A Method for Identification of Machine-tool Dynamics under Machining

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Abstract

Dynamic properties of machine-tool structures are likely to change under machining conditions. Thus, the dynamic parameters obtained by traditional experimental modal analysis in the static state may not characterize accurately the dynamics of the machine tool structure in operation. This paper proposes a new method of so-called AEMA (Active Excitation Modal Analysis) to identify the dynamic modal parameters of a machine tool structure during machining. A random cutting excitation technique realized by cutting a specially designed workpiece is proposed to provide strong and evenly distributed excitation within the frequency range of interest. The surface of the workpiece has a long narrow random zigzag width, which randomizes the resulting cutting forces. The LSCE (Least Square Complex Exponential) method is employed to estimate the modal parameters from just the measured responses. Then an algorithm based on two novel tools, the harmonic frequency fence and the spectrum abruptness ratio, is presented to eliminate the harmonic modes attributed to AC power and rotation frequency. The abruptness ratio is used to detect the basic frequency, and then the fence filters out the harmonic modes caused by peaks at integer multiples of the basic frequency through narrow frequency fence slots followed by a damping ratio limit. Finally, the proposed AEMA method is experimentally validated and shows satisfactory results.

Keywords: Dynamics; Machine tool; Modal analysis

1. Introduction

Estimation of the dynamics of a machine tool structure is generally done by forced vibration tests (FVTs), namely impact or shaker tests, with experimental modal analysis (EMA) when the machine tool is at rest. However, significant changes in dynamics are expected to occur due to spindle rotation and changes of boundary conditions between the resting state and the machining state [1,2,3]. Thus, the static results of EMA may fail to characterize the machine-tool dynamics during machining accurately. Therefore, there are limits to the application of EMA in identifying the dynamics of machine tools during machining.

Operational modal analysis or output-only modal analysis (OMA) is a powerful tool for the identification of dynamic modal parameters in ambient vibration tests (AVTs) in the case of civil engineering structures. Since the artificial excitation produced by impact hammers or heavy shakers is replaced by freely available ambient forces, AVTs are much more practical and economical. Furthermore, as structures are characterized under real operating conditions, the identified results are associated with realistic states of vibration rather than with artificially generated vibration states, as is the case when FVTs are used [4]. However, the ambient excitation is too low to excite the machine tool effectively. Minis [5] proposed a technique that provides a strong, broadband excitation by interrupted cutting of pseudo-randomly distributed teeth and channels; however, the input cutting forces must be measured in order to identify structure dynamics.

In order to improve the excitation, Bin Li et al. developed so-called active experimental modal analysis (AEMA) methods to excite the structure by the machine tool itself based on OMA. One excitation technique [6] is realized by the interrupted cutting of a narrow workpiece step while the spindle rotates randomly. The resultant cutting force, like random pulses, excites the structure effectively within the frequency range of interest. Another technique [7,8] uses the
inertia force caused by random running of the worktable to excite the structure. However, both techniques which run under special random conditions, are still different from normal machining performance. Zaghibani and Songmene [3] first applied OMA to machine tools under normal milling operations. However, natural frequencies are quite difficult to distinguish from tooth-passing frequencies and their harmonics. This paper proposes a new AEMA method to extract machine tool dynamic parameters during normal machining. A novel random cutting excitation technique is presented to provide strong, broadband excitation, after which an algorithm is developed to eliminate the harmonic modes attributed to AC power and rotation frequencies. The proposed AEMA method is experimentally validated and shows satisfactory results.

2. Background of AEMA

Although OMA is a powerful tool for online identification of structural dynamics, the input excitation, often consisting of ambient forces, may be too low to excite all the modes of interest of the mechanical equipment. The key of the active experimental modal analysis (AEMA) methods is to excite the mechanical structure by itself to perform OMA. The AEMA methods make use of the working load of the structure rather than the ambient forces to cause excitation. Then the dynamic parameters can be estimated from output responses like OMA. The load can be the inertial forces of the moving components, the impact forces between components, etc. Since the excitation is created by the structure itself, the input can be controlled actively. Thus, in addition to the advantages of OMA, AEMA can be used to intentionally adjust the energy level and effective frequency range of the excitation, which results in an improved signal-to-noise ratio. It is especially applicable for automatically controlled equipment, like the widely used CNC machine tools. In such cases, the inertial forces of the slider or the cutting forces between the tool and the workpiece can both be a good source of excitation.

Fig. 1 is the schematic diagram of the inertial force excitation technique in [7,8]: the slider accelerates or decelerates randomly and continuously in a small area. The inertial forces, as shown in Fig.1b, are proven to be square-wave pulses in [9]. Fig. 2 shows the AEMA method that employs cutting forces to excite the structure in [6]: the tool rotates randomly to cut a step of the workpiece symmetrically. However, both methods run under special random conditions rather than normal machining operations. For this reason, alternatives have to be investigated to excite the machine tool structure during normal operation to apply AEMA.

3. New random cutting excitation technique

This section introduces a new random cutting excitation technique based on the normal cutting of a workpiece with a long narrow random zigzag width, resulting in random impulse-like cutting forces. Fig.3 shows the schematic diagram of this technique; the random narrow width can be stretched into a random polyline or random spline curve. The tool rotates while the worktable feeds constantly to cut the random-going width; therefore, the machine runs normally while randomizing the cutting forces. Fig.4 shows the model of the random impulse excitation signal.
The above signal of the cutting impulses can be described by a mathematical function $f_i(t)$, where

$$f_i(t) = \begin{cases} \frac{A_i}{\omega i} e^{-j\omega_i t} & \text{for } t \in [t_i, t_i + \tau_i] \\ 0 & \text{for } t \notin [t_i, t_i + \tau_i] \end{cases}$$

where $t_i$ (for $i = 0, 1, 2, ..., n$) is the start moment of the $i$th impulse, $\tau_i$ (for $i = 1, 2, ..., n$) is the time duration of that impulse, $A_i$ is its amplitude, $\Delta t_i$ is the duration time between adjacent impulses $i$ and $j$, and $n$ is the number of impulses included in one measurement, which can be represented by the number of impulses per unit time (referred to as density $n$). In general, $A_i$, $\tau_i$, and $\Delta t_i$ are all random variables that determine the random characteristics of the signal. The spectrum of $f_i(t)$ is

$$F_n(\omega) = \int_{t_1}^{t_n} f_i(t) e^{j\omega t} dt = \sum_{i=1}^{n} \frac{A_i}{\omega i} e^{-j\omega_i t} \left( e^{j\omega t} - 1 \right)$$

(2)

According to the Euler formula, $e^{-j\omega t} = \cos(-\omega t) + j\sin(-\omega t) = -\cos\omega t - j\sin\omega t$, and remembering that $\cos\omega t = 1 - 2\sin^2(\omega t/2)$, $\sin\omega t = 2\sin(\omega t/2)\cos(\omega t/2)$, So equation (2) becomes

$$F_n(\omega) = \sum_{i=1}^{n} \frac{2A_i}{\omega i} \sin\frac{\omega_i t}{2} e^{-j\left[1-\frac{\omega_i^2}{\omega^2}\right]t}$$

(3)

Letting $u_i = -\omega(t_i + t_0/2)$ and $v_i = \sin(\omega t_0/2)$, and remembering $\omega = 2\pi f$, the power spectral density (PSD) $G_n(\omega)$ of $f_i(t)$ is

$$G_n(f) = F_n^*(f)F_n(f) = \left( \frac{1}{\tau f} \right)^2 \sum_{i=1}^{n} A_i v_i e^{j\omega_i} \times \sum_{i=1}^{n} A_i v_i e^{-j\omega_i}$$

(4)

Expanding equation (4),

$$G_n(f) = \sum_{i=1}^{n} (A_i v_i)^2 + \sum_{i=1}^{n} \sum_{j=i}^{n} A_i A_j v_i v_j \cos(u_i - u_j)$$

(5)

It can be seen that $G_n(f)$ is probably proportional to the square of the impulse amplitude $A_i$, and to the number of impulses $n$ (or impulse density $n$). It should be noted that the zeros of $v_i$ are $f = k\omega_0$ ($k = 0, \pm1, \pm2, ...$). If $t_i$ has a Gaussian distribution (with mean $\mu$ and standard deviation $\sigma$), most of its values will fall into the range $[\mu \pm 3\sigma]$. Moreover, if $\mu$ is far larger than $\sigma$, most values of $t_i$ will be close to $\mu$ leading to $t_i = t_j = \mu$ ($i \neq j$). Thus, the bandwidth of the first spectral lobe (the first zero) of $G_n(f)$, referred to as $BW_{1st}$, approximates the inverse of the mean $\mu$.

$$BW_{1st} = \frac{1}{\mu}$$

(6)

The force amplitude $A_i$ is assumed to be proportional to the shear area (referred to as $A_i$). In this case, $A_i$ consists of two phases similar to those shown in Fig.2, the flank and the button, and is

$$A_i = \frac{a_d a_s}{\sin \alpha} + a_s T_u$$

(7)

where $a_d$ is the axial depth, $a_s$ is the width of the step, $T_u$ is the width of the blade, and $\alpha$ is the angle of tool cutting edge. The duration $\tau$ of each pulse is

$$\tau = \frac{a_d}{v} = \frac{60a_d}{\pi n D}$$

(8)

where $n$ is the speed of rotation, and $D$ is the diameter of the cutter. Then the first lobe $BW_{1st}$ of the excitation is

$$BW_{1st} = \frac{1}{\tau} = \frac{\pi n D}{60a_d}$$

(9)

Finally, Table 1 summarizes the relationship between the excitation and the cutting parameters. According to Table 1, the power $E_r$ and effective frequency range $BW_{1st}$ of the excitation can be adjusted by the cutting parameters.

<table>
<thead>
<tr>
<th>Power $E_r$</th>
<th>Band $BW_{1st}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E = A^2$</td>
<td>$A = a_d, a_s$</td>
</tr>
<tr>
<td></td>
<td>$BW_{1st} = 1/\tau$</td>
</tr>
<tr>
<td>$E = \rho$</td>
<td>$\tau = a_a, a_n$</td>
</tr>
<tr>
<td></td>
<td>$a_a, n$</td>
</tr>
</tbody>
</table>

4. Harmonic elimination algorithm

The loads of many mechanical structures are often complex in that they are typically a combination of harmonic components, originating from the rotating and reciprocating parts, and the broadband excitation, all of which contribute to the total response. During the inline measurement of the total response, the AC power may also leak into the signal acquisition system and contaminate the measurements. Therefore, the measured signals may consist of three parts – the response to broadband excitations, the response to harmonic excitations and the AC components. Here the latter two are called the harmonic components. Since both of them will create similar problems, the harmonics are identified as structural modes. For this reason, when applying AEMA to real cases, it is necessary to have some additional tools for the separation of harmonic and structural modes. Fortunately, in the PSD of such responses, the harmonic components present some properties which are useful for identification. In addition to the basic frequency, the peaks of harmonic components will exist at integer multiples of the frequency. Also, the peaks appear abruptly and vary greatly in amplitude,
which is very different from the peaks related to the structural modes. This indicates that the damping ratio related to the harmonic modes will be much smaller. To sum up, the major properties of the harmonic modes are

- The peaks appear at integer multiples of basic frequency;
- Peaks appear abruptly and vary greatly in amplitude;
- The damping ratio will be much smaller.

According to the first point, a so-called frequency fence is proposed to separate the harmonic modes from the structural modes. The frequency fence is a mode filter which allows the modes caused by peaks at integer multiples of the basic frequency \( f_0 \) to pass through narrow frequency slots \( [f_{ref} - \Delta \omega, f_{ref} + \Delta \omega] \). The frequency fence can be employed to eliminate the harmonic modes. The frequency fence is plotted on the stability diagram, which is a standard tool in modal analysis used for selection of physical modes. Since some physical modes, which are close to the harmonic modes, may also pass through these slots, a damping ratio limit \( \zeta_{min} \) is utilized to separate harmonic modes close to the physical modes. If one mode passes through the fence and its damping ratio is also smaller than \( \zeta_{min} \), it is identified as a harmonic mode. Once the basic frequency \( f_0 \) is known, the width \( = 2 \Delta \omega \) of slots determines the preciseness of the fence. If the slots are too wide, many physical modes may also pass, which will cause structural modes to be wrongly eliminated. In contrast, if the slots are too narrow, some harmonic modes will slip out, leading to additional modes. The suggested value of \( \Delta \omega \) is 0.15 Hz.

In some cases, the basic frequency is not known or even changing. Besides, there may be many basic frequencies in complex cases. Then the center (or reference) frequencies \( f_{ref} \) of the fence slots are difficult to set. Fortunately, in all cases the harmonic components will produce abrupt peaks on the PSD of the responses. Since the first-order derivative of the PSD describes the varying trend of the spectral, it will jump to a high level and show peaks at the harmonic frequencies while having the same level elsewhere. Then the first-order derivative, here called the abruptness ratio, can be used as an indicator to determine the center and harmonic frequencies. After the harmonic frequencies are identified, the frequency fence can be employed to eliminate the harmonic modes. The harmonic modes elimination algorithm, shown in Fig. 6, can be treated as a post-process after all the modes are estimated.

5. Experimental verification

5.1. Realization of the random cutting excitation

A common aluminium alloy workpiece with one zigzag narrow width prepared for excitation is shown in Fig. 7. The reference line of the zigzag width is connected by many points. All the points are evenly distributed with a distance of 5 millimetre in y direction. The distances in the x direction between each point and the centre line are produced randomly. 30 values of a random variable \( N_i \) having the uniform distribution over \([-15, 15]\) are first generated. Then 30 values between -40 and 40 for random variable \( N \) denoting the x coordinate of each point are represented by \( N \pm N_i \). The minimum of these values is -30, the maximum is 32, the mean \( \mu \) is -4.7, and the standard deviation \( \sigma \) is 18.7. Then a parallel line, 5 mm away from the reference line in the x direction, is produced to envelop the zigzag narrow step with the reference line. The step width \( a_x \) in the normal direction, namely the cutting width, varies from 1.6 to 5.1 mm. The actual cutting width may differ a little.

The proposed excitation technique is conducted by

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Fig. 5. Frequency fence with typical stability diagram.

Fig. 6. Harmonic modes elimination algorithm.

Fig. 7. Designed workpiece for excitation.
symmetrically end-milling of the narrow zigzag width while the table is feeding along the +y direction. A representative stiff face milling cutter was used during the machining. The cutter diameter \( D \) is 80 mm. Only one indexical carbide insert was secured to the cutter and engaged in the milling operation. If the frequency range of interest is 0 ~ 250 Hz, \( BW_{1st} \) should be 400 Hz. According to equation (9) \( BW_{1st} = 500, D = 80, a_w =1.6 ~ 5.3 \), and the rotation speed of the tool should be within 191 ~ 633 rpm.

5.2. Experimental setup and measurements

The proposed random cutting excitation technique has been verified on a 3-axis vertical milling center XHK5140. During the machining, the cutting forces were measured using a three-axis 9253B23 Kistler table dynamometer. Fig. 8 shows the experimental setup. Two three-axis accelerometers of type PCB 356A15 were mounted on the slide, the worktable, the base, the headstock, and the column while another accelerometer was attached to the spindle and the workpiece respectively to measure the vibrations. Three channels of the table dynamometer were replaced by another three-axis accelerometer on the tool during impact testing. The impact tests were conducted under the same experimental setup and the tool tip was tapped in \( x \) and \( y \) directions using an impact hammer of type PCB-086D05 (referred to HPCB). All the 39 signals were collected by the acquisition system LMS SCADAS Mobile SCM05 simultaneously at a sampling rate of 1024 Hz. Table 2 presents some of the tests during the experimental study.

<table>
<thead>
<tr>
<th>Case</th>
<th>Test #</th>
<th>Feed (mm/min)</th>
<th>Revolution speed (rpm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3</td>
<td>600</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>600</td>
<td>200</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>600</td>
<td>400</td>
</tr>
<tr>
<td></td>
<td>Test #</td>
<td>Tap point</td>
<td>Tap direction</td>
</tr>
<tr>
<td>B</td>
<td>15</td>
<td>Tool:1</td>
<td>( x ) and ( y )</td>
</tr>
</tbody>
</table>

5.3. Results and discussion

Fig. 9 shows the cutting force signals in \( x, y \), and \( z \) directions, namely \( F_x, F_y \), and \( F_z \) and their PSD in test #3. The PSD of the impact force, \( P_x \) and \( P_y \), in \( x \) and \( y \) directions are also presented. It can be seen that the first lobe \( BW_{1st} \) of the PSD of the cutting excitation is about 430 Hz, which is a little lower than the expected 500 Hz. The cutting forces appear to be random impulses in the time domain and demonstrate greater power than impact forces almost within the whole first lobe. The PSD of accelerations of points Y-Slide:3 and Headstock:12 of test #15 (impact excitation) and test #3 (random cutting excitation) are shown in Fig. 10 and Fig.11, respectively. Since the structural modes in all three directions are of interest while the tool tip can hardly be tapped in the \( z \) direction, only the signals of the \( x \) and \( y \) directions are presented here for comparison, while signals of all directions within the effective cutting period would later be used in the modal parameters estimation. It can be observed that the PSDs of the acceleration signals under the cutting excitation have a similar trend to the ones of the hammer impact outputs. The former is generally 10 dB higher than the latter leading to a clearer presentation of the peaks. The modes in both directions are excited well in the cutting test compared with the impact test. Of course, there are also some differences presented in the PSD, which may be attributed to the change of dynamics between the static state (impact tests) and the machining state (cutting excitation tests). Generally, the effectiveness of the proposed random cutting excitation technique is proven to be satisfactory.

Only acceleration signals were used to estimate the modal parameters of the machine tool during machining through the LSCE method. The harmonics elimination algorithm is treated as a post-process of the results of LSCE. Table 3 summarizes the parameters of the first six modes obtained from AEMA and the impact tests. It is clear that there is a great difference in the damping ratio between the machining and the rest state.

![Fig. 8. Experimental setup.](image)

![Fig. 9. (a) Random impulses cutting forces; (b) PSD of excitations.](image)
6. Conclusions

AEMA is a new method to identify the dynamics of mechanical structures, especially machine tools, while the structures are in operation. This paper proposes a new AEMA method based on cutting a specially designed workpiece to provide strong and evenly distributed excitation within the frequency range of interest. The surface of the workpiece has a long narrow random zigzag width, which randomizes the resulting cutting forces. Then an algorithm based on two novel tools, the harmonic frequency fence and the spectrum abruptness ratio, is presented to eliminate the harmonic modes attributed to AC power and rotation frequency. The frequency fence is a set of narrow frequency fence slots at integer multiples of the basic frequency. The abruptness ratio is used to detect the harmonic frequencies when they are unknown or complex. A damping ratio limit is employed to separate harmonic modes close to the physical modes. Finally, the proposed AEMA method is experimentally validated and shows satisfactory results although the robustness need to be further investigated.

This new AEMA method can be treated as an easy and economical tool to calibrate the dynamics of machine tools under machining so the results can be used in accurate prediction of chatter stability diagram etc. In the future, the authors will focus on the following study: (1) prediction of the random impulse-like cutting forces to replace measurement so the frequency response function can be estimated with only responses in OMA; (2) the receptance coupling technique together with the proposed AEMA method to estimate the FRF at the tool tip.

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References