



Holographic Chaplygin gas model

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Received 19 January 2007; received in revised form 8 March 2007; accepted 15 March 2007

Available online 20 March 2007

Editor: M. Cvetič

Abstract

In this Letter we consider a correspondence between the holographic dark energy density and Chaplygin gas energy density in FRW universe. Then we reconstruct the potential and the dynamics of the scalar field which describe the Chaplygin cosmology.

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1. Introduction

The type Ia supernova observations suggests that the universe is dominated by dark energy (DE) with negative pressure which provides the dynamical mechanism of the accelerating expansion of the universe [1–3]. The strength of this acceleration is presently matter of debate, mainly because it depends on the theoretical model implied when interpreting the data.

An approach to the problem of DE arises from the holographic principle that states that the number of degrees of freedom related directly to entropy scales with the enclosing area of the system. It was shown by 't Hooft and Susskind [4] that effective local quantum field theories greatly overcount degrees of freedom because the entropy scales extensively for an effective quantum field theory in a box of size L with UV cut-off Λ . As pointed out by [5], attempting to solve this problem, Cohen et al. showed [6] that in quantum field theory, short distance cut-off Λ is related to long distance cut-off L due to the limit set by forming a black hole. In other words the total energy of the system with size L should not exceed the mass of the same size black hole, i.e. $L^3 \rho_\Lambda \leq LM_p^2$ where ρ_Λ is the quantum zero-point energy density caused by UV cut-off Λ and M_p denotes the Planck mass ($M_p^2 = 1/8\pi G$). The largest L is required to saturate this inequality. Then its holographic energy density is given by $\rho_\Lambda = 3c^2 M_p^2 / 8\pi L^2$ in which c is a free dimensionless parameter and coefficient 3 is for convenience. As an application of the holographic principle in cosmology, it was studied

by [7] that the consequence of excluding those degrees of freedom of the system which will never be observed by the effective field theory gives rise to IR cut-off L at the future event horizon. Thus in a universe dominated by DE, the future event horizon will tend to a constant of the order H_0^{-1} , i.e. the present Hubble radius. On the basis of the cosmological state of the holographic principle, proposed by Fischler and Susskind [8], a holographic model of dark energy (HDE) has been proposed and studied widely in the literature [9,10]. In HDE, in order to determine the proper and well-behaved system's IR cut-off, there are some difficulties that must be studied carefully to get results adapted with experiments that claim our universe has accelerated expansion. For instance, in the model proposed by [9], it is discussed that considering the particle horizon, as the IR cut-off, the HDE density reads

$$\rho_\Lambda \propto a^{-2(1+\frac{1}{c})}, \quad (1)$$

that implies $w > -1/3$ which does not lead to an accelerated universe. Also it is shown in [11] that for the case of closed universe, it violates the holographic bound.

The problem of taking apparent horizon (Hubble horizon)—the outermost surface defined by the null rays which instantaneously are not expanding, $R_A = 1/H$ —as the IR cut-off in the flat universe was discussed by Hsu [12]. According to Hsu's argument, employing the Friedmann equation $\rho = 3M_p^2 H^2$ where ρ is the total energy density and taking $L = H^{-1}$ we will find $\rho_m = 3(1 - c^2)M_p^2 H^2$. Thus either ρ_m or ρ_Λ behave as H^2 . So the DE results as pressureless, since ρ_Λ scales like matter energy density ρ_m with the scale factor a as a^{-3} . Also,

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taking the apparent horizon as the IR cut-off may result in a constant parameter of state w , which is in contradiction with recent observations implying variable w [13]. On the other hand, taking the event horizon, as the IR cut-off, gives results compatible with observations for a flat universe.

In a very interesting paper Kamenshchik, Moschella, and Pasquier [14] have studied a homogeneous model based on a single fluid obeying the Chaplygin gas equation of state

$$P = \frac{-A}{\rho}, \quad (2)$$

where P and ρ are respectively pressure and energy density in comoving reference frame, with $\rho > 0$; A is a positive constant. This equation of state has raised a certain interest [15] because of its many interesting and, in some sense, intriguingly unique features. Some possible motivations for this model from the field theory points of view are investigated in [16]. The Chaplygin gas emerges as an effective fluid associated with d-branes [17] and can also be obtained from the Born–Infeld action [18].

In the present Letter, we suggest a correspondence between the holographic dark energy scenario and the Chaplygin gas dark energy model. We show this holographic description of the Chaplygin gas dark energy in FRW universe and reconstruct the potential and the dynamics of the scalar field which describe the Chaplygin cosmology.

2. Chaplygin gas as holographic dark energy

Here we consider the Friedmann–Robertson–Walker universe with line element

$$ds^2 = -dt^2 + a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right), \quad (3)$$

where k denotes the curvature of space $k = 0, 1, -1$ for flat, closed and open universe, respectively. A closed universe with a small positive curvature ($\Omega_k \sim 0.01$) is compatible with observations [19,20]. We use the Friedmann equation to relate the curvature of the universe to the energy density. The first Friedmann equation is given by

$$H^2 + \frac{k}{a^2} = \frac{1}{3M_p^2} [\rho_\Lambda + \rho_m]. \quad (4)$$

Define as usual

$$\Omega_m = \frac{\rho_m}{\rho_{\text{cr}}} = \frac{\rho_m}{3M_p^2 H^2}, \quad \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{\text{cr}}} = \frac{\rho_\Lambda}{3M_p^2 H^2},$$

$$\Omega_k = \frac{k}{a^2 H^2}. \quad (5)$$

Inserting the equation of state (2) into the relativistic energy conservation equation, leads to a density evolving as

$$\rho_\Lambda = \sqrt{A + \frac{B}{a^6}}, \quad (6)$$

where B is an integration constant.

Now following [21] we assume that the origin of the dark energy is a scalar field ϕ , so

$$\rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi) = \sqrt{A + \frac{B}{a^6}}, \quad (7)$$

$$P_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi) = \frac{-A}{\sqrt{A + \frac{B}{a^6}}}. \quad (8)$$

Then, one can easily derive the scalar potential and kinetic energy term as

$$V(\phi) = \frac{2a^6(A + \frac{B}{a^6}) - B}{2a^6 \sqrt{A + \frac{B}{a^6}}}, \quad (9)$$

$$\dot{\phi}^2 = \frac{B}{a^6 \sqrt{A + \frac{B}{a^6}}}. \quad (10)$$

Now we suggest a correspondence between the holographic dark energy scenario and the Chaplygin gas dark energy model. In non-flat universe, our choice for holographic dark energy density is

$$\rho_\Lambda = 3c^2 M_p^2 L^{-2}. \quad (11)$$

As it was mentioned, c is a positive constant in holographic model of dark energy ($c \geq 1$) and the coefficient 3 is for convenient. L is defined as the following form:

$$L = ar(t), \quad (12)$$

here a is scale factor and $r(t)$ is relevant to the future event horizon of the universe. Given the fact that

$$\int_0^{r_1} \frac{dr}{\sqrt{1 - kr^2}} = \frac{1}{\sqrt{|k|}} \text{sinn}^{-1}(\sqrt{|k|}r_1)$$

$$= \begin{cases} \sin^{-1}(\sqrt{|k|}r_1)/\sqrt{|k|}, & k = 1, \\ r_1, & k = 0, \\ \sinh^{-1}(\sqrt{|k|}r_1)/\sqrt{|k|}, & k = -1, \end{cases} \quad (13)$$

one can easily derive

$$L = \frac{a(t) \text{sinn}[\sqrt{|k|}R_h(t)/a(t)]}{\sqrt{|k|}}, \quad (14)$$

where R_h is event horizon. Therefore while R_h is the radial size of the event horizon measured in the r direction, L is the radius of the event horizon measured on the sphere of the horizon.¹

¹ As I have discussed in introduction, in non-flat case the event horizon cannot be considered as the system's IR cut-off, because if we use R_h as IR cut-off, the holographic dark energy density is given by

$$\rho_\Lambda = 3c^2 M_p^2 R_h^{-2}. \quad (15)$$

When there is only dark energy and the curvature, $\Omega_\Lambda = 1 + \Omega_k$, and $c = 1$, we find [22]

$$\dot{R}_h = \frac{1}{\sqrt{\Omega_\Lambda}} - 1 = \frac{1}{\sqrt{1 + \Omega_k}} - 1 < 0, \quad (16)$$

Since we have

$$\frac{\Omega_k}{\Omega_m} = a \frac{\Omega_{k0}}{\Omega_{m0}} = a\gamma, \quad (17)$$

where $\gamma = \Omega_{k0}/\Omega_{m0}$, we get $\Omega_k = \Omega_m a\gamma$ and

$$\Omega_m = \frac{1 - \Omega_\Lambda}{1 - a\gamma}. \quad (18)$$

Hence, from the above equation, we get

$$\frac{1}{aH} = \frac{1}{H_0} \sqrt{\frac{a(1 - \Omega_\Lambda)}{\Omega_{m0}(1 - a\gamma)}}. \quad (19)$$

Combining Eqs. (14) and (19), and using the definition of Ω_Λ , we obtain

$$\begin{aligned} \sqrt{|k|} \frac{R_h}{a} &= \text{sinn}^{-1} \left[c\sqrt{|\gamma|} \sqrt{\frac{a(1 - \Omega_\Lambda)}{\Omega_\Lambda(1 - a\gamma)}} \right] \\ &= \text{sinn}^{-1} (c\sqrt{|\Omega_k/\Omega_\Lambda|}). \end{aligned} \quad (20)$$

Using definitions $\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{cr}}$ and $\rho_{cr} = 3M_p^2 H^2$, we get

$$HL = \frac{c}{\sqrt{\Omega_\Lambda}}. \quad (21)$$

Now using Eqs. (14), (21), we obtain²

$$\dot{L} = \frac{c}{\sqrt{\Omega_\Lambda}} - \frac{1}{\sqrt{|k|}} \cosn(\sqrt{|k|}R_h/a), \quad (23)$$

where

$$\frac{1}{\sqrt{|k|}} \cosn(\sqrt{|k|x}) = \begin{cases} \cos(x), & k = 1, \\ 1, & k = 0, \\ \cosh(x), & k = -1. \end{cases} \quad (24)$$

By considering the definition of holographic energy density ρ_Λ , and using Eqs. (21), (23) one can find:

$$\dot{\rho}_\Lambda = -2H \left[1 - \frac{\sqrt{\Omega_\Lambda}}{c} \frac{1}{\sqrt{|k|}} \cosn(\sqrt{|k|}R_h/a) \right] \rho_\Lambda. \quad (25)$$

Substitute this relation into following equation

$$\dot{\rho}_\Lambda + 3H(1 + w_\Lambda)\rho_\Lambda = 0, \quad (26)$$

we obtain

$$w_\Lambda = - \left[\frac{1}{3} + \frac{2\sqrt{\Omega_\Lambda}}{3c} \frac{1}{\sqrt{|k|}} \cosn(\sqrt{|k|}R_h/a) \right]. \quad (27)$$

If we establish the correspondence between the holographic dark energy and Chaplygin gas energy density, then using

² while we know that in this situation we must be in de Sitter space with constant EoS.

² Now we see that the above problem is solved when R_h is replaced with L . According to Eqs. (5), (11), the ratio of the energy density between curvature and holographic dark energy is

$$\frac{\Omega_k}{\Omega_\Lambda} = \frac{\sin^2 y}{c^2}, \quad (22)$$

when there is only dark energy and the curvature, $\Omega_\Lambda = 1 + \Omega_k$, and $c = 1$, we find $\Omega_\Lambda = \frac{1}{\cos^2 y}$, in this case according to Eq. (23) $\dot{L} = 0$, therefore, as one expected in this de Sitter space case, the dark energy remains a constant.

Eqs. (6), (11) we have

$$B = a^6(9c^4 M_p^4 L^{-4} - A). \quad (28)$$

Also using Eqs. (2), (6), (27) one can write

$$\begin{aligned} w &= \frac{P}{\rho} = \frac{-A}{\rho^2} = \frac{-A}{A + \frac{B}{a^6}} \\ &= - \left[\frac{1}{3} + \frac{2\sqrt{\Omega_\Lambda}}{3c} \frac{1}{\sqrt{|k|}} \cosn(\sqrt{|k|}R_h/a) \right]. \end{aligned} \quad (29)$$

Substitute B in the above equation, we obtain following relation for A :

$$A = 3c^4 M_p^4 L^{-4} \left[1 + \frac{2\sqrt{\Omega_\Lambda}}{c} \frac{1}{\sqrt{|k|}} \cosn(\sqrt{|k|}R_h/a) \right]. \quad (30)$$

Then B is given by³

$$B = 6c^4 M_p^4 L^{-4} a^6 \left[1 - \frac{\sqrt{\Omega_\Lambda}}{c} \frac{1}{\sqrt{|k|}} \cosn(\sqrt{|k|}R_h/a) \right]. \quad (34)$$

Now we can rewritten the scalar potential and kinetic energy term as following

$$\begin{aligned} V(\phi) &= 2c^2 M_p^2 L^{-2} \left[1 + \frac{\sqrt{\Omega_\Lambda}}{2c} \frac{1}{\sqrt{|k|}} \cosn(\sqrt{|k|}R_h/a) \right] \\ &= 2H^2 M_p^2 \Omega_\Lambda \left[1 + \frac{\sqrt{\Omega_\Lambda}}{2c} \frac{1}{\sqrt{|k|}} \cosn(\sqrt{|k|}R_h/a) \right], \end{aligned} \quad (35)$$

$$\dot{\phi} = \frac{cM_p}{L} \sqrt{2 \left[1 - \frac{\sqrt{\Omega_\Lambda}}{c} \frac{1}{\sqrt{|k|}} \cosn(\sqrt{|k|}R_h/a) \right]}. \quad (36)$$

Considering $x(\equiv \ln a)$, we have

$$\dot{\phi} = \phi' H. \quad (37)$$

Then using Eqs. (21), (36), derivative of scalar field ϕ with respect to $x(\equiv \ln a)$ is as

$$\phi' = M_p \sqrt{2\Omega_\Lambda \left[1 - \frac{\sqrt{\Omega_\Lambda}}{c} \frac{1}{\sqrt{|k|}} \cosn(\sqrt{|k|}R_h/a) \right]}. \quad (38)$$

Consequently, we can easily obtain the evolutionary form of the field

³ As one can see in this case the A and B can change with time. Similar situation can arise when the cosmological constant has dynamic, see for example Eq. (12) of [14], according to this equation

$$A = \Lambda(\Lambda + \rho_m) \quad (31)$$

therefore, if Λ vary with time [23], A does not remain constant.

In the flat universe case L replace with event horizon R_h , in this case Eqs. (30), (34) take following simple form respectively

$$A = 3c^4 M_p^4 R_h^{-4} \left(1 + \frac{2\sqrt{\Omega_\Lambda}}{c} \right), \quad (32)$$

$$B = 6c^4 M_p^4 R_h^{-4} a^6 \left(1 - \frac{\sqrt{\Omega_\Lambda}}{c} \right). \quad (33)$$

Substitute the present value for a , Ω_Λ and R_h , one can obtain the values of A and B in present time.

$$\phi(a) - \phi(a_0) = \int_0^{\ln a} M_p \sqrt{2\Omega_\Lambda \left[1 - \frac{\sqrt{\Omega_\Lambda}}{c} \frac{1}{\sqrt{|k|}} \operatorname{cosn}(\sqrt{|k|} R_h/a) \right]} dx, \quad (39)$$

where a_0 is the present time value of the scale factor.

3. Conclusions

It is fair to claim that the simplicity and reasonable nature of HDE provide a more reliable framework for investigating the problem of DE compared with other models proposed in the literature [24–26]. For instance the coincidence or “why now?” problem is easily solved in some models of HDE based on this fundamental assumption that matter and holographic dark energy do not conserve separately, but the matter energy density decays into the holographic energy density [27].

Within the different candidates to play the role of the dark energy, the Chaplygin gas, has emerged as a possible unification of dark matter and dark energy, since its cosmological evolution is similar to an initial dust like matter and a cosmological constant for late times. Inspired by the fact that the Chaplygin gas possesses a negative pressure, people [28] have undertaken the simple task of studying a FRW cosmology of a universe filled with this type of fluid.

In this Letter we have associated the holographic dark energy in FRW universe with a scalar field which describe the Chaplygin cosmology. We have shown that the holographic dark energy can be described by the scalar field in a certain way. Then a correspondence between the holographic dark energy and Chaplygin gas model of dark energy has been established, and the potential of the holographic scalar field and the dynamics of the field have been reconstructed.

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