau^*} M$ is a boundary set in τ^*. Hence by [7, 2.3 of Section 2] it is a discrete subspace of (S,τ^*) and is therefore not connected. By [7, 1.3, 1.1(2) and 1.1(3) of Section 1] for the set U there exists an open set V in τ such that $U \subseteq V$ and $\mathrm{Cl}_{\tau^*} U = \mathrm{Cl}_{\tau^*} V$. Consequently $V \cap \mathrm{Int}_{\tau^*} \mathrm{Cl}_{\tau^*} M = \emptyset$, while $V \cap M \neq \emptyset$ because M is connected in τ, hence dense in τ. Since the subspace $\mathrm{Cl}_{\tau^*} M \setminus \mathrm{Int}_{\tau^*} \mathrm{Cl}_{\tau^*} M$ is discrete in τ^* it follows that the set $V \cap \mathrm{Cl}_{\tau^*} M$ is open and closed in $\mathrm{Cl}_{\tau^*} M$ which is impossible. Therefore M is dense.

The space (S,τ^*) is biconnected because if M, N are connected subsets containing more than one point then both M, N are dense and hence open, by [7, 2.2 of Section 2]. Therefore $M \cap N \neq \emptyset$. $\square$

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References

[1] P. Erdős, Some remarks on connected sets, Bull. Amer. Math. Soc. 50 (1944) 442–446.
[2] G. Gruenhage, Spaces in which the non-degenerate connected sets are the cofinite sets, to appear.
[3] S. Iliadis and V. Tzannes, Spaces on which every continuous map into a given space is constant, Canad. J. Math. 38 (1986) 1281–1296.
[4] V. Kannan and M. Rajagopalan, Constructions and applications of Rigid spaces I, Adv. Math. 29 (1978) 89–130.
[5] K. Kuratowski, Topology (Academic Press, New York, 1966).
[6] E.W. Miller, Concerning biconnected sets, Fund. Math. 24 (1937) 123–133.
[7] J. Mioduszewski and L. Rudolf, H-closed and extremally disconnected Hausdorff spaces, Dissertationes Math. 66 (1969).
[8] M.E. Rudin, A connected subset of the plane, Fund. Math. 46 (1958) 15–24.
[9] P.M. Swingle, Two types of connected sets, Bull. Amer. Math. Soc. 37 (1931) 254–258.
[10] P. Urysohn, Über die Mächtigkeit der zusammenhängenden Mengen, Math. Ann. 94 (1925) 262–295.
[11] S. Watson, Problems I wish I could solve, in: J. van Mill and G.M. Reed, eds., Open Problems in Topology (North-Holland, Amsterdam, 1990).