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Linguistic rating of procedural knowledge in computer algebra system learning environment: Fuzzy multi factorial evaluation approach

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Abstract

This paper proposes linguistic rating of students’ views on procedural knowledge under Computer Algebra System (CAS) learning environment using multi factorial evaluation approach. The experiment involved a survey which consists of six factors for the purpose of providing a rating. The fuzzy synthetic multiplication was used to analyse the data collected from one hundred and sixty four students sampled from a secondary school in Terengganu, Malaysia. The evaluation indicates that students were ‘strongly agree’ and ‘agree’ that the procedural knowledge is still needed despite the presence of a CAS.

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1. Introduction

Computer algebra systems (CASs) have been proliferated in the algebra classes at the higher school level and tertiary education with the purpose of enhancing learning outcomes. Aside from its actual efficacy in promoting learning process, one of the most useful contributions of CAS is the extent to which it provokes in the relationship between pedagogical principles and practice. One of the pedagogical considerations to be looked into is the way of teachers managing teaching process and how students react to the CAS technology. In CAS environment, it is notably accepted that teacher imparting knowledge not purely via physical computer screen but more importantly the way the students use cognitive domain rigorously. It is the same as the students deeply engage in algebraic operations with the help from CAS. It is anticipated that the CAS will bring about several benefits to the students and teachers. It has been suggested that the role of teacher must change in the sense that it is no longer sufficient for teachers merely to impart content knowledge. At the same time, students are expected to acquire knowledge efficiently and certainly are accumulated views toward such the first ever technology that comes to their learning environment.

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minimum) values of quadratic functions by the method of completing the square (CTS). Students have to express the standard form of algebraic quadratic function in CTS form before determining the maximum (or minimum) value of the quadratic function. This dependency on procedural skills could be carried out easily using CASs. The symbolic manipulating technological tools, CASs have been used extensively by the students and teachers all over the world [1]. However, in Malaysia this technology has not been used widely in teaching and learning purposes. Not many reports on the use of CAS in the teaching of secondary school mathematics in Malaysia have been published so far. From data analysing point of view, most of computer based education were analysed using descriptive and inferential statistics [2], [3], [4], [5]. Recently an attempt has been made to apply fuzzy sets theory in data analysing of computer based education research. For example, Abdullah [6], [7], [8] applied fuzzy set based model in analyzing students’ perceptions on learning algebra and role of teacher in the presence of a computer algebra system. This present study attempts to give a brief view on the learning CTS in traditional approach versus the learning in the environments of a computer algebra system using another fuzzy sets decision making based method. Specifically, the objective of this study is to rate views of students toward procedural knowledge of CTS in a CAS learning environment using fuzzy set based multi factorial evaluation approach.

2. Computer Algebra System: Pedagogical Perspectives

Computer algebra system of today is a prototypical example of a cognitive technology, a media that helps “transcend the limitation of mind … in thinking, learning and problem solving activities” [9]. As a cognitive technology, CAS is a tool that may afford students access to higher level mathematics processes. With the usage of CAS, students can generate and manipulate symbolic expressions that were otherwise time-consuming, too effortful, or too complicated for students to handle in the course of problem solving. These capabilities can be used for a range of instructional purposes, and the potential impact of the CAS on the knowing and learning of mathematics depends on the roles it assumes particular mathematics instructions [10].

Cognitive technologies, like CAS plays substantially different roles in mathematics instruction than in instruction in other fields. Pea [9] proposed thinking about the role of technology (in this case a CAS) as either that of amplifier or that of a reorganiser. As a cognitive technology, the CAS affords the user the opportunity to transcend the limitations of the mind through its capacity to generate a larger number and a greater range of examples for students to encounter. When CAS is used primarily to generate additional examples, it can extend the existing curriculum, facilitating students’ learning of mathematics content very similar to what they would have learned without CAS and leaving the goals and sequence of the curriculum basically intact. Technology that is used to extend the existing curriculum is called an amplifier.

A second way in which CAS serves as a cognitive technology is as a reorganiser, changing the fundamental nature and arrangement of the curriculum. CAS as a reorganiser in research of mathematics instruction has been to facilitate adjustment in the balance, the sequence, the priorities assigned to concept and procedures in mathematics curriculum. In some of these studies students in introductory algebra classes [10], [11] used packages with CAS facility for almost all of the routine symbolic manipulation, for production of graphs and tables of values, and for curve fitting. In these studies the nature of the algebra curriculum were fundamentally changed. Time traditionally spent on the development and refinement of students’ abilities to perform routine procedures was reallocated to the development of the ability to interpret symbolic results and to recognise and use concept of algebra.

3. Experiment

One hundred and sixty four Form Four students from a secondary school in the east coast of Peninsular Malaysia participated in this study. The students were placed at a Multimedia Lab equipped with 30 workstations, one workstation for teachers, a Liquid Crystal Display (LCD) projector, a screen, and a white board. Every workstation had been installed with CAS software.

At the first stage of the experiment, the students have been taught the maximum (or minimum) values of quadratic function through the traditional paper and pencils method. They learn how to find maximum (or minimum) values of quadratic function through CTS. Every class session took 40 minutes to complete the prescribed learning objectives. At this stage, there was no computer aided in teaching even though they were sitting in the multimedia lab.
The second stage of research began with the learning of CTS using computers. The students were studying the same knowledge as the first stage but using a different medium. They were using a CAS to get the maximum (or minimum) values of quadratic functions. Three commands or steps were created in these activities and named as Sym, Mn and Plot. Sym is a command to find the corresponding values of \( x \) and subsequently being substituted in the next command, Mn. The Mn command yields the maximum (or minimum) values of quadratic functions. The values from these two commands (Mn and Sym), then brought to visualise into the graphical form by using the command Plot. The experiment setup and samples of student’s exploration in activity notebook can be retrieved from Abdullah [6],[7].

After completing the lessons, they were asked to respond to a survey. A simple six-factor survey was used to establish the students’ views about the procedural knowledge approach in learning algebra. The survey comes with five scales descriptive grade of agreement ‘strongly disagree’ (SDA), ‘disagree’ (DA), ‘neutral’ (N), ‘agree’ (A), and ‘strongly agree’ (SA) for each factor. The six factors and its description can be seen in Table 1. Six class sessions were held to accommodate the complete student cohort. Data collected were then analysed with a fuzzy multi factorial evaluation approach.

4. Multi Factorial Evaluation

Kantardzic [12] proposed an evaluation process which has been called as multi factorial evaluation. Multi factorial evaluation is a good example of the application of fuzzy set theory to decision making processes. Its purpose is to provide synthetic evaluation of an object relative to an objective in a fuzzy decision environment that has many factors.

Let \( U = \{u_1, u_2, u_3, \ldots, u_n\} \) be a set of objects for evaluation.
Let \( F = \{f_1, f_2, f_3, \ldots, f_m\} \) be the set of basic factors in the evaluation process.
Let \( E = \{e_1, e_2, e_3, \ldots, e_p\} \) be a set of descriptive grades or qualitative classes used in the evaluation.

For every object \( u \) in the set of \( U \), there is a single factor evaluation matrix \( R(u) \) with dimension \( m \times p \), which is usually the result of survey. This matrix may be interpreted and used as a two-dimensional membership function for the fuzzy relation \( F \times E \).

Within the preceding three elements, \( F, E, \) and \( R \), the evaluation result \( D(u) \) for a given object \( u_1^1 \ldots \ldots u_i^\ldots \ldots u_n^\ldots \); \( U \) can be derived using the basic fuzzy processing procedure: the product of fuzzy relations through max-min composition. Max-min composition is defined using the fuzzy set operations of union and intersection. The union of two fuzzy sets \( A \) and \( B \) is a fuzzy set \( C \), written as \( C = A \lor B \) whose membership function \( \mu_C(x) \) is related to those of \( A \) and \( B \) by

\[
\mu_C(x) = \max(\mu_A(x), \mu_B(x)) = \mu_A(x) \lor \mu_B(x), \forall x \in X
\]  

(1)

A more intuitive but equivalent definition of the union of two fuzzy sets \( A \) and \( B \) is the smallest fuzzy set containing both \( A \) and \( B \).

The intersection of fuzzy sets can be defined analogously. The intersection of two fuzzy sets \( A \) and \( B \) is a fuzzy set \( C \), written as \( C = A \land B \) whose membership function \( \mu_C(x) \) is related to those of \( A \) and \( B \) by

\[
\mu_C(x) = \min(\mu_A(x), \mu_B(x)) = \mu_A(x) \land \mu_B(x), \forall x \in X
\]

(2)

As in the case of the union sets, it is obvious that the intersection of \( A \) and \( B \) is the largest fuzzy set that is contained in both \( A \) and \( B \).

An additional input to the process is the weight vector \( W(u) \) for the evaluation factors, which can be viewed as a fuzzy set for a given input \( u \). (see Figure 1).
This schematic structure of evaluation is tested to a CAS pedagogical experiment data.

5. Computational Steps and Results

Computational steps specifically for the experiment are proposed based on the evaluation in Section 4.

Step 1: Identify factors, object of evaluation and scores of descriptive grade.

Percentages of responses from students in five descriptive grades for six factors are presented in Table 1.

<table>
<thead>
<tr>
<th>Factors</th>
<th>Factors description</th>
<th>Students’ response in percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1$</td>
<td>CTS methods are one of the enjoyable methods of getting maximum (or minimum) values of quadratic functions.</td>
<td>SDA 29.6 DA 13.0 N 11.1 A 27.8</td>
</tr>
<tr>
<td>$f_2$</td>
<td>CTS methods are still needed even though computer methods are available</td>
<td>SDA 22.2 DA 37.0 N 5.6 A 18.5</td>
</tr>
<tr>
<td>$f_3$</td>
<td>CTS methods complement knowledge that acquired from computers</td>
<td>SDA 8.4 DA 2.9 N 10.7 A 29.3</td>
</tr>
<tr>
<td>$f_4$</td>
<td>CTS methods need to be learned because steps are clearly shown</td>
<td>SDA 28.6 DA 30.6 N 10.0 A 12.2</td>
</tr>
<tr>
<td>$f_5$</td>
<td>I am more confident with the answers done through procedures in CTS methods</td>
<td>SDA 15.1 DA 23.2 N 35.2 A 14.2</td>
</tr>
<tr>
<td>$f_6$</td>
<td>Computer generated methods are not the perfect methods in finding the maximum (or minimum) values of quadratic functions</td>
<td>SDA 16.7 DA 27.8 N 16.7 A 20.4</td>
</tr>
</tbody>
</table>

Step 2: Create evaluation matrix

Data from Table I are translated into the evaluation matrix.

Evaluation matrix $R$ is given as
Step 3: Define weight vector

The weight vector with respect to the six factors is defined as

\[
W(u) = \frac{R_i(u)_{\text{max}}}{\sum_{i=1}^{6} R_i(u)_{\text{max}}} \quad \text{with the condition,} \quad \sum_{i=1}^{6} W_i = 1
\]

for \( i = 1,2,3,4,5,6 \) in which \( R_i(u)_{\text{max}} \) is the maximum membership score in a single-factor evaluation vector. Hence,

\[
W(u) = \{0.142, 0.177, 0.233, 0.146, 0.169, 0.133\}
\]

Step 4: Obtain product of fuzzy relations

Using the multi factorial evaluation, rating of the factors of the procedural knowledge approach can be computed. Multiplication of matrices \( W(u) \) and \( R(u) \) is based on max-min composition of fuzzy relation, where the resulting evaluation is in the form of a fuzzy set \( D(u) = [d_1, d_2, d_3, d_4, d_5] \)

\[
D(u) = W(u) \otimes R(u)
\]

\[
= \begin{bmatrix}
0.142 & 0.177 & 0.233 & 0.146 & 0.169 & 0.133 \\
0.185 & 0.296 & 0.130 & 0.111 & 0.278 \\
0.222 & 0.370 & 0.056 & 0.185 & 0.167 \\
0.084 & 0.029 & 0.107 & 0.293 & 0.487 \\
0.286 & 0.306 & 0.100 & 0.122 & 0.186 \\
0.151 & 0.232 & 0.352 & 0.142 & 0.151 \\
0.167 & 0.278 & 0.167 & 0.204 & 0.167 \\
\end{bmatrix}
\]

\[
= [d_1, d_2, d_3, d_4, d_5]
\]

\[
= [0.177, 0.177, 0.169, 0.233, 0.233]
\]
The decision matrix $D(u)$ has two maxima.

**Step 5: Rating consensus factors**

Using the equation (1) and (2), consensus ratings are obtained as $d_4 = 0.233$ and $d_5 = 0.233$. The value of $d_4$ and $d_5$ represent the result of synthetic evaluation which corresponds to the linguistics variable rating $e_4$ (agree) and $e_5$ (strongly agree). It concludes that factors of the procedural knowledge approach to CTS in the presence of a CAS should be rated as ‘agree’ and ‘strongly agree’.

**6. Conclusion**

The procedural knowledge approach has been regarded as one of the traditional methods in learning algebra. The six factors in the procedural knowledge approach in the presence of computer algebra system were analysed using multi factorial evaluation to set a new rating. In this study, students pooled their views in the category of ‘strongly agree’ and ‘agree’ that procedural skills still needed in the presence of computer algebra system. Despite the efficiency of computer algebra system, students firmly agreed that they still need procedural knowledge. Therefore this finding does not support the idea that routine symbolic manipulation can be successfully offloaded to computer algebra system [10]. This finding meant to verify the procedural knowledge approach still relevant even though in the presence of computerised algebra learning environment. This finding offers some thoughts on the implication for developing a fully computer algebra in teaching and learning.

**References**


