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Bending Analysis of Folded Laminated Plates by the FSDT Meshfree Method

L. X. Peng[1a], K. M. Liew[2] and S. Kitipornchai[2]

[1] *College of Civil Engineering and Architecture, Guangxi University, China*
[2] *Department of Building and Construction, City University of Hong Kong, China*

Abstract

A meshfree method is developed in this paper to study the flexure behavior of folded laminated plates. A folded laminated plate is considered as an assembly of laminates. Based on the first-order shear deformation theory (FSDT) and the moving-least square approximation, the stiffness equations of the laminates are derived. A treatment is implemented to modify the equations, and the equations are then superposed to obtain the governing equation of the entire folded laminated plate. No mesh is required in the determination of the stiffness equations of the laminates, and therefore the proposed method is more flexible than the finite element methods. The convergence and accuracy of the proposed method are examined by computing several numerical examples, and good agreement is observed between the present results and those given by ANSYS.

Keywords: meshfree method, FSDT, folded plate, laminate, bending

1. Introduction

Folded plate structures are found in a wide range of engineering applications, such as roofs, sandwich plate cores, ship hulls, and cooling towers, because of their high strength-to-weight ratio, easy forming, and low cost. The folds of the structures increase their effective thicknesses and give them the same load carrying capacity as thicker plates. Despite the wide variety of applications for folded laminated plates, relatively few studies have analyzed these structures, which is the motive for this paper.

The study of folded plate structures has quite a long history. In the early days, the beam method and the theory that ignores relative joint displacement were introduced, although they were weak in dealing with generalized folded plate problems (Task Committee 1963). Gaafar (1954), Yitzhaki (1958) and Whitney et

[a] Corresponding author and Presenter: Email: penglx@gxu.edu.cn

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al. (1959) considered the relative joint displacement in their methods and opened the door to more precise solutions. Goldberg and Leve (1957) used both the two-dimensional theory of elasticity and the two-way slab theory to derive the stiffness of the individual slab of a folded plate. Yitzhaki and Reiss (1962) chose the moments along the joints of the folded plates as variables and applied the slope deflection method to analyze the folded plates. Since the development of computer technologies, numerical methods have played a major role in structural analysis in engineering. The finite element methods (FEM) (Liu and Huang 1992) have been introduced to solve folded plate problems. However, few studies of laminated composite folded plates are available. Guha-Niyogi et al. (1999) and Lee et al. (2004) have conducted such studies, using dynamic analysis and FEMs. As alternatives to FEMs, powerful meshfree methods have been established (Belytschko et al. 1994; Chen et al. 1996; Liew et al. 2006). The meshfree methods base their approximate solutions to problems entirely on the points that are distributed on the problem domain. No mesh or any other interrelationship among the points is required, which makes these methods more flexible and applicable.

The objective of this paper is to propose a meshfree method based on the FSDT (Reddy and Miravete 1995) for the elastic bending analysis of folded laminated plate structures. Such structures are regarded as assemblies of laminated flat plates. The meshfree method is used to obtain the stiffness equations of the laminated flat plates, and the equations are then superposed to give the governing equation of the entire folded plate. Several numerical examples are computed with the proposed method. A comparison of the present solutions with the solutions given by the finite element structural analysis software (ANSYS) is carried out to demonstrate the accuracy and convergence of the proposed method.

2. Formulation for Laminated Flat Plates

The meshfree model of a laminate in the local coordinate is shown in Figure 1, in which a set of nodes is distributed on the mid-plane of the laminate, and the degree of freedom (DOF) of every node of the plate is (u_0, v_0, w, φ_x, φ_y), where u_0, v_0, w are the translations along the x, y, and z directions, and φ_x, φ_y are the rotations about the y and x axes, respectively. The laminate is assumed to have N layers.

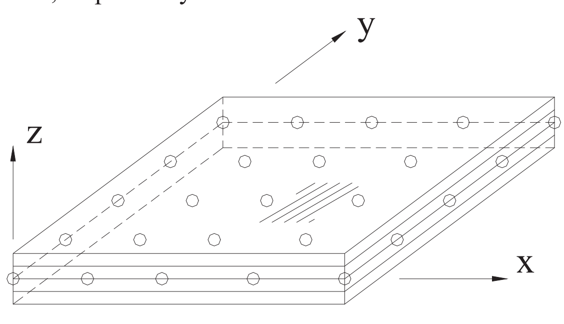

Figure 1. Meshfree model of a laminated composite plate.

2.1. Displacement approximation

According to the FSDT and the moving-least square approximation (Belytschko et al. 1994), the displacements of the laminate can be approximated by

$$\begin{cases} u(x,y,z) = u_0(x,y) - z\varphi_x(x,y) = \sum_{I=1}^{n} N_I(x,y)u_{0I} - z\sum_{I=1}^{n} N_I(x,y)\varphi_{xI} \\ v(x,y,z) = v_0(x,y) - z\varphi_y(x,y) = \sum_{I=1}^{n} N_I(x,y)v_{0I} - z\sum_{I=1}^{n} N_I(x,y)\varphi_{yI} \\ w(x,y) = \sum_{I=1}^{n} N_I(x,y)w_I \end{cases} \tag{1}$$

where n is the number of nodes of the plate. Here, φ_x and φ_y are independent of the deflection w, and they are not the derivatives of deflection w. The shape functions $N_I(x,y)$ are computed by the moving-least square approximation.

The strains are defined as

$$\boldsymbol{\kappa} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} u_{0,x} - z\varphi_{x,x} \\ v_{0,y} - z\varphi_{y,y} \\ u_{0,y} + v_{0,x} - z(\varphi_{x,y} + \varphi_{y,x}) \end{bmatrix} = \sum_{I=1}^{n} \mathbf{B}_I^b \boldsymbol{\delta}_I, \quad \boldsymbol{\gamma} = \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{bmatrix} w_{,x} - \varphi_x \\ w_{,y} - \varphi_y \end{bmatrix} = \sum_{I=1}^{n} \mathbf{B}_I^s \boldsymbol{\delta}_I, \tag{2}$$

where

$$\mathbf{B}_I^b = \begin{bmatrix} \mathbf{B}_{0I} & -z\mathbf{B}_{1I} \end{bmatrix} = \begin{bmatrix} N_{I,x} & 0 & 0 & -zN_{I,x} & 0 \\ 0 & N_{I,y} & 0 & 0 & -zN_{I,y} \\ N_{I,y} & N_{I,x} & 0 & -zN_{I,y} & -zN_{I,x} \end{bmatrix}, \quad \mathbf{B}_{0I} = \begin{bmatrix} N_{I,x} & 0 \\ 0 & N_{I,y} \\ N_{I,y} & N_{I,x} \end{bmatrix},$$

$$\mathbf{B}_{1I} = \begin{bmatrix} 0 & N_{I,x} & 0 \\ 0 & 0 & N_{I,y} \\ 0 & N_{I,y} & N_{I,x} \end{bmatrix} \qquad \mathbf{B}_I^s = \begin{bmatrix} \mathbf{0} & \mathbf{B}_{2I} \end{bmatrix} = \begin{bmatrix} 0 & 0 & N_{I,x} & -N_I & 0 \\ 0 & 0 & N_{I,y} & 0 & -N_I \end{bmatrix},$$

$$\mathbf{B}_{2I} = \begin{bmatrix} N_{I,x} & -N_I & 0 \\ N_{I,y} & 0 & -N_I \end{bmatrix},$$

$\boldsymbol{\delta}_I = \begin{bmatrix} u_{0I} & v_{0I} & w_I & \varphi_{xI} & \varphi_{yI} \end{bmatrix}^{\mathrm{T}}$. '$_{,x}$' refers to the derivatives of x, '$_{,y}$' refers to the derivatives of y, and δ_I are the nodal parameters of the plate.

2.2. Derivation of the stiffness equation in local coordinates

When subjected to a point load P and a uniformly distributed load q, the potential energy of the laminate is

$$\Pi = \frac{1}{2}\iint\int_{-h/2}^{h/2} \boldsymbol{\kappa}^{\mathrm{T}}\mathbf{D}\boldsymbol{\kappa}\,\mathrm{d}z\,\mathrm{d}x\,\mathrm{d}y + \frac{1}{2}\iint \boldsymbol{\gamma}^{\mathrm{T}}\mathbf{A}_s\boldsymbol{\gamma}\,\mathrm{d}x\,\mathrm{d}y - \iint qw\,\mathrm{d}x\,\mathrm{d}y - Pw \tag{3}$$

where

$$\mathbf{D}=\begin{bmatrix}\bar{Q}_{11}^{(k)} & \bar{Q}_{12}^{(k)} & \bar{Q}_{16}^{(k)}\\ \bar{Q}_{21}^{(k)} & \bar{Q}_{22}^{(k)} & \bar{Q}_{26}^{(k)}\\ \bar{Q}_{61}^{(k)} & \bar{Q}_{62}^{(k)} & \bar{Q}_{66}^{(k)}\end{bmatrix},\quad \tilde{\mathbf{A}}_s=\begin{bmatrix}\tilde{A}_{55} & \tilde{A}_{45}\\ \tilde{A}_{45} & \tilde{A}_{44}\end{bmatrix},\quad \tilde{A}_{ij}=k_c\int_{-h/2}^{h/2}\bar{Q}_{ij}^{(k)}\,\mathrm{d}z=k_c\sum_{k=1}^{N}\bar{Q}_{ij}^{(k)}\left(z_{k+1}-z_k\right),$$

$(i, j = 4, 5)$

$\bar{Q}_{ij}^{(k)}$ $(i, j = 1, 2, 6, 4, 5)$ are the material stiffnesses of the kth layer of the laminate that are defined in the reference (Reddy and Miravete 1995), and k_c=5/6 is the shear correction factor. h is the thickness of the laminate and ($z_{k+1}-z_k$) denotes the thickness of the kth layer. The substitution of equations (1) and (2) into equation (3) gives

$$\Pi=\frac{1}{2}\boldsymbol{\delta}^{\mathrm{T}}\mathbf{K}\boldsymbol{\delta}-\boldsymbol{\delta}^{\mathrm{T}}\mathbf{f} \tag{4}$$

where

$$\begin{aligned}\mathbf{K}_{IJ}&=\iint\int_{-h/2}^{h/2}\begin{bmatrix}\mathbf{B}_{0I}^{\mathrm{T}}\\ -z\mathbf{B}_{1I}^{\mathrm{T}}\end{bmatrix}\mathbf{D}\begin{bmatrix}\mathbf{B}_{0J} & -z\mathbf{B}_{1J}\end{bmatrix}\mathrm{d}z\,\mathrm{d}x\,\mathrm{d}y+\iint\begin{bmatrix}\mathbf{0}\\ \mathbf{B}_{2I}^{\mathrm{T}}\end{bmatrix}\tilde{\mathbf{A}}_s\begin{bmatrix}\mathbf{0} & \mathbf{B}_{2J}\end{bmatrix}\mathrm{d}x\,\mathrm{d}y\\ &=\iint\int_{-h/2}^{h/2}\begin{bmatrix}\mathbf{B}_{0I}^{\mathrm{T}}\mathbf{D}\mathbf{B}_{0J} & -z\mathbf{B}_{0I}^{\mathrm{T}}\mathbf{D}\mathbf{B}_{1J}\\ -z\mathbf{B}_{1I}^{\mathrm{T}}\mathbf{D}\mathbf{B}_{0J} & z^2\mathbf{B}_{1I}^{\mathrm{T}}\mathbf{D}\mathbf{B}_{1J}\end{bmatrix}\mathrm{d}z\,\mathrm{d}x\,\mathrm{d}y+\iint\begin{bmatrix}\mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{B}_{2I}^{\mathrm{T}}\tilde{\mathbf{A}}_s\mathbf{B}_{2J}\end{bmatrix}\mathrm{d}x\,\mathrm{d}y,\end{aligned} \tag{5}$$

$$\left[\mathbf{f}\right]_I=\iint q\mathbf{C}_I^{\mathrm{T}}\,\mathrm{d}x\,\mathrm{d}y+P\mathbf{R}_I^{\mathrm{T}}, \tag{6}$$

$\mathbf{C}_I=\begin{bmatrix}0 & 0 & N_I(x,y) & 0 & 0\end{bmatrix}$, $\mathbf{R}_I=\begin{bmatrix}0 & 0 & N_I(x_p,y_p) & 0 & 0\end{bmatrix}$, and (x_p, y_p) is the point where the concentrated load applies. Taking $\left(\tilde{\mathbf{A}},\tilde{\mathbf{B}},\tilde{\mathbf{H}}\right)=\int_{-h/2}^{h/2}\mathbf{D}\left(1,z,z^2\right)\mathrm{d}z$, we have

$$\mathbf{K}_{IJ}=\iint\begin{bmatrix}\mathbf{B}_{0I}^{\mathrm{T}}\tilde{\mathbf{A}}\mathbf{B}_{0J} & -\mathbf{B}_{0I}^{\mathrm{T}}\tilde{\mathbf{B}}\mathbf{B}_{1J}\\ -\mathbf{B}_{1I}^{\mathrm{T}}\tilde{\mathbf{B}}\mathbf{B}_{0J} & \mathbf{B}_{1I}^{\mathrm{T}}\tilde{\mathbf{H}}\mathbf{B}_{1J}+\mathbf{B}_{2I}^{\mathrm{T}}\tilde{\mathbf{A}}_s\mathbf{B}_{2J}\end{bmatrix}\mathrm{d}x\,\mathrm{d}y, \tag{7}$$

and for the symmetric laminates, $\tilde{\mathbf{B}}=0$. Invoking $\delta\Pi = 0$ results in the following linear system of $\boldsymbol{\delta}$, which governs the elastic bending behavior of the laminates:

$$\mathbf{K}\boldsymbol{\delta}=\mathbf{f} \tag{8}$$

3. Formulation for Folded Laminated Plates

Since a folded laminated plate is modeled as an assembly of laminates, the next step is to superpose the stiffness equations of the laminates to obtain the stiffness equation of the entire structure. However, as pointed out by the authors (Liew et al. 2006), direct superposition of the stiffness equations may lead to failure of the analysis. Therefore, here we follow the same route as that in the reference (Liew et al. 2006) to modify the stiffness equations before the superposition.

3.1. Modification of the stiffness equations

The matrices and vectors in the stiffness equation are modified as

$$\tilde{\mathbf{K}} = \mathbf{\Lambda}^{-1}\mathbf{K}\mathbf{\Lambda}^{-\mathrm{T}}, \quad \tilde{\mathbf{f}} = \mathbf{\Lambda}^{-1}\mathbf{f}, \quad \tilde{\boldsymbol{\delta}} = \mathbf{\Lambda}^{\mathrm{T}}\boldsymbol{\delta}, \tag{9}$$

where the transformation matrix Λ was defined in the reference (Liew et al. 2006) and the nodal parameter $\boldsymbol{\delta}$ is modified to the nodal displacement $\tilde{\boldsymbol{\delta}}$. Therefore, the modified stiffness equation is

$$\tilde{\mathbf{K}}\tilde{\boldsymbol{\delta}} = \tilde{\mathbf{f}} \tag{10}$$

After the modification, the essential boundary conditions can be enforced directly, as well.

3.2. The governing equation of the entire folded plate

Because the stiffness equation (10) is established in the local coordinates, transformation of the equation to the global coordinate is required. The stiffness matrix and the force vector in the global coordinates are

$$\bar{\mathbf{K}} = \mathbf{T}\tilde{\mathbf{K}}\mathbf{T}^{\mathrm{T}}, \quad \bar{\mathbf{f}} = \mathbf{T}\tilde{\mathbf{f}}, \tag{11}$$

where $\mathbf{T}$ is the a $6n \times 6n$ coordinate transformation matrix derived in the reference (Liew et al. 2006). The superposition of the stiffness matrices $\bar{\mathbf{K}}$ and force vectors $\bar{\mathbf{f}}$ of the laminates in the global coordinates gives us the governing equation of the entire folded laminated plate:

$$\mathbf{K}_G\boldsymbol{\delta}_G = \mathbf{f}_G. \tag{12}$$

4. Results and Discussion

In the following sections, several numerical examples are computed with the proposed method, and the results are compared to those given by ANSYS. In all of the following examples, all of the plies are assumed to have the same thickness and orthotropic material properties: $E_1 = 2.5\times10^7$ Pa, $E_2 = 1\times10^6$ Pa, $G_{12} = G_{13} = 5\times10^5$ Pa, $G_{23} = 2\times10^5$ Pa, and $\mu_{12} = 0.25$. In ANSYS, we modeled the folded laminated plate structures as shells and chose the linear layered structural shell element SHELL99 to discretise the structures.

4.1. Validation Studies

An example is employed to show the convergence of the proposed method, and the effect that the domain of influence of the nodes has on this convergence. A folded laminated plate that is made up of two identical square laminates is subjected to a uniformly distributed load (q = 10 Pa), which is applied vertically (Figure 2, α = 90°). The global coordinates (x-y-z) of the entire folded plate and the local coordinates (x1-y1) and (x2-y2) of the laminates are defined in Figure 2. The lamination scheme is (-45°/45°/45°/-45°). The folded plate is clamped at sides a and b.

We take the number of the nodes that discretise the flat plate and the scaling factor $d_{\max}$ as variables to carry out the convergence study. The scaling factor $d_{\max}$ denotes the size of the compact support of the nodes. The central deflections of flat plate A, calculated by the proposed method under a different $d_{\max}$, are shown in Figure 3, and compared with the results from the ANSYS (5581 nodes). Figure 3 shows that all of the present results converge to the ANSYS result when the number of nodes increases. A larger support size can achieve relatively better convergence characteristics.

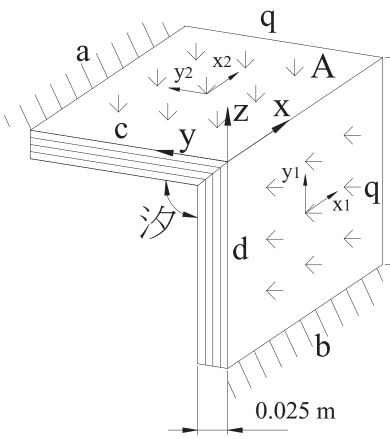

Figure 2: A folded laminated plate that is made up of two identical square laminates.

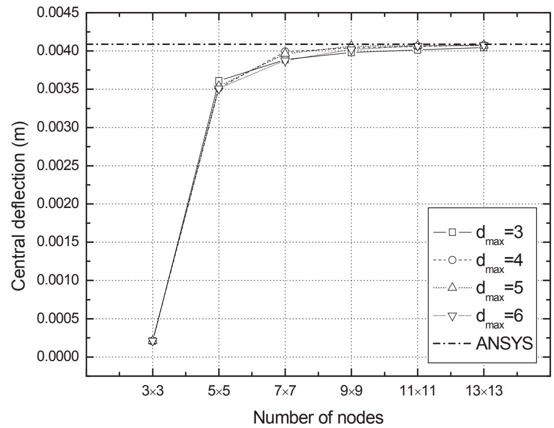

Figure 3: Variation of the central deflections of laminate A.

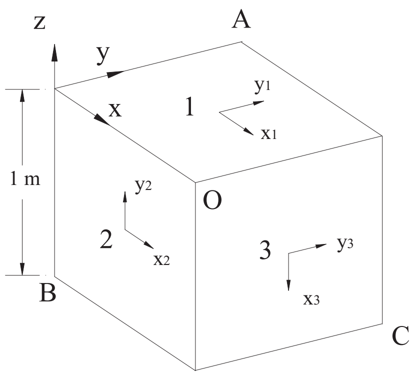

Figure 4. Half of a box structure.

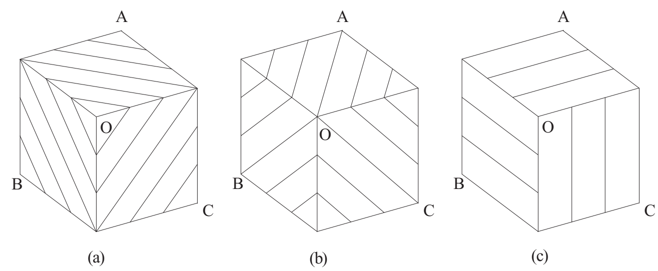

Figure 5. Lamination schemes of the laminates

4.2. Half of a box structure

Three square laminates that are identical in their dimensions are joined with each other vertically and form half of a box structure (Figure 4). The thickness of every laminate is 0.05 m. The structure is pinned at points A, B, and C. All of the DOFs, except the rotations of these points, are set to zero. The same uniformly distributed load (q = 10 Pa) as in 4.1 is applied vertically to every laminate.

Three lamination schemes are considered. Case 1: laminate 1 is taken to be (45°/-45°/-45°/45°) and laminates 2 and 3 to be (-45°/45°/45°/-45°), as is demonstrated in Figure 5a; Case 2: laminate 1 is taken to be (-45°/45°/45°/-45°) and laminates 2 and 3 to be (45°/-45°/-45°/45°), as is demonstrated in Figure 5b; and Case 3: laminate 1 is taken to be (90°/0°/0°/90°) and laminates 2 and 3 to be (0°/90°/90°/0°), as is shown in Figure 5c.

The deflection along x = 0.5 m of laminate 1 in Cases 1 and 2, and x = 0.5 m and y = 0.5 m in Case 3, are computed by both the proposed method and ANSYS and shown in Figures 6 to 7 (because of symmetry, the deflection along the y = 0.5 m of laminate 1 in Cases 1 and 2 can be obtained accordingly).

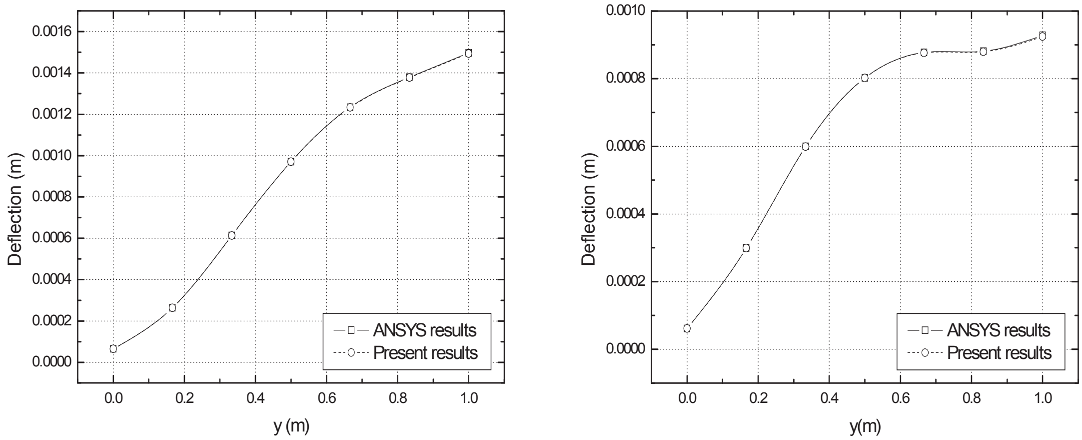

Figure 6. Deflections along x = 0.5 m of laminate 1 of Case 1 (left, Figure 5a) and Case 2 (right, Figure 5b).

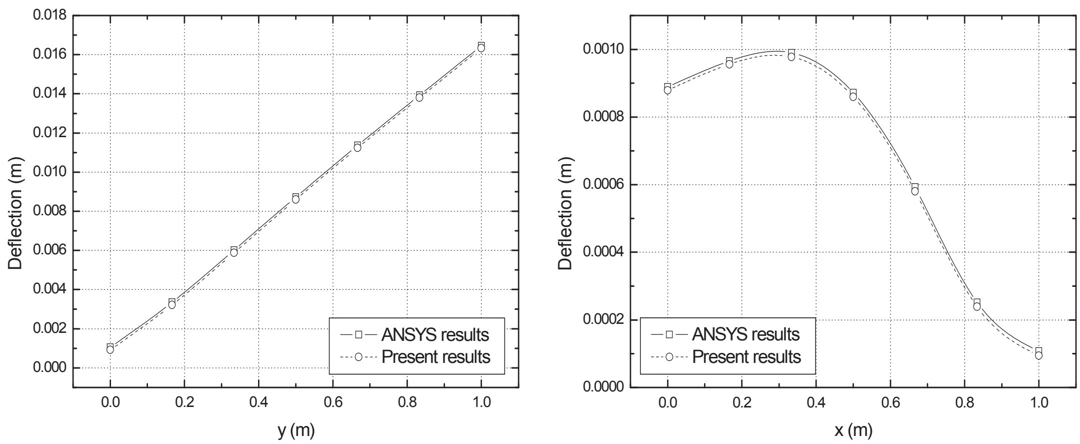

Figure 7. Deflections along x = 0.5 m (left) and y = 0.5 m (right) of laminate 1 of Case 3 (Figure 5c).

In ANSYS, 3025 nodes are used to discretise the structure. The agreement of the two sets of results is good. It is also noted that, for the same composite material that is used to make the structure, the lamination scheme of Case 2 (Figure 5b) leads to the smallest deflection. Therefore, the lamination scheme of Case 2 is the best choice when constructing such a box structure using laminated plates, whereas Case 3 is apparently the worst choice, because the deflection along x = 0.5 m is the largest of the three.

5. Conclusions

This paper proposes a meshfree method based on FSDT for the elastic bending analysis of folded laminated plate structures. The folded laminated plates are considered to be assemblies of flat symmetrical laminates. The global stiffness equation of the folded plate is formed by superposing the stiffness equations of the laminates derived with the meshfree method. The convergence and accuracy of the proposed method are demonstrated by a comparison of the solutions of several examples with those given by ANSYS. Good agreement between the two sets of results is observed. The proposed method can also be employed to analyze box beams and closed structures.

Acknowledgements

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