Available online at www.sciencedirect.com

ScienceDirect

Procedia Engineering 14 (2011) 2949–2956

Procedia Engineering

www.elsevier.com/locate/procedia

The Twelfth East Asia-Pacific Conference on Structural Engineering and Construction

A Genetic Algorithm Based Form-Finding for Tensegrity Structure

M. Yamamoto[1a], B. S. Gan[2b], K. Fujita[1] and J. Kurokawa[1]

[1]*Department of Architecture, Graduate School of Engineering, Nihon University, Japan*
[2]*Department of Architecture, College of Engineering, Nihon University, Japan*

Abstract

A tensegrity structure consists of a set of continuous cables in tension and a set of discontinuous struts in compression. The tensegrity structure can be classified into self-stressed and pre-stressed structures. Present paper interest is in the self-stressed tensegrity structures, since they can free standing without any support while maintaining their self-equilibrium state.

In the process form-finding of a tensegrity structure, some constraints are usually introduced for geometry and/or member to ensure uniqueness of the solution. The tensegrity structures are indeterminate problems in most cases. In this paper, a genetic algorithm based form-finding for tensegrity structures is presented to assist designers to obtain tensegrity structures with less design variables.

A novel and versatile numerical form-finding procedure which requires only a minimal knowledge of initial structure configuration is adopted. The procedure needs only the prototype of each member, i.e. either compression or tension, and the connectivity information of members. The connectivity of members and its prototype information are encoded to form an individual population used in genetic algorithm searching problems. As for the fitness evaluation to each population, the existence of self-stressed state in each population is sought. At the end, some numerical examples are given to show the efficiency of the present study and its ability in searching new configurations of tensegrity structures.

Keywords: Tensegrity, Form-finding, Genetic Algorithm, Self-stressed.

[a] Presenter: Email: g22230@cc.ce.nihon-u.ac.jp
[b] Corresponding author: Email: buntara@arch.ce.nihon-u.ac.jp

1877–7058 © 2011 Published by Elsevier Ltd. Open access under CC BY-NC-ND license.
doi:10.1016/j.proeng.2011.07.371

1. Introduction

A tensegrity structure consists of a set of continuous cables in tension and a set of discontinuous struts in compression. Tensegrity structures are usually associated with pin-jointed mechanism which is stabilized by the action of pre-stress. Generally, the tensegrity structures are statically indeterminate. The tensegrity structure can be classified into self-stressed and pre-stressed structures. Present paper interest is in the self-stressed tensegrity structures, since they can free standing without any support while maintaining their self-equilibrium state. In this paper, a genetic algorithm based form-finding of tensegrity structures is presented to assist designers to search tensegrity structures with less design variables.

2. Form-Finding of Tensegrity Structures

Most of existing form-finding procedures (Motro 2003; Tibert and Pellegrino 2003) require initial assumptions on the length of member, geometry or symmetrical information of the structure. Here, a novel and versatile numerical form-finding procedure (Estrada et al. 2006) which requires only a minimal knowledge of initial structure configuration is adopted. The procedure requires only a prototype of tension coefficient, i.e. either compression or tension, and connectivity information of members. In (Estrada et al. 2006), the form-finding of tensegrity structure is found by a full iteration procedure until a state of self-stress is obtained. Full iterative process in form-finding does not fit with the GA because the iteration will violate the evolutionary process. In present study, one time iteration is used to search a self-stress state of tensegrity structures.

3. Genetic Algorithm

Genetic Algorithm (GA) is one among the trendy computation algorithms for searching problems based on the mechanics of natural selection and genetics (Goldberg 1999). The most crucial effort in developing a form-finding method for a tensegrity structure is how to define the input parameters which have to be encoded into genes to form a chromosome of an individual population. In this study, connectivity matrix and prototype tension coefficient are encoded into two different chromosomes to form an individual population with different genetic information.

3.1. Equilibrium equation of tensegrity structure

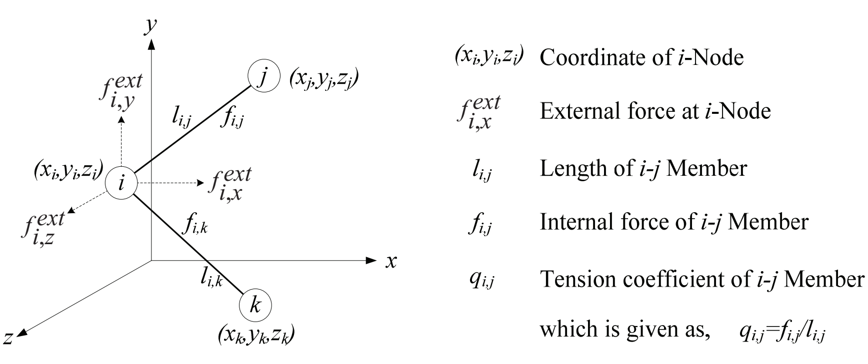


Figure 1: Equilibrium at node i.

Figure 1 is used to illustrate the equations of static equilibrium of a reference node *i* connected to nodes *j* and *k* by members *i,j* and *i,k*, respectively.

The tension coefficient (Southwell 1920), or force density (Schek 1974) coefficient is often used to simplify the equilibrium equations which are given by the following equations.

$$
\begin{aligned}
(q_{i,j}+q_{i,k})x_i - q_{i,j}x_j - q_{i,k}x_k &= f_{i,x}^{ext} \\
(q_{i,j}+q_{i,k})y_i - q_{i,j}y_j - q_{i,k}y_k &= f_{i,y}^{ext} \\
(q_{i,j}+q_{i,k})z_i - q_{i,j}z_j - q_{i,k}z_k &= f_{i,z}^{ext}
\end{aligned} \tag{1}
$$

For a tensegrity structure which is self-stressed, no external load is to be applied. Therefore, the equilibrium equation can be written as

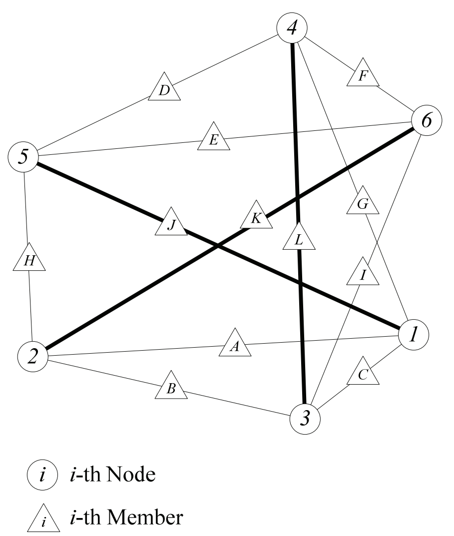
$$
A \cdot t = \begin{pmatrix} C^T diag(Cx) \\ C^T diag(Cy) \\ C^T diag(Cz) \end{pmatrix} \cdot t = 0 \tag{2}
$$

where A is the equilibrium matrix, t is the tension coefficient vector and C is the connectivity matrix of $b \times n$ size which is given by

$$
C_{i,j} = \begin{Bmatrix} +1 & \text{if } j \text{ is the initial node of member } i \\ -1 & \text{if } j \text{ is the terminal node of member } i \\ 0 & \text{otherwise} \end{Bmatrix} \tag{3}
$$

here, b is the number of members and n is number of nodes.

3.2. Encoding scheme for individual population



Node Number

$$
\begin{array}{c} \\ \boldsymbol{C} = \end{array}
\begin{array}{c}
\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\
\begin{bmatrix}
1 & -1 & 0 & 0 & 0 & 0 \\
0 & 1 & -1 & 0 & 0 & 0 \\
1 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & -1 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 \\
0 & 0 & 0 & 1 & 0 & -1 \\
1 & 0 & 0 & -1 & 0 & 0 \\
0 & 1 & 0 & 0 & -1 & 0 \\
0 & 0 & 1 & 0 & 0 & -1 \\
1 & 0 & 0 & 0 & -1 & 0 \\
0 & 1 & 0 & 0 & 0 & -1 \\
0 & 0 & 1 & -1 & 0 & 0
\end{bmatrix}
\end{array}
\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \\ I \\ J \\ K \\ L \end{array}
\qquad
\boldsymbol{t} = \left\{ \begin{array}{c} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ -1 \\ -1 \\ -1 \end{array} \right\}
\begin{array}{c} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{array}
$$

Connectivity Matrix *b* x *n*

Tension Coefficient Vector *b* x *1*

Figure 2: Encoding scheme for triplex tensegrity structure example.

Here, a triplex tensegrity structure is used to illustrate the encoding scheme. Figure 2 shows the encoding for the triplex tensegrity as an individual population which is presented by two separated chromosomes: connectivity matrix and prototype tension coefficient as genetic information.

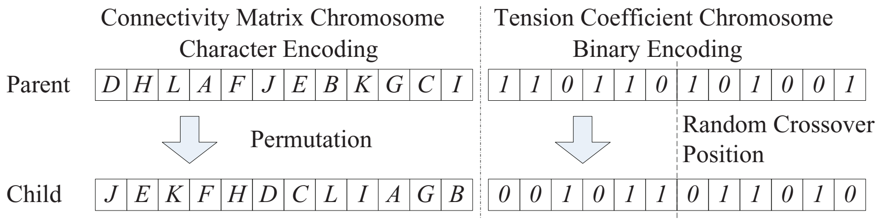


Figure 3: Genetic operation for triplex tensegrity structure example.

The connectivity matrix is encoded into character based chromosome. The binary based chromosome is used for encoding the tension coefficient vector, in which the value of 1 represents the tensile member and the value of 0, represents the compressive member. Figure 3 shows genetic operation used in the present study. Permutation in reproduction process is used in the connectivity matrix chromosome to ensure that there are no members having the same connectivity.

3.3. Solution procedure

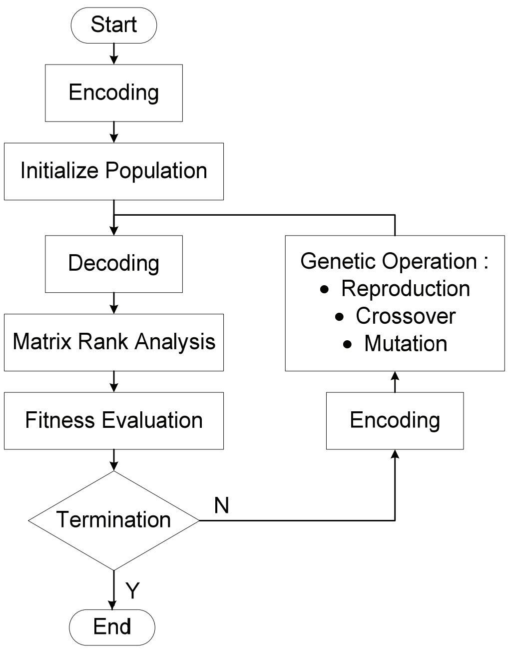


Figure 4: Solution procedure for searching a tensegrity structure algorithm.

Figure 4 gives an overview of the present search process for a tensegrity structure. The process keeps reproducing generations until the tensegrity structure is found.

3.4. Fitness and penalty functions

In this study, one fitness function combined with two constraints which are transformed into penalty functions (Michalewicz 1994) to evaluate the fitness of each individual in the population are adopted. The fitness function is determined from the result of seeking the state of self-stress of an individual tensegrity structure. The state of self-stress of a tensegrity structure is evaluated from connectivity matrix and vector of tension coefficient which are decoded from the individual population. The fitness evaluation for an individual population is defined from a summation of three fitness function as below.

$$f = f_1 + f_2 + f_3 \tag{4}$$

3.4.1. Equilibrium and State of Self-Stress

The fitness of an individual population is evaluated by using the following procedures. The force density matrix, D, is constructed from the vector of tension coefficients, t, and the connectivity matrix, C, from each individual population.

$$D = C^T diag(t)\, C = UVU^T \tag{5}$$

By performing the Schur decomposition (Meyer 2000), the first ($d+1$) columns of the matrix U contain the basis of nodal coordinates calculated from the tension coefficient vector. The d is a parameter for dimensional space. The approximation of coordinates can be obtained from

$$\begin{bmatrix} x & y & z \end{bmatrix} = \begin{bmatrix} u_1 & u_2 & . & . & u_{d+1} \end{bmatrix} \cdot T \text{, with } T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}. \tag{6}$$

The set of approximated coordinates, x, y and z, are then used to compute the equilibrium matrix A.

A Singular Value Decomposition (Meyer 2000) analysis of matrix A in equation (2) by using the coordinates from equation (6) will result in singular vectors of tension coefficient, t.

$$A = UVW^T \tag{7}$$

The resulting singular values of diagonal matrix V follows the decreasing sequence of $diag(v_{1,1} \ldots v_{end,end})$. When zeroes in the diagonal matrix V obtained, it reduces the number of rank of the matrix which further imply the number of states of self-stress which are exist for the tensegrity structure. Here, the state of self-stress of the diagonal matrix V is used as the fitness value for the individual population. To narrow the searching space, only one rank of deficiency is sought which gives only one state of self-stress of the tensegrity structure. Deficiency of one rank is found when the value of right bottom diagonal in the matrix is considerably small and approaching zero.

$$f_1 = V(end, end) \tag{8}$$

3.4.2. Connectivity at a Node

To eliminate any infeasible solution resulting from very few member connections at a node, a predefined minimum number of members, NC, at a node is imposed by the following constraint evaluation function.

$$f_2 = \sum_{j=1}^{n} \left| \left(NC - \sum_{i=1}^{b} \left| C(i,j) \right| \right) \right| \tag{9}$$

Equation (9) will result in zero value as the best fitness if the number of members at a node equal to *NC*, elsewhere will give fitness value larger than zero.

3.4.3. Separation between Struts

One of the typical characteristic of tensegrity structure is that all the compressive strut members have to be independent and separated each other to guarantee that all the strut members are in compression. In order to eliminate the connection which has two struts, the following constraint evaluation function which is transformed into fitness value is adopted.

$$f_3 = \sum_{j=1}^{n} \left| \left(NS - \sum_{i=1}^{b} CT(i,j) \right) \right| \text{ where, } CT = diag(t)^T \times \left| C \right| \tag{10}$$

here *NS* can be obtained from $NS = NC\text{-}2$. Equation (10) will give the best fitness value of zero only if there are no two strut members connected at a node, otherwise will give value larger than zero.

4. Examples

The conventional GA (Goldberg 1999) such as reproduction, crossover, mutation and elitism operators are used for implementing GA in the present study. The GA is run with several individuals in population, where one individual is reserved as elite after each generation. The GA is run with mutation probability of 0.2 value.

The input parameters for this example are n = 6 and 10 nodes (two examples), d = 3 dimensional, and NC = 4 is a predefined number of member connectivity's at a node. Figure 5 and 6 show the results of form-finding of tensegrity structures from 6 and 10 nodes examples.

Summary of parameters used in the examples shown in Figure 5 and 6 are given in Table 1.

Table 1: Summary for different population size

Case	*6-Node Example*		*10-Node Example*	
	Population Size	Required Generation	Population Size	Required Generation
I	10	134	30	5,837
II	20	55	40	2,581
II	30	2	50	1,480

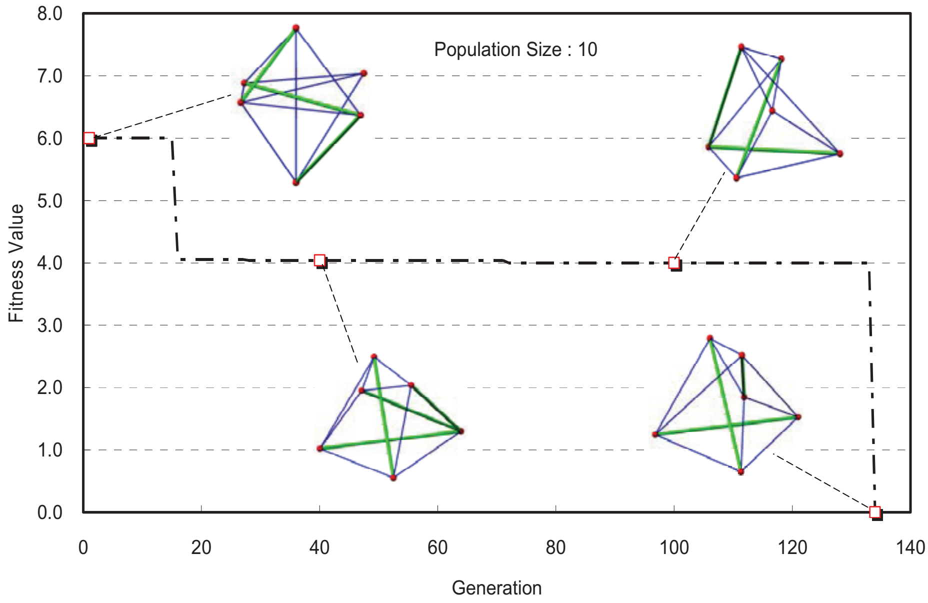


Figure 5: The 6-node result by using 10 population size.

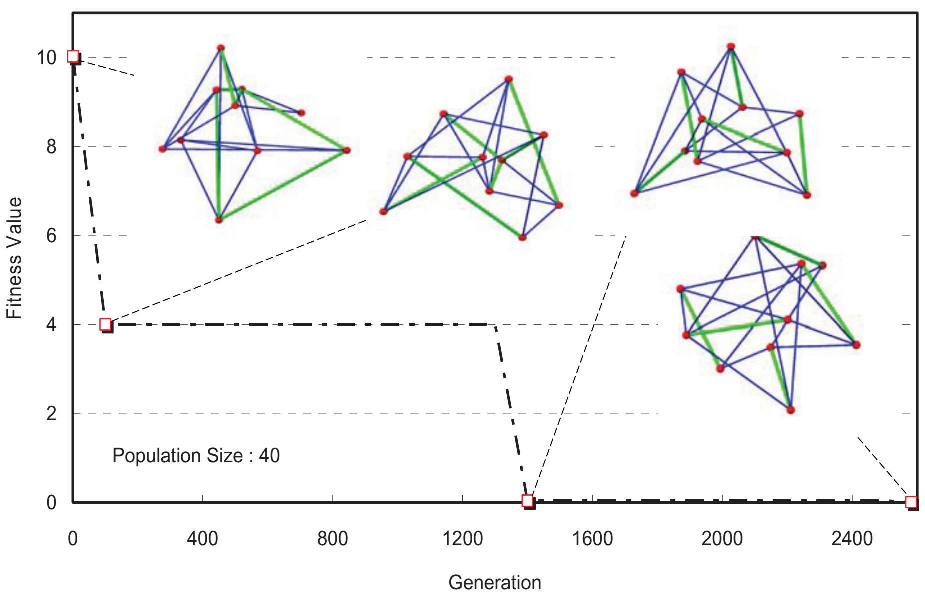


Figure 6: The 10-node result by using 40 population size.

5. Conclusions

From the results shown in the previous examples, several conclusions can be drawn as follow.

- By adopting the form-finding procedure (Estrada et al. 2006) which requires only less information such as dimensional space, connectivity matrix and tension coefficient, the implementation of GA to the form-finding of tensegrity structures problems becomes very easy and straightforward. In the present study, the new idea of encoding the connectivity matrix and tension coefficient into genetic information of an individual population accommodates the implementation of GA process in searching for a self-stressed tensegrity structure. Finally, with only a few constraints defined and number of nodes required, a self-stressed tensegrity structure could be sought by using the evolutionary process.
- The more number of nodes used in the GA simulation, the more generation process is required until the form-finding of the tensegrity structure is achieved.
- The application of the GA to the form-finding of tensegrity structure is highly depend on the need of the designer. Hence, addition of some constraints, criteria or fitness functions to the present proposed method is recommended to achieve results as desired.

References

[1] Estrada GG, Bungartz HJ, and Morhdieck C (2006). Numerical form-finding of tensegrity structures. International Journal of Solids and Structures. 43, pp.6855-6868.

[2] Goldberg DE (1999). Genetic Algorithms: in Search, Optimization & Machine Learning. Addison Wesley Longman, Inc. 20th Edition.

[3] Meyer CD (2000). Matrix Analysis and Applied Linear Algebra. SIAM.

[4] Michalewicz Z (1994). Genetic Algorithms + Data Structures = Evolution Programs. Springer-Verlag, 2nd Edition.

[5] Motro R (2003). Tensegrity: Structural Systems for the Future. Kogan Page Science.

[6] Schek HJ (1974). The force density method for form finding and computation of general networks. Computer Methods in Applied Mechanics and Engineering. 3, pp.115-134.

[7] Southwell RV (1920). Primary stress-determination in space frames. Engineering, CIX, pp.165-168.

[8] Tibert AG and Pellegrino S (2003). Review of form-finding methods for tensegrity structures. International Journal of Space Structures 18(4), pp.209-223.