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Complex principle component analysis on dynamic correlation structure in price index data

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Abstract

We carry out multivariate time series analysis on price indices of individual goods and services collected over the last 35 years in Japan. Adoption of the complex principal component analysis (CPCA) enables us to have a new insight into dynamic correlation structure involved in the price data. The CPCA is based on complexification of real data using the Hilbert transformation; lead-lag relations between individual prices manifest in a form of instantaneous phases of the complex time series. The correlation matrix in the CPCA is purified by adopting the random matrix theory as a null hypothesis for removal of statistical noises. We identify four significant eigenmodes for price movement which are free from seasonal variations. Each of them has different characteristics of dynamical correlations and is shown to be responsive to different economic events.

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Keywords: Consumer price index (CPI), Hilbert transformation, Complex principal component analysis (CPCA), Random matrix theory (RMT)

1. Introduction

In econophysics principal component analysis (PCA) and random matrix theory (RMT) was successfully combined to detect correlations hidden in financial markets[? ? ?], and now the combined method is widely used in analyzing multivariate time series data of various complex systems[?]. The RMT serves as a theoretically sound criterion to determine if eigenmodes of the correlation matrix are statistically significant; this is the critical issue that the PCA always encounters. However, the PCA assisted by the RMT is not so capable of extracting correlation structures with lead/lag relations, because it totally depends on the equal-time correlation matrix. Correlations between time series data are not always present in a simultaneous manner.

In order to explore dynamic correlations in climate data, the complex principal component analysis (CPCA) was developed by meteorologists[? ? ?]. The CPCA is based on complexification of real data using the Hilbert transformation. Lead/lag relations in original data manifest in a form of instantaneous phases of the complex time series thus constructed. Recently, the RMT has been extended so that it works as a null hypothesis for the CPCA. If time series data have appreciable autocorrelations, however, the RMT criterion tends to predict more significant modes than it should do. This is because autocorrelations deceive us by giving rise to spurious cross-correlations for time series of finite length, especially in the case that their length is comparable with the number of species of data. To over-

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come such limitation of the RMT, the rotational random shuffling (RRS) method was worked out. This is a numerical method which destroys cross-correlations with autocorrelations preserved in time series data.

Recently, the CPCA assisted by the RMT or the RRS has been applied to various multivariate data including stock market data[?], world-wide financial data of markets and currencies[?], and individual prices constituting the consumer price index (CPI) in Japan[?]. In this study, we analyze monthly Japanese data of individual prices of consumers and corporate goods collected for the period of 1980-2014. We like to elucidate dynamic correlation structures in the economic system such as comovement and lead/lag relations of the prices. The analysis period encompasses enforcement of consumption tax law (3% in April, 1989), subsequent consumption tax increase (to 5% in April, 1997 and to 8% in April, 2014), sub-prime mortgage crisis (2007-2009), and the Lehman's bankruptcy (September, 2008). We expect that they must have acted as shocks on prices. We thereby investigate how these economic events impact on the price changes of individual goods and services. In fact, the investigation elucidates that each of the significant eigenmodes for the price dynamics has different characteristics in their reaction to the shocks.

2. Complex Principal Component Analysis

Let us suppose that we have N different time series $x_\mu(t)$ $(\mu = 1, \cdots, N)$ of length T $(t = 1, \cdots, T)$. We first derive complex time series $\xi_\mu(t)$ out of $x_\mu(t)$ through the relation,

$$\xi_\mu(t) = x_\mu(t) + iy_\mu(t)\ , \tag{1}$$

where the imaginary part $y_\mu(t)$ is Hilbert transform of $x_\mu(t)$ defined by

$$y_\mu(t) = -\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{x_\mu(u)}{t-u}\mathrm{d}u\ . \tag{2}$$

The integration over u in Eq. (**??**) should be interpreted as Cauchy's principal integration[1]. We then construct the complex correlation matrix $\tilde{\boldsymbol{C}}$ from the complex time series $\{\xi_\mu(t)\}$:

$$\tilde{\boldsymbol{C}} = \frac{1}{T}\boldsymbol{\Xi}\boldsymbol{\Xi}^\dagger, \tag{3}$$

where $\boldsymbol{\Xi}$ denotes $N \times T$ data matrix whose component is $\xi_\mu(t)$ and $\boldsymbol{\Xi}^\dagger$ is Hermite conjugate of $\boldsymbol{\Xi}$.

The complex principal component analysis (CPCA) computationally amounts to the eigenvalue problem for $\tilde{\boldsymbol{C}}$. Since $\tilde{\boldsymbol{C}}$ is a Hermitian matrix, its eigenvalues are real and furthermore positive definite because of the dyadic form (**??**). On the other hand, the components of the eigenvectors are complex. The absolute values and the phases of the eigenvector components provide us with information on strength of correlations and lead-lag relationships embedded in multivariate time series. The correlation matrix $\tilde{\boldsymbol{C}}$ is expressible in terms of its eigenvalues and eigenvectors as

$$\tilde{\boldsymbol{C}} = \sum_{\ell=1}^{N}\lambda_\ell\boldsymbol{\alpha}_\ell\boldsymbol{\alpha}_\ell^\dagger\ , \tag{4}$$

where λ_ℓ and $\boldsymbol{\alpha}_\ell$ are the ℓ-th eigenvalue and its associated eigenvector, respectively, and we align the eigenvalues in descending order, that is, $\lambda_1 > \lambda_2 > \cdots > \lambda_N$.

We perform a basis conversion of the time series data with the eigenvectors $\boldsymbol{\alpha}_\ell$ which form an orthonormal complete basis set:

$$\boldsymbol{\xi}(t) = \sum_{\mu=1}^{N}\xi_\mu(t)\boldsymbol{e}_\mu = \sum_{\ell=1}^{N}a_\ell(t)\boldsymbol{\alpha}_\ell\ , \tag{5}$$

where

$$a_\ell(t) = \boldsymbol{\alpha}_\ell^\dagger\cdot\boldsymbol{\xi}(t)\ . \tag{6}$$

[1] In the actual calculations, we used a discretized version of the Hilbert transformation.[?]

We refer to the coefficient $a_\ell(t)$ as *mode signal* of the ℓth eigenmode. The mode signals represent temporal behavior of the eigenmodes. In addition, we define relative mode intensity $I_\ell(t)$ by

$$I_\ell(t) = \frac{|a_\ell(t)|^2}{\sum_{\ell=1}^{N} |a_\ell(t)|^2} , \tag{7}$$

which calculates the fractional contribution of each eigenmode to the overall strength of price fluctuations at each instant of time; we note the following equality,

$$\boldsymbol{\xi}(t)^\dagger \cdot \boldsymbol{\xi}(t) = \sum_{\mu=1}^{N} |\xi_\mu(t)|^2 = \sum_{\ell=1}^{N} |a_\ell(t)|^2 . \tag{8}$$

3. Random Matrix Theory and Rotational Random Shuffling

It is a crucial issue for the CPCA as well as the PCA how to identify eigenmodes which are statistically significant, i.e., mode signals representing systemic co-movements in the system under study, not noises. The random matrix theory (RMT) serves as a sound null hypothesis for such a statical significance test. Arai and Iyetomi extended the RMT for the CPCA. According to them, the eigenvalue spectrum $\rho(\lambda)$ of the complex correlation matrix obtained from random data corresponding to the actual data is given as

$$\rho(\lambda) = \frac{Q}{2\pi} \frac{\sqrt{(\lambda_+ - \lambda)(\lambda - \lambda_-)}}{\lambda} , \tag{9}$$

with

$$\lambda_\pm = \left(1 \pm \sqrt{\frac{1}{Q}}\right)^2 , \tag{10}$$

where $Q = T/(2N) > 1$ and the limit of $N, T \to \infty$ with Q kept finite is assumed. We thus see that the eigenvalues predicted by RMT are confined in $[\lambda_-, \lambda_+]$. If we find eigenvalues for the actual correlation matrix which are larger than λ_+, we can identify those eigenvalues and associated eigenvectors as representing statistically meaningful correlations. Although the formula (**??**) is mathematically exact for a random correlation matrix of infinite size, in practice, it is applicable[?] to the finite matrices with $N \gtrsim 50$.

As remarked in the Introduction, autocorrelations involved in multivariate data reduce the usefulness of the RMT in removing statistical noises from them. The rotational random shuffling (RRS) method provides[?,?] us with a null hypothesis alternative to the RMT in such a case. We impose the periodic boundary condition on each time series to make a “ring” in the time direction and randomly shuffle the data in a rotational way. The randomization destroys only cross-correlations preserving autocorrelations. This gives us a robuster null hypothesis than the RMT. However, we have to numerically compute the eigenvalue spectrum of the complex correlation matrix for the randomized data in the RRS.

4. Data set

We study the Japanese monthly data of the following four categories of individual prices which were collected for the period, January 1980 through December 2014:

- Consumer Price Index (CPI)[?] with 45 items,
- Corporate Goods Price Index (CGPI)[?] with 23 items,
- Import Price Index (IPI)[?] with 10 items,
- US Dollar to Japanese Yen Exchange Rate (USD/JPY)[?].

The totally 79 items as shown in Tables **??** and **??** were combined into a set of multivariate time series data with length of 420 months. Assuming the prices basically obey geometric brownian motion, we took the first difference of logarithm of the time series. The preprocessed data passed the unit root test for stationarity. However, some items such

Table 1. List of items and their abbreviations for CPI in the middle-level classification.

ID	Abbreviation	Item
1	CPI-CER	Cereals
2	CPI-FSF	Fish & seafood
3	CPI-MET	Meats
4	CPI-DPE	Dairy products & eggs
5	CPI-VSW	Vegetables & seaweeds
6	CPI-FRU	Fruits
7	CPI-OFS	Oils, fats & seasonings
8	CPI-CAC	Cakes & candies
9	CPI-CFD	Cooked food
10	CPI-BEV	Beverages
11	CPI-ALB	Alcoholic beverages
12	CPI-MOH	Meals outside the home
13	CPI-REN	Rent
14	CPI-REM	Repairs & maintenance
15	CPI-ELE	Electricity
16	CPI-GAS	Gas
17	CPI-OFL	Other fuel & light
18	CPI-HDG	Household durable goods
19	CPI-IFN	Interior furnishings
20	CPI-BED	Bedding
21	CPI-DUT	Domestic utensils
22	CPI-NDG	Domestic non-durable goods
23	CPI-DSR	Domestic services
24	CPI-CLO	Clothes
25	CPI-SSU	Shirts, sweaters & underwear
26	CPI-FTW	Footwear
27	CPI-OCL	Other clothing
28	CPI-SCL	Services related to clothing
29	CPI-MHF	Medicines & health fortification
30	CPI-SAP	Medical supplies & appliances
31	CPI-MSR	Medical services
32	CPI-PUT	Public transportation
33	CPI-PRT	Private transportation
34	CPI-COM	Communication
35	CPI-SCF	School fees
36	CPI-TBS	School textbooks & reference books for study
37	CPI-RDG	Recreational durable goods
38	CPI-REG	Recreational goods
39	CPI-BRM	Books & other reading materials
40	CPI-RSR	Recreational services
41	CPI-PCS	Personal care services
42	CPI-TAR	Toilet articles
43	CPI-PEE	Personal effects
44	CPI-TBC	Tobacco
45	CPI-OMS	Other miscellaneous

as Vegetables & seaweeds and Cloths have significant seasonal components in their prices' fluctuations. To address this issue, we also prepared seasonally adjusted data by taking year-to-year change of the original time series. The procedure is the most primitive way to remove seasonal components from time series data. As its side effect, however, 12-month moving average is inevitably brought into the analysis.

A more detailed analysis was carried out in the previous paper[?], using price data of 830 items at small-level classification in Japan. We distinguish this work from the previous one by carefully treating seasonal variations and investigating relationship between price dynamics and external shocks.

5. Results and Discussion

We computed eigenvalues of the complex correlation matrix $\tilde{\boldsymbol{C}}$ constructed from the price data with and without seasonal adjustment. In Fig. **??** we show the results in a form of the probability distribution. The left panel of Fig. **??**

Table 2. List of items and their abbreviations for GCPI and IPI in the middle-level classification together with US Dollar to Japanese Yen Exchange Rate.

ID	Abbreviation	Item
		CGPI
46	CGPI-FBT	Food, beverages, tobacco & feedstuffs
47	CGPI-TET	Textile products
48	CGPI-LWD	Lumber & wood products
49	CGPI-PAP	Pulp, paper & related products
50	CGPI-CHE	Chemicals & related products
51	CGPI-PEC	Petroleum & coal products
52	CGPI-PLA	Plastic products
53	CGPI-CSC	Ceramic, stone & clay products
54	CGPI-IRS	Iron & steel
55	CGPI-NFM	Nonferrous metals
56	CGPI-MET	Metal products
57	CGPI-GPM	General purpose machinery
58	CGPI-PDM	Production machinery
59	CGPI-BOM	Business oriented machinery
60	CGPI-ECD	Electronic components & devices
61	CGPI-EME	Electrical machinery & equipment
62	CGPI-ICE	Information & communications equipment
63	CGPI-TPE	Transportation equipment
64	CGPI-MIP	Other manufacturing industry products
65	CGPI-AFF	Agriculture, forestry & fishery products
66	CGPI-MIN	Minerals
67	CGPI-EGW	Electric power, gas & water
68	CGPI-SCW	Scrap & waste
		IPI
69	IPI-FFS	Foodstuffs & feedstuffs
70	IPI-TET	Textiles
71	IPI-MET	Metals & related products
72	IPI-LWD	Wood, lumber & related products
73	IPI-PEC	Petroleum, coal & natural gas
74	IPI-CHE	Chemicals & related products
75	IPI-GBM	General purpose, production & business oriented machinery
76	IPI-ELE	Electric & electronic products
77	IPI-TPE	Transportation equipment
78	IPI-OPG	Other primary products & manufactured goods
79	USD/JPY	US Dollar to Japanese Yen Exchange Rate

shows the top 6 largest eigenvalues exceeds the upper limit λ_+ of the eigenvalue distribution of the RMT for the original data. The right panel of the figure confirms the top 4 largest eigenvalues are beyond λ_+ for the seasonally adjusted data. The eigenvectors associated with those eigenvalues are regarded as manifestation of statistically meaningful correlations among individual prices.

Also we carried out the CPCA on the same data but with the RRS preprocessing (sampled 1000 times). Figure **??** compares the eigenvalue distributions with the corresponding results as given in Fig. **??**. The spillover of the eigenvalues across λ_+ indicates the data have appreciable autocorrelations. Clearly, the isolated peak around $\lambda = 4.5$ in the left panel of Fig. **??** arises from seasonal components involved in some of the price fluctuations, which well mimic cross-correlations; we observe no such a peak in the right panel of Fig. **??**. Parallel (rank-by-rank) comparison of the actual eigenvalues with those obtained with the RRS identifies the same number of significant eigenmodes as counted in each panel of Fig. **??**.

In Table **??** we spell out similarity between the two sets of eigenvectors. The one is a set of the significant eigenvectors $\boldsymbol{\alpha}_\ell$ ($\ell = 1, \cdots, 6$) obtained for the original data and the other, that of the significant eigenvectors $\tilde{\boldsymbol{\alpha}}_m$ ($m = 1, \cdots, 4$) for the seasonally adjusted data. The similarity is measured by calculating the inner product of $\boldsymbol{\alpha}_\ell$ and $\tilde{\boldsymbol{\alpha}}_m$ for all pairs. The first 2 eigenvectors in the two sets are in excellent agreement with each other. The 4th eigenvector $\boldsymbol{\alpha}_4$ moderately agrees with $\tilde{\boldsymbol{\alpha}}_3$. Similarity between $\boldsymbol{\alpha}_5$ and $\tilde{\boldsymbol{\alpha}}_4$ is rather marginal; $\boldsymbol{\alpha}_5$ is also partly similar to $\tilde{\boldsymbol{\alpha}}_3$. On the other hand, the remainder in the set $\{\boldsymbol{\alpha}_m\}$, $\boldsymbol{\alpha}_3$ and $\boldsymbol{\alpha}_6$, have no notable counterparts in the set $\{\tilde{\boldsymbol{\alpha}}_m\}$. To understand the reason, we calculated power spectrum of the mode signals associated with $\boldsymbol{\alpha}_\ell$ ($\ell = 1, \cdots, 6$) as shown in Fig. **??**. We observe the third and the sixth mode signals have large peaks corresponding to seasonal variations. We

Table 3. Similarity between the statistically significant eigenvectors $\boldsymbol{\alpha}_\ell$ ($\ell = 1, \cdots, 6$) obtained for the original data and those $\tilde{\boldsymbol{\alpha}}_m$ ($m = 1, \cdots, 4$) for the seasonally adjusted data. The coefficient $|\boldsymbol{\alpha}_\ell^* \cdot \tilde{\boldsymbol{\alpha}}_m|^2$ is listed for each pair as a similarity measure. The third and the sixth eigenmodes in the original data have no counterparts in the seasonally adjusted data.

$\ell \backslash m$	1	2	3	4
1	**0.926**	0.008	0.004	0.002
2	0.006	**0.934**	0.002	0.004
3	0.000	0.018	0.054	0.019
4	0.020	0.002	**0.483**	0.170
5	0.001	0.001	0.251	**0.286**
6	0.010	0.002	0.030	0.101

thus see that $\boldsymbol{\alpha}_3$ and $\boldsymbol{\alpha}_6$ mainly describe seasonal components of fluctuations in the original price data, not involved in the seasonally adjusted data.

Let us delve into the significant eigenmodes in the original data which are free from seasonal variations, that is, the first, the second, the fourth, and the fifth eigenmodes. The eigenvector components of those modes are plotted on complex plane in Fig. **??**.

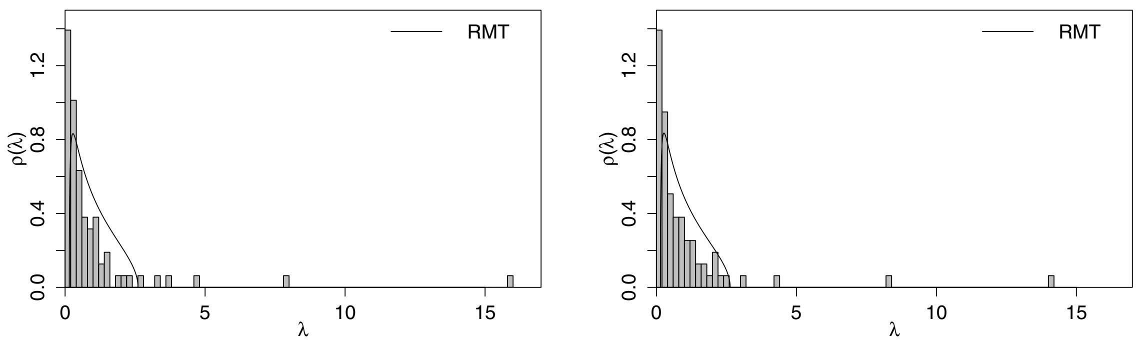

Fig. 1. Probability density $\rho(\lambda)$ of the eigenvalues of $\tilde{\boldsymbol{C}}$ constructed from the original price data (left) and the seasonally adjusted data (right). The solid curve depicts the reference result, Eq. (**??**), predicted by the RMT.

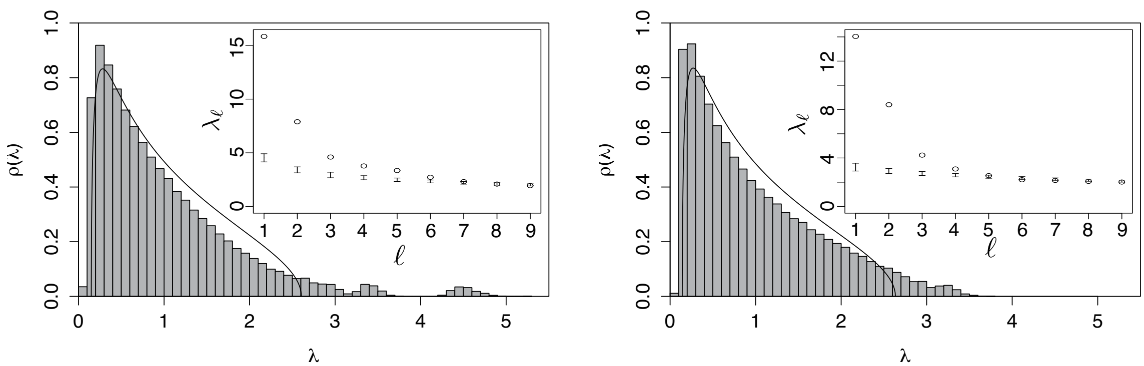

Fig. 2. Same as Fig. **??**, but for the data preprocessed by the RRS. The inset in each panel shows parallel comparison of the nine largest eigenvalues (open circles) in Fig. **??** with the corresponding results (dots with 3σ error bars) in the RRS.

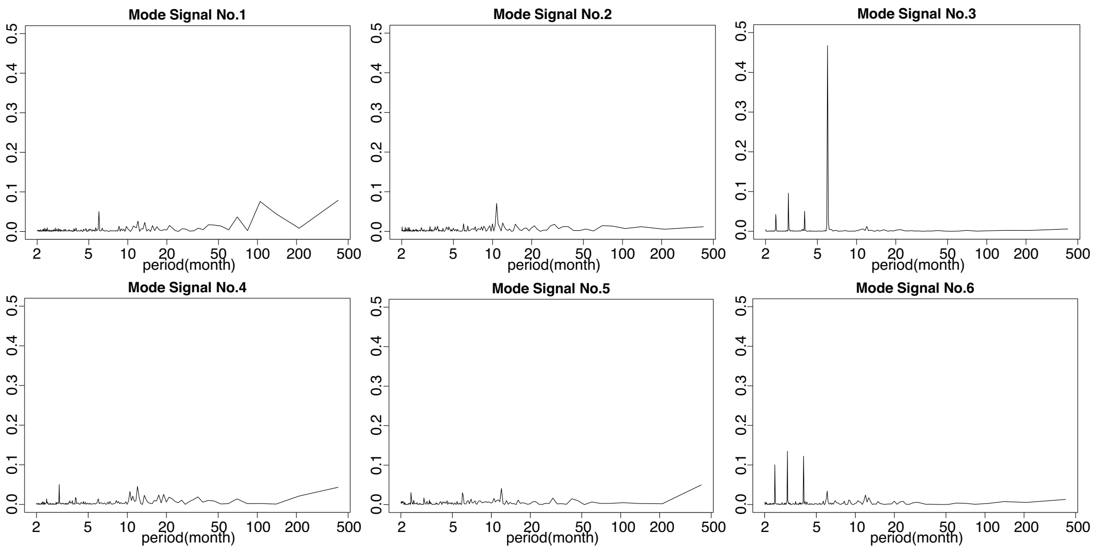

Fig. 3. Power spectral density of the mode signal $a_\ell(t)$ of the statistically significant eigenmodes ($\ell = 1, \cdots, 6$) obtained for the original data.

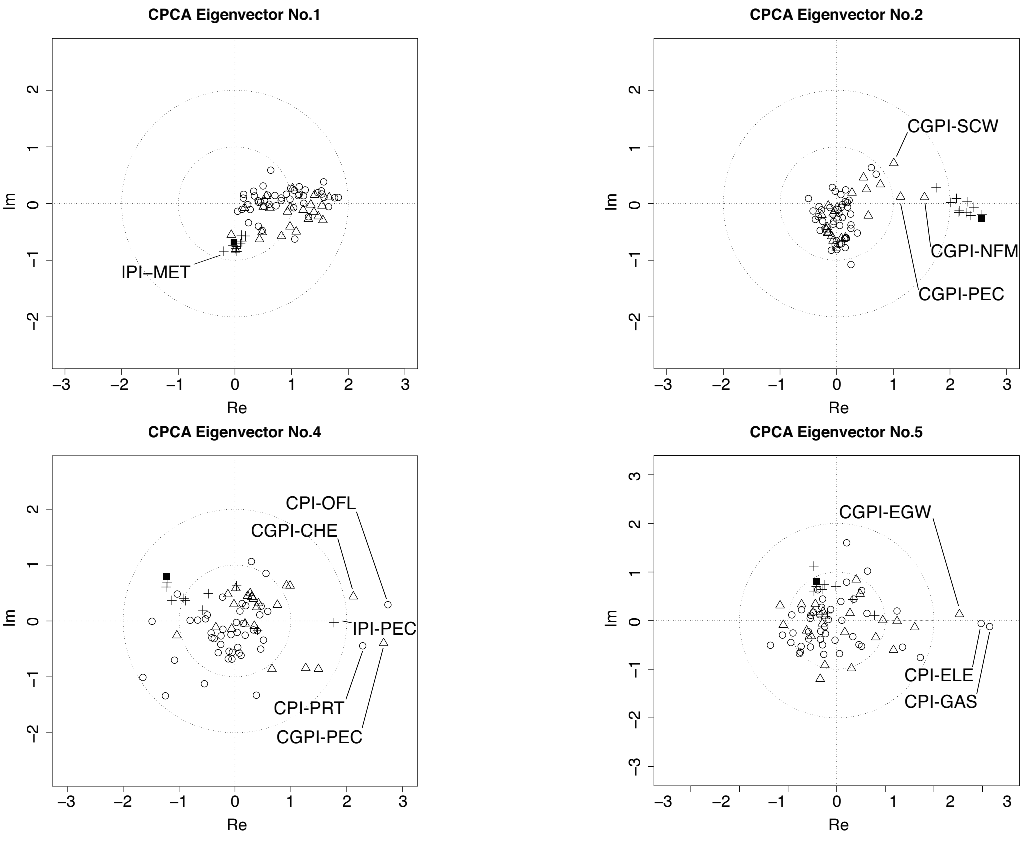

Fig. 4. Distribution of the eigenvector components of the statistically significant modes without seasonal variations obtained for the original data on complex plane; CPI (open circles), CGPI (triangles), IPI (pluses), and USD/JPY (filled square). Time development corresponds to the clockwise direction; for instance, IPI-MET leads to all other items in the first mode.

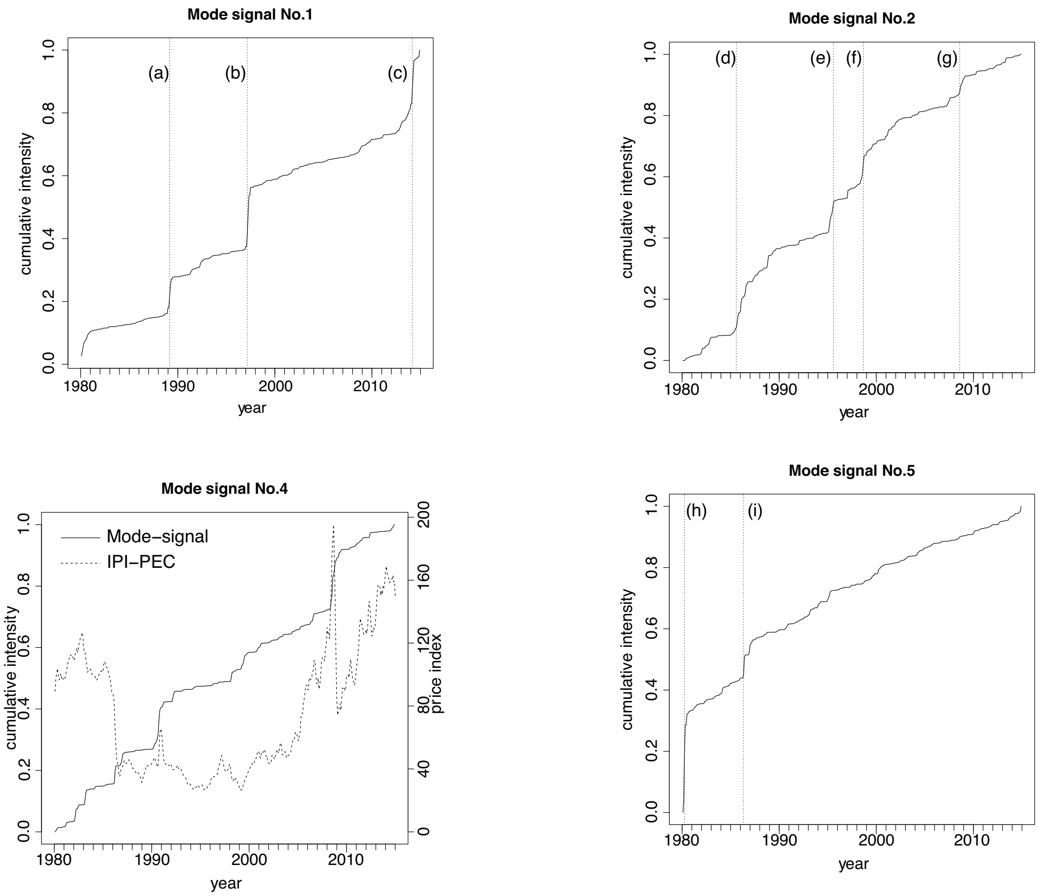

Fig. 5. Temporal accumulation of the relative mode intensity for the significant eigenmodes which are free from seasonal components. The vertical dotted lines signify prominent economical events for Japan: (a) Consumption tax law enforcement in April 1989 (3%); (b) and (c) Consumption tax rate increase in April 1997 (to 5%) and in April 2014 (to 8%); (d) Plaza Accord in September 1985; (e) Record high of the yen in September 1995; (f) Surge of the yen in October 1998 due to the Asian currency crisis; (g) Bankruptcy of Lehman Brothers in September 2008; (h) and (i) Great revision of public utility charges in April 1980 and in April 1986.

The panel in Fig. **??** for the eigenvector components of the first eigenmode clearly demonstrates a collective motion of individual prices with a narrow band of variation of their phases; many of the same category items move coherently. This indicates dynamics of individual prices are mutually connected, not just random fluctuations. Furthermore, Table **??** shows the phase θ and the absolute value of each component of the first eigenvector on the complex plane. Broadly speaking, the items of IPI occupy a leading position and rise/fall of their prices gradually propagates to items of CGPI and finally to those of CPI. The direction of the propagation of price changes from raw materials to final consumer goods is very natural from an industrial point of view. We also define cumulative intensity $S_\ell(t)$ by

$$S_\ell(t) = \frac{\sum_{t'=1}^{t} I_\ell(t')}{\sum_{t'=1}^{T} I_\ell(t')}. \tag{11}$$

The results are shown in Fig. **??**. We find the first mode is strongly exerted by the consumption tax law enforcement (3%) and subsequent consumption tax rate increases to 5% and then to 8%.

The second eigenmode is also manifestation of a collective motion of individual prices. Especially, the yen-dollar exchange rate, IPI's, and some of CGPI such as CGPI-PEC, CGPI-SCW and CGPI-NFM lead the other prices. We remark this mode highly contrasts with the first mode. Because many prices of CPI and CGPI are dynamically coupled to the leading prices almost in quadrature and with negative correlations, that is, rise of the leading prices giving rise

Table 4. Phases $\Delta\theta$ of individual prices in the first eigenvector measured relative to the phase of IPI-MET, a price leader to all others. To squeeze the list, we do not list items with the absolute value (Abs) of their eigenvector components less than 0.5 for $\Delta\theta < 0.5$ and those with Abs less than 0.8 for $\Delta\theta > 0.5$. On the whole, the IPI items first go ahead and then the CGPI's follow and the CPI's bring up the rear.

$\Delta\theta$ [rad/π]	Abs	ID	Items
0.000	0.864	71	IPI-MET
0.036	0.556	68	CGPI-SCW
0.058	0.737	72	IPI-LWD
0.065	0.685	79	USD/JPYF
0.078	0.813	55	CGPI-NFM
0.079	0.795	78	IPI-OPG
0.084	0.759	74	IPI-CHE
0.086	0.845	69	IPI-FFS
0.118	0.718	75	IPI-GBM
0.130	0.687	73	IPI-PEC
0.131	0.688	70	IPI-TET
0.137	0.569	77	IPI-TPE
0.173	0.601	76	IPI-ELE
0.265	0.765	48	CGPI-LWD
0.315	0.697	51	CGPI-PEC
0.326	0.672	17	CPI-OFL
0.329	0.577	33	CPI-PRT
0.379	1.001	50	CGPI-CHE
0.403	1.229	43	CPI-PEE
0.437	1.189	47	CGPI-TET
0.447	1.053	54	CGPI-IRS
0.507	1.324	49	CGPI-PAP
0.514	1.580	56	CGPI-MET
0.517	1.319	3	CPI-MET
0.524	0.938	59	CGPI-BOM
0.525	1.488	53	CGPI-CSC
0.539	1.403	52	CGPI-PLA
0.544	1.208	58	CGPI-PDM
0.551	1.040	16	CPI-GAS
0.552	0.980	66	CGPI-MIN
0.563	1.653	22	CPI-NDG
0.566	1.537	46	CGPI-FBT
0.569	1.339	57	CGPI-GPM
0.574	1.213	7	CPI-OFS
0.592	1.763	21	CPI-DUT
0.592	1.829	28	CPI-SCL
0.594	1.667	64	CGPI-MIP
0.596	1.575	12	CPI-MOH
0.599	0.975	26	CPI-FTW
0.608	1.414	61	CGPI-EME
0.608	1.547	9	CPI-CFD
0.610	1.219	30	CPI-SAP
0.613	1.128	42	CPI-TAR
0.615	1.473	8	CPI-CAC
0.619	1.143	23	CPI-DSR
0.620	1.542	14	CPI-REM
0.631	0.922	11	CPI-ALB
0.634	1.263	10	CPI-BEV
0.643	1.075	20	CPI-BED
0.650	1.612	41	CPI-PCS
0.655	1.053	63	CGPI-TPE
0.655	1.172	19	CPI-IFN
0.658	1.021	29	CPI-MHF
0.812	0.863	13	CPI-REN

to drop of the CGPI and CPI followers and vice versa. The accumulated relative intensity of the mode signal given in Fig. **??** shows that the second mode is reacts sensitively to the epoch-making economic events including Plaza Accord (September, 1985), record high of the yen (September, 1995), surge of the yen due to the Asian currency crisis (October 1998), and bankruptcy of Lehman Brothers (September, 2008). On the other hand, the second mode is completely silent on the consumption tax shocks. Also this fact evidently discriminates the second mode from the first mode.

In the fourth eigenmode as shown by Fig. **??**, we observe that CPI-OFL, CGPI-PEC, CPI-PRT, CGPI-CHE and IPI-PEC move together with almost the same phases. And the relative intensity of the corresponding mode signal in Fig. **??** shows the fourth mode is strongly affected by IPI-PEC. We thus see that the mode represents primary effects on the price system of changes in import prices of natural resources.

The panel for the fifth eigenmode in Fig. **??** shows the mode is dominated by the CPI-ELE, CPI-GAS and CGPI-EGW. Figure **??** shows the relative intensity of the corresponding mode signal is highly enhanced in 1980 and 1986. In fact, electricity and gas had great price revisions at those times.

6. Summary

We attempted to extract dynamical correlation structures hidden in the Japanese price system by applying the CPCA to the individual price data. The RRS method enabled us to identify four statistically significant eigenmodes which are free from seasonal variations. The first two dominant modes represent collective motions of individual prices with different correlation properties. The remaining modes illuminate dynamics of specific prices. Also we adopted the idea of response theory to characterize those modes. In fact, each of them shows different reaction characteristics to prominent economic shocks. We can not rule out the possibility that the mode signals just accidentally coincide with the shocks. A randomization test is in progress to ascertain that the coincidences are statistically significant.

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