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Anomalous dimension of Dirac's gauge-invariant nonlocal order parameter in Ginzburg–Landau field theory

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Abstract

In a Ginzburg–Landau theory with n fields, the anomalous dimension of the gauge-invariant nonlocal order parameter defined by the long-distance limit of Dirac's gauge-invariant two-point function is calculated. The result is exact for all n to first order in $\epsilon \equiv 4 - d$, and for all $d \in (2, 4)$ to first order in $1/n$, and coincides with the previously calculated gauge-dependent exponent in the Landau gauge.

1. Introduction

An outstanding problem in gauge theories is the construction of physical correlation functions or propagators of the charged matter fields. As such objects involve fields located at different points in spacetime, the standard forms, expressed solely in terms of the matter fields, are in general not gauge invariant and, consequently, not physical. The principle of gauge invariance by itself does not yield a unique prescription, and various solutions have been proposed a long time ago, notably by Dirac [1] and by Schwinger [2]. These proposals have been used to investigate important physical problems such as anomalies, quark potentials, and order parameters distinguishing the different phases of gauge theories. Recently, the issue has received considerable attention in the context of high-temperature superconductors, where massless Dirac fermions coupled to a dynamical gauge field were put forward as an effective theory for studying the unusual properties of the normal state of underdoped materials [3].

In this Letter we contribute to this issue by showing that different gauge-invariant proposals for correlation functions lead to different physical results. We do so by considering the Abelian Higgs or Ginzburg–Landau model,

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which describes a great variety of physical systems, ranging from scalar QED, superconductors over liquid crystals to cosmic strings and vortex lines in superfluids [4]. The model consists of a $|\phi|^4$-theory coupled minimally to the electromagnetic gauge field A_μ, with $\mu = 1, \ldots, d$. Its Hamiltonian is

$$\mathcal{H} = \left|(\partial_\mu + ieA_\mu)\phi\right|^2 + m^2|\phi|^2 + \lambda|\phi|^4 + \frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2\alpha}(\partial_\mu A_\mu)^2, \tag{1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The scalar field ϕ has $n/2$ complex components and a $\mathrm{O}(n)$-symmetric self-interaction with coupling constant λ. The coefficients e and m denote electric charge and mass parameters of the complex ϕ field, respectively. The last term with parameter α fixes a Lorentz-invariant gauge. We use mostly the notation of statistical field theory in d space dimensions. The results apply, however, equally to quantum field theory in d spacetime dimensions in the Euclidean formulation.

In the following, we will work at criticality by setting $m = 0$. The free correlation function $G(x - x') = \langle\phi(x)\phi^\dagger(x')\rangle_0$ of the scalar field is the Fourier transform of $1/k^2$:

$$G(x) = \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{\mathrm{e}^{\mathrm{i}k\cdot x}}{k^2} = \frac{\Gamma(d/2 - 1)}{4\pi^{d/2}} \frac{1}{x^{d-2}}. \tag{2}$$

For the free correlation function $D_{\mu\nu}(x - x') = \langle A_\mu(x) A_\nu(x')\rangle_0$ of the gauge field, we must Fourier transform

$$D_{\mu\nu}(q) = \frac{1}{q^2}\left[\delta_{\mu\nu} - (1 - \alpha)\frac{q_\mu q_\nu}{q^2}\right], \tag{3}$$

and obtain

$$D_{\mu\nu}(x) = \frac{\Gamma(d/2 - 1)}{8\pi^{d/2} x^{d-2}}\left[(1 + \alpha)\delta_{\mu\nu} + (d - 2)(1 - \alpha)\frac{x_\mu x_\nu}{x^2}\right]. \tag{4}$$

In the presence of interactions, the expectation value $\langle\phi(x)\phi^\dagger(x')\rangle$ is an unphysical quantity since it is not invariant under gauge transformations

$$\phi(x) \to \mathrm{e}^{\mathrm{i}e\Lambda(x)}\phi(x), \qquad A_\mu(x) \to A_\mu(x) - \partial_\mu \Lambda(x). \tag{5}$$

In fact, it vanishes identically due to Elitzur's theorem [5]. A gauge-invariant correlation function was first proposed by Dirac [1]. Adapted to our purposes, it reads

$$\mathsf{G}(x - x') = \left\langle\phi(x)\mathrm{e}^{\mathrm{i}e\int \mathrm{d}^d z\, J_\mu(z) A_\mu(z)}\phi^\dagger(x')\right\rangle. \tag{6}$$

The average denoted by angle brackets is taken with respect to the full Hamiltonian (1), and the external current $J_\mu(z)$ satisfies the equations

$$\partial_\mu J_\mu(z) = \delta(z - x') - \delta(z - x), \qquad \partial^2 J_\mu(z) = 0, \tag{7}$$

where the first ensures the conservation of the external current in the presence of a source of strength $+1$ at x and a sink of strength -1 at x'.[1] The explicit form of the external current (see Fig. 1) is $J_\mu(z) = J'_\mu(z - x') - J'_\mu(z - x)$, where

$$J'_\mu(z) = -i\int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{k_\mu}{k^2} \mathrm{e}^{ik\cdot z} = -\frac{\Gamma(d/2 - 1)}{4\pi^{d/2}} \partial_\mu \frac{1}{z^{d-2}}. \tag{8}$$

[1] When the model (1) is viewed as a quantum field theory in d spacetime dimensions, the source and sink in Eq. (7) correspond to an instanton and antiinstanton, respectively. This is different from Dirac's original construction, where that equation only refers to the $d - 1$ spatial components J_i of the current, taking the form $\partial_i J_i(z) = \delta(z - x') - \delta(z - x)$ and $\partial_i^2 J_j(z) = 0$. The source and sink then represent electric charges which generate an electric dipole field, that is needed to satisfy Gauss's law.

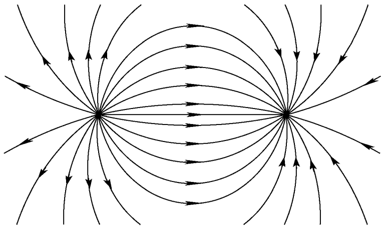

Fig. 1. Flow lines corresponding to the Dirac proposal for the external current $J_\mu(z)$ specified by Eqs. (7) in two dimensions.

Being nonlocal, the current in Eq. (6) is more properly denoted by $J_\mu(z; x, x')$. At the critical point, the gauge-invariant correlation function (6) is expected to have the power behavior

$$\mathsf{G}(x) \sim \frac{1}{x^{d-2+\eta_{\mathrm{GI}}}}, \tag{9}$$

with the Fisher exponent η_{GI}. In the ordered phase, the correlation function (6) has the large-distance behavior

$$\mathsf{G}(x-x') \underset{|x-x'|\to\infty}{\longrightarrow} |\tilde{\phi}|^2, \tag{10}$$

where $\tilde{\phi}(x)$ is the nonlocal order parameter

$$\tilde{\phi}(x) \equiv \mathrm{e}^{-\mathrm{i}e\int \mathrm{d}^d z\, J'_\mu(z-x) A_\mu(z)} \phi(x). \tag{11}$$

Since $J'_\mu(z)$ is a total derivative (see Eq. (8)), $\tilde{\phi}(x)$ reduces after a partial integration in the Landau gauge $\partial_\mu A_\mu = 0$ to the local form $\phi(x)$ [6]. In other words, J_μ becomes invisible in this gauge and the value for η_{GI} is expected to coincide with the gauge-dependent result for $\langle \phi(x)\phi^\dagger(x')\rangle$ obtained in the gauge $\alpha = 0$.

The purpose of this Letter is to determine η_{GI} to first order in $\epsilon \equiv 4-d$ and also for all $d \in (2,4)$ to first order in $1/n$.

Note that of the two equations in (7), only the source equation is needed for gauge invariance of (6). This can also be solved by the δ-function on a line L running from x' to x,

$$J_\mu(z) = \delta_\mu(z; L) \equiv \int_L \mathrm{d}s\, \frac{\mathrm{d}\bar{x}_\mu(s)}{\mathrm{d}s} \delta\big(z - \bar{x}(s)\big). \tag{12}$$

For a straight line L with $\bar{x}_\mu(s) = x'(1-s) + sx$, $s \in [0,1]$, this leads to Schwinger's gauge-invariant correlation function [2]

$$\big\langle \phi(x) \mathrm{e}^{-\mathrm{i}e\int_x^{x'} \mathrm{d}\bar{x}_\mu A_\mu(\bar{x})} \phi^\dagger(x') \big\rangle, \tag{13}$$

whose critical properties we studied in Ref. [11]. The Schwinger construction can be thought of having all the external current originating from the source at x and terminating at the sink at x' squeezed into an infinitely thin line along the shortest path connecting the two points. In the disordered phase, the current lines have a finite line tension, and this correlation function vanishes exponentially for large distances. Because the finite line tension exponentially suppresses larger loops, only a few small current loops are present in this phase. Upon approaching the critical point, the line tension vanishes and current loops can grow without energy cost. Their proliferation signals the onset of superconductivity [7–10].

The dependence of Eq. (13) on the shape of L can also be seen more formally. We observe that a deformation of L is a new type of gauge transformation discussed extensively in Refs. [4,8]

$$\delta_\mu(z; L) \to \delta_\mu(z; L') = \delta_\mu(z; L) + \partial_\nu \delta_{\mu\nu}(z; S), \tag{14}$$

where $\delta_{\mu\nu}(z; S)$ is the δ-function on the surface S swept out in the deformation $L \to L'$. Under this gauge transformation, the correlation function (13) changes by a nontrivial phase

$$\bar{\phi}(x) \underset{L\to L'}{\longrightarrow} \mathrm{e}^{-\mathrm{i}e\int d^d z\, F_{\mu\nu}(z)\delta_{\mu\nu}(z;S)}\bar{\phi}(x). \tag{15}$$

Due to the different physical content of the correlation functions involved, the critical behavior to be derived here for Dirac's correlation function (6) will be quite different from that of Schwinger's (13) calculated in our previous note [11].

2. ϵ-expansion

Perturbation theory yields via Wick's theorem, three perturbative corrections to (6) to lowest order in e^2:

$$\mathsf{G}(x-x') = G + T_0 + T_1 + T_2, \tag{16}$$

where $T_{0,1,2}$ contain zero, one, and two factors of the flow field J_μ.

The term T_0 is calculated by standard methods. Infrared divergences are avoided by evaluating Feynman diagrams at a finite external momentum κ. Being the only scale available, κ is used to render dimensionful parameters such as e^2 dimensionless: $\hat{e}^2 = e^2\kappa^{d-4}$. The result is the well-known gauge-dependent contribution [12,13]:

$$\eta_\phi = \frac{\alpha-3}{8\pi^2}\hat{e}_*^2 = \frac{6}{n}(\alpha-3)\epsilon. \tag{17}$$

The lowest-order ϵ-expansion on the right hand is obtained by inserting for $\hat{e}^2$ the charge $\hat{e}_*^2 = 48\pi^2\epsilon/n$ at the infrared-stable fixed point, which at one loop exists only for $n > 12(15+4\sqrt{15}) \approx 365.9$. At two loop, different resummation techniques suggest the existence of a fixed point for the physical case $n = 2$ [14,15].

Next, we calculate the last term in Eq. (16):

$$T_2(x-x') = -\frac{e^2}{2}G(x-x')\int \mathrm{d}^d z\,\mathrm{d}^d z'\, J_\mu(z)D_{\mu\nu}(z-z')J_\nu(z') \tag{18}$$

which splits in a separate scalar and gauge part. Several integrations by part reduce the integrals in coordinate spacetime to the generic form

$$\int \mathrm{d}^d z\,\frac{1}{|z-x|^{d-2}}\frac{1}{|z-x'|^{d-p}} = \frac{2\pi^{d/2}}{p}\frac{\Gamma(d/2-1-p/2)}{\Gamma(d/2-1)\Gamma(d/2-p/2)}\frac{1}{|x-x'|^{d-2-p}}, \tag{19}$$

and we obtain, with the abbreviations $\partial_\mu \equiv \partial/\partial x_\mu$, $\partial'_\mu \equiv \partial/\partial x'_\mu$,

$$T_2(x-x') = \frac{e^2}{64\pi^{3d/2}}\Gamma^3(d/2-1)G(x-x')\biggl(\partial_\mu\partial'_\mu\int \mathrm{d}^d z\,\mathrm{d}^d z'\,\frac{1}{|z-x|^{d-2}|z-z'|^{d-2}|z'-x'|^{d-2}} - \frac{1}{2}\frac{1-\alpha}{d-4}\partial^2\partial'^2\int \mathrm{d}^d z\,\mathrm{d}^d z'\,\frac{1}{|z-x|^{d-2}|z-z'|^{d-4}|z'-x'|^{d-2}}\biggr). \tag{20}$$

In the limit of small $\epsilon = 4-d$, this reduces to

$$T_2(x-x') = -\hat{e}^2\frac{\alpha}{8\pi^2}G(x-x')\ln\bigl(\kappa|x-x'|\bigr). \tag{21}$$

Comparison with an expansion of (9) in powers of η_{GI} gives a contribution to the Fisher exponent proportional to the gauge-fixing parameter α:

$$\eta_2 = \frac{\alpha}{8\pi^2}\hat{e}_*^2. \tag{22}$$

This result can be checked by considering the ratio of the correlation functions $\langle\phi(x)\phi^\dagger(x')\rangle$ and $\langle \mathrm{e}^{-\mathrm{i}e\int \mathrm{d}^d z\, J_\mu(z) A_\mu(z)}\rangle$. Adapting an argument given in Ref. [16], one can show that this ratio and, consequently, the combination $\eta_\phi - \eta_2$ should be independent of the gauge-fixing parameter α. The expressions (17) and (22) indeed fulfill this requirement. In our previous study [11], we found for the Schwinger correlation function as only difference an additional contribution to η_2 independent of α.

We are left with the calculation of the second, or mixed term in Eq. (16), which reads explicitly

$$T_1(x-x') = e^2 \int \mathrm{d}^d z\, \mathrm{d}^d z' \left[G(x-z) \frac{\overleftrightarrow{\partial}}{\partial z_\mu} G(z-x') \right] J_\nu(z') D_{\mu\nu}(z-z'), \tag{23}$$

where the right-minus-left derivative $\overleftrightarrow{\partial}_\mu = \partial_\mu - \overleftarrow{\partial}_\mu$ operates only within the square brackets. To logarithmic accuracy, we can write [16]

$$T_1(x-x') \approx e G(x-x') \int \mathrm{d}^d z\, \mathrm{d}^d z' \left[\frac{\partial}{\partial z_\mu} G(z-x') - \frac{\partial}{\partial z_\mu} G(x-z) \right] J_\nu(z') D_{\mu\nu}(z-z'). \tag{24}$$

Both terms in the square brackets give the same contribution. Proceeding in the same way as before, we find

$$T_1 = -2T_2, \tag{25}$$

and therefore as contribution to the Fisher exponent

$$\eta_1 = -2\eta_2 = -\frac{\alpha}{4\pi^2} \hat{e}_*^2. \tag{26}$$

This contribution, which is again proportional to the gauge-fixing parameter α, is identical to the one found for the Schwinger correlation function [11]. Added together, we obtain for the manifestly gauge-invariant correlation function (6)

$$\eta_{\mathrm{GI}} = \eta_\phi + \eta_1 + \eta_2 = -\frac{3}{8\pi^2} \hat{e}_*^2 = -\frac{18}{n} \epsilon. \tag{27}$$

As expected, this result for the nonlocal Dirac order parameter coincides with the value for η_ϕ obtained in the Landau gauge ($\alpha = 0$). When α is considered a running coupling constant of the theory (1), the Landau gauge emerges as a fixed point $\alpha_* = 0$ of the renormalization group [17]. This is a special case of the more general result [18] that $\alpha = 0$ is always a fixed point when considering a gauge-fixing term of the form $(L_\mu A_\mu)^2/2\alpha$. The choice $L_\mu = \partial_\mu$ then leads to the Landau gauge, while the choice $L_\mu = n_\mu$, with n_μ a constant vector, leads to the axial gauge.

The expression (27) is to be contrasted with $\eta_{\mathrm{GI}} = -(3/4\pi^2)\hat{e}_*^2$ we [11] derived for Schwinger's gauge-invariant correlation function (13). It follows that the exponent (27) characterizing Dirac's correlation function is less negative than the one characterizing Schwinger's. The latter coincides with η_ϕ in the gauge $\alpha = -3$. For this value of α, the external current line connecting x to x' has no effect. A similar observation in the context of quantum chromodynamics was made in Ref. [19].

3. Large-*n* expansion

The leading contribution in $1/n$ generated by fluctuations in the gauge field is obtained by dressing its correlation function with arbitrary many bubble insertions, and adding the infinite set of Feynman graphs [20]. The resulting geometric series leads to the following change in the prefactor of the correlation function (3):

$$\frac{1}{q^2} \to \frac{1}{q^2 + ne^2[c(d)/2(d-1)]\, q^{d-2}}, \tag{28}$$

where the second term in the denominator dominates the first for small q if $d \in (2,4)$. The constant $c\,(d)$ stands for the 1-loop integral

$$c(d) = \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{1}{k^2(k+p)^2}\bigg|_{p^2=1} = \frac{\Gamma(2-d/2)\Gamma^2(d/2-1)}{(4\pi)^{d/2}\Gamma(d-2)}, \tag{29}$$

where analytic regularization is used as before to control ultraviolet divergences. To leading order in $1/n$, the value of η_ϕ for $d \in (2,4)$ reads [21,22]

$$\eta_\phi = \frac{2}{n} \frac{4-d-(d-1)[4(d-1)-d\,\alpha]}{(4\pi)^{d/2}c(d)\Gamma(d/2+1)}, \tag{30}$$

which depends on the gauge-fixing parameter α. For $d = 4-\epsilon$, this result reduces to Eq. (17) obtained to first order in ϵ.

We next consider the gauge-invariant version of this. The term T_2 in (18) can be evaluated as before. To extract the dependence on $\ln(|x-x'|)$ it will be useful to replace q^{d-2} by $q^{d-2+\delta}$ in Eq. (28), with a dummy parameter δ, which will be taken to zero at the end. Then the large-n limit of the gauge-field correlation function becomes

$$D_{\mu\nu}(q) = \frac{2}{ne^2} \frac{d-1}{c(d)} \frac{1}{q^{d-2+\delta}} \left[\delta_{\mu\nu} - (1-\alpha)\frac{q_\mu q_\nu}{q^2}\right], \tag{31}$$

or in coordinate spacetime

$$D_{\mu\nu}(x) = \frac{8}{ne^2} \frac{d-1}{c(d)} \frac{1}{(4\pi)^{d/2}\Gamma(d/2)} \frac{1}{x^{2-\delta}} \left[\frac{1}{2}(d-3+\alpha)\delta_{\mu\nu} + (1-\alpha)\frac{x_\mu x_\nu}{x^2}\right]. \tag{32}$$

Proceeding in the same way as before, we find after various integrations by parts

$$\begin{aligned} T_2(x-x') = {} & \frac{1}{n} \frac{1}{2^{d+1}\pi^{3d/2}} \frac{d-1}{c(d)} \Gamma(d/2-1) G(x-x') \\ & \times \left(\partial_\mu \partial'_\mu \int \mathrm{d}^d z\, \mathrm{d}^d z' \, \frac{1}{|z-x|^{d-2}|z-z'|^{2-\delta}|z'-x'|^{d-2}} \right. \\ & \left. + \frac{1}{d-2} \frac{1-\alpha}{\delta} \partial^2 \partial'^2 \int \mathrm{d}^d z\, \mathrm{d}^d z' \, \frac{1}{|z-x|^{d-2}|z-z'|^{-\delta}|z'-x'|^{d-2}} \right). \end{aligned} \tag{33}$$

Using the integral formula (19), we obtain for η_2:

$$\eta_2 = \alpha \frac{4}{n} \frac{d-1}{(4\pi)^{d/2}c(d)\Gamma(d/2)}. \tag{34}$$

This large-n result valid for all $d \in (2,4)$ is once more proportional to the gauge-fixing parameter α, just as for small ϵ in Eq. (22). The result can again be easily checked by noting that the combination $\eta_\phi - \eta_2$ is independent of the gauge-fixing parameter α. As for small ϵ, the result for the Schwinger correlation function differs only by an α-independent contribution to η_2.

For the mixed term T_1, we also find for large n the relation (25) between the two contributions T_1 and T_2, and thus $\eta_1 = -2\eta_2$. This expression for η_1 is identical to the one for the Schwinger correlation function. Adding the three contributions together, we arrive at

$$\eta_{\mathrm{GI}} = \frac{2}{n} \frac{(7-4d)d}{(4\pi)^{d/2}c(d)\Gamma(d/2+1)}, \tag{35}$$

independent of the gauge-fixing parameter α. This result, valid for all $d \in (2,4)$, is the leading contribution in $1/n$. As expected, it coincides with the value (30) for η_ϕ obtained in the Landau gauge ($\alpha = 0$). This should be compared to the critical exponent for the Schwinger correlation function [11] (13) which coincides with Eq. (30) obtained in the *traceless gauge* $\alpha = 1 - d$, of which $\alpha = -3$ found in the ϵ-expansion is a special case. In this gauge, where the correlation function $D_{\mu\nu}$ is traceless, the external current line connecting x to x' becomes invisible. Although less than for Schwinger's correlation function, $\eta_{\rm GI}$ found here is negative for small ϵ and all n or for $d \in (2, 4)$ and large n. In a recent Monte Carlo study [23] of the three-dimensional lattice model ($n = 2$) in the London limit where $|\phi| = \text{const}$, the large negative value $\eta_\phi = -0.79(1)$ was obtained in the Landau gauge.

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