Available online at www.sciencedirect.com
ScienceDirect
Procedia Economics and Finance 30 (2015) 690 – 701
Procedia Economics and Finance
www.elsevier.com/locate/procedia

3rd Economics & Finance Conference, Rome, Italy, April 14-17, 2015 and 4th Economics & Finance Conference, London, UK, August 25-28, 2015

# Time Preference and the Intertemporal Substitution of Labour: Böhm-Bawerk Reconsidered

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**Abstract**

This paper shows that the introduction of the intertemporal substitution of labour to the Böhm-Bawerkian system may reinforce the first reason for interest. However, its operation is conditioned upon the second reason. Since people may underestimate (discount) satisfaction not only from future consumption goods but also from future leisure, they may allocate more work to the future. This generates an increasing profile in their real income stream, even if the real wage rate is constant over time. The first reason and the second reason might be closely interconnected; with the second reason being the underlying determinant and the first cause having a subsidiary effect. This paper also shows that the intertemporal substitution in labour may generate a concave "investment opportunity curve", which is usually associated with the third reason of interest and the productivity element in the Böhm-Bawerk theory. Thus, the underestimation of future wants supported by the intertemporal substitution of labour could be the primary source of interest—all curves in the Fisherian diagram may be based on these two phenomena, i.e. on utility.

Peer-review under responsibility of IISES-International Institute for Social and Economics Sciences.
*Keywords:* Intertemporal substitution of labour; time preference; Böhm-Bawerk; interest rate

## 1. Introduction

More than one hundred years ago, Böhm-Bawerk (1891) presented a general theory, aimed at explaining the interest phenomenon in the market economy. In his magnum opus, *Positive Theory of Capital*, interest was identified with a value premium of present goods over future goods, and three famous causes were introduced to elucidate this premium. The first cause was based on the assumption of better provision with future goods, giving present goods higher marginal utility and consequently a value advantage over future goods. The second reason stressed the inherent inclination of human beings to underestimate future wants, scaling down the importance of future goods. The third cause, thoroughly studied by Böhm-Bawerk, was linked to a higher productivity of roundabout methods of production and the resulting technical superiority of present goods.


Peer-review under responsibility of IISES-International Institute for Social and Economics Sciences.
doi:10.1016/S2212-5671(15)01318-0

This paper shows that the introduction of the intertemporal substitution of labour to the Böhm-Bawerkian system may reinforce the first reason for interest. However, its operation is conditioned upon the second reason. Since people may underestimate (discount) satisfaction not only from future consumption goods but also from future leisure, they may allocate more work to the future. This generates an increasing profile in their real income stream, even if the real wage rate is constant over time. As a result, the first reason and the second reason might be closely interconnected; with the second reason being the underlying determinant and the first cause having a subsidiary effect. This paper also shows that the intertemporal substitution in labour may generate a concave "investment opportunity curve", which is usually associated with the third reason of interest and the productivity element in the Böhm-Bawerk theory. Thus, the underestimation of future wants supported by the intertemporal substitution of labour could be the primary source of interest—all curves in the Fisherian (1930) diagram may be based on these two phenomena.

## 2. The model

Consider a representative consumer maximizing a logarithmic additively separable life-time utility function (1) in a simple two-period model. Her utility depends on consumption C and leisure time H in both periods. Future utilities are discounted by subjective discount rate $\rho$, which incorporates the Böhm-Bawerkian idea of the second cause of interest—underestimation of future wants (Olson and Bailey, 1981; Becker and Mulligan, 1997; Frederick et al., 2002).

The relative weight of consumption and leisure in the utility function is represented by parameter $b$. This parameter might also play a role that distinguishes discounting of utility from consumption and from leisure. Alternatively, both terms might be discounted by a different subjective discount rate (i.e. $\rho_C$ and $\rho_H$).*

Equation (2) represents individual's intertemporal budget constraint. $W_0$ and $W_1$ stand for the real wage earned exogenously in period one and two, respectively. $W_0$ and $W_1$ might be understood as parameters in a linear production function $Y_t=A_tL_t$, i.e. $W_0 = dY_0/dL_0 = A_0$ and $W_1 = dY_1/dL_1 = A_1$. Furthermore, labour can be used only in short production processes, i.e. in the creation of the given period output (in earning the given period income).

$$U = \ln C_0 + b\ln H_0 + \frac{1}{1+\rho}\ln C_1 + \frac{b}{1+\rho}\ln H_1 \tag{1}$$

$$C_0 + \frac{1}{1+r}C_1 = W_0L_0 + \frac{1}{1+r}W_1L_1 \tag{2}$$

The time constraint in both periods, with the time endowment normalized to 1, is given by:

$$L_0 + H_0 = 1 \tag{3}$$

$$L_1 + H_1 = 1 \tag{4}$$

Substituting (3) and (4) into (1), the lifetime utility function can be written as:

$$U = \ln C_0 + b\ln(1-L_0) + \frac{1}{1+\rho}\ln C_1 + \frac{b}{1+\rho}\ln(1-L_1) \tag{5}$$

The Lagrange function, along with the first order conditions for consumption, is:

* The model that follows is an extension of a textbook model from Romer (2006).

$$\mathrm{L} = \ln C_0 + b\ln(1-L_0) + \frac{1}{1+\rho}\ln C_1 + \frac{b}{1+\rho}\ln(1-L_1) + \lambda(\mathrm{W}_0\mathrm{L}_0 + \frac{1}{1+r}\mathrm{W}_1\mathrm{L}_1 - C_0 - \frac{1}{1+r}C_1) \tag{6}$$

$$\frac{\partial L}{\partial C_0} = \frac{1}{C_0} - \lambda = 0 \tag{7}$$

$$\frac{\partial L}{\partial C_1} = \frac{1}{1+\rho}\frac{1}{C_1} - \lambda\frac{1}{1+r} = 0 \tag{8}$$

(7) and (8) imply:

$$\frac{C_1}{C_0} = \frac{1+r}{1+\rho} \tag{9}$$

Equation (9) represents the Euler (consumption) equation for this problem.

FOCs for labour are given by:

$$\frac{\partial L}{\partial L_0} = \frac{-b}{1-L_0} + \lambda W_0 = 0 \tag{10}$$

$$\frac{\partial L}{\partial L_1} = \frac{1}{1+\rho}\frac{-b}{1-L_1} + \lambda\frac{W_1}{1+r} = 0 \tag{11}$$

(10) and (11) imply:

$$\frac{1-L_1}{1-L_0} = \frac{W_0}{W_1}\frac{1+r}{1+\rho} \tag{12}$$

Equation (12) represents the Euler (employment) equation for this problem. It describes the optimal allocation of leisure (labour) over time. This system has five unknowns ($C_0$, $C_1$, $L_0$, $L_1$,$\lambda$) in five equations (7,8,10,11,2). Leisure time can be then easily determined from time constraints (3) and (4).

(7) and (10) imply:

$$\frac{1}{C_0} = \frac{b}{1-L_0}\frac{1}{W_0} \tag{13}$$

It is easier to solve this problem for the leisure time. Hence (13) becomes:

$$\frac{1}{C_0} = \frac{b}{H_0}\frac{1}{W_0} \tag{14}$$

Similar manipulations can be done with (8) and (11), which yields:

$$\frac{1}{C_1} = \frac{b}{H_1}\frac{1}{W_1} \tag{15}$$

Substituting (14) and (15) into (2) and using time constraints (3) and (4), equation (2) becomes:

$$\frac{W_0}{b}H_0 + \frac{W_1}{b(1+r)}H_1 = \mathrm{W}_0(1-\mathrm{H}_0) + \frac{1}{1+r}\mathrm{W}_1(1-\mathrm{H}_1) \tag{16}$$

Using (12), equation (16) takes the form:

$$\frac{W_0}{b}H_0 + \frac{W_0}{b(1+\rho)}H_0 = \mathrm{W}_0 - \mathrm{W}_0\mathrm{H}_0 + \frac{1}{1+r}\mathrm{W}_1 - \frac{\mathrm{W}_0}{1+\rho}\mathrm{H}_0 \tag{17}$$

A simple rearrangement of terms above gives us:

$$H_0\left[\frac{W_0}{b} + \mathrm{W}_0 + \frac{W_0}{b(1+\rho)} + \frac{\mathrm{W}_0}{1+\rho}\right] = \mathrm{W}_0 + \frac{1}{1+r}\mathrm{W}_1 \tag{18}$$

$$H_0^* = \frac{1 + \frac{1}{1+r}\frac{\mathrm{W}_1}{\mathrm{W}_0}}{1 + \frac{1}{b} + \frac{1}{1+\rho} + \frac{1}{b(1+\rho)}} \tag{19}$$

Optimum $H_1$ is, using (19) and (12):

$$H_1 = \frac{\mathrm{W}_0}{\mathrm{W}_1}\frac{1+r}{1+\rho} \times \frac{1 + \frac{1}{1+r}\frac{\mathrm{W}_1}{\mathrm{W}_0}}{1 + \frac{1}{b} + \frac{1}{1+\rho} + \frac{1}{b(1+\rho)}} \tag{20}$$

$$H_1^* = \frac{1+(1+r)\dfrac{W_0}{W_1}}{(2+\rho)+\dfrac{1+\rho}{b}+\dfrac{1}{b}} \tag{21}$$

Thus, leisure time (labour) in the present increases (decreases) and leisure time (labour) in the future decreases (increases) when the interest rate falls, the relative intertemporal wage $W_1/W_0$ rises, or when the subjective discount rate grows. Since $L^* = (1–H^*)$ in every period, the individual's labour income in the given period is $Y_t=W_tL_t^*$.

For a constant wage over time ($W_1 = W_0$), the income stream is smoothed when the interest rate is equal to the subjective discount rate (see equation 12). A lower interest rate moves the endowment closer to the vertical axis in Figure 1. The concave "PPF" (which should rather be called the income endowment frontier in this case), sketched as a dashed line in Figure 1, depends only on the utility of leisure time. As a result, the equilibrium real rate of interest will depend only on subjective phenomena, not on productivity.

A decrease in the interest rate and the resulting change in the budget line of an individual are presented in Figure 1. As can be seen, a lower interest rate moves the income endowment point closer to the vertical axis from $A^1$ to $A^2$. The budget line is also flatter. In standard analysis, the budget line rotates around the endowment point A. When the intertemporal substitution in labour is present, however, the pivot point itself is being moved.†

An increase in the interest rate decreases the growth rate in income over time, because it is more profitable to work in the present and relax in the future. Thus, at the individual level, an inverse relationship between the interest rate and the shape of the income stream exists. The phenomenon of the intertemporal substitution of labour brings about a new channel that partly offsets the impact of the income stream on the interest rate presented in the Böhm-Bawerkian analysis. To find the ultimate impact on the interest rate, though, we have to analyse the optimum consumption stream in this model.‡

Substituting the Euler consumption equation (9) and equations (14) and (15) into the intertemporal budget constraint (2), we get:

$$C_0 + \frac{1}{1+\rho}C_0 = W_0(1-\frac{bC_0}{W_0}) + \frac{W_1}{1+r}(1-\frac{b}{W_1}\frac{1+r}{1+\rho}C_0) \tag{22}$$

$$\frac{(1+\rho)C_0 + C_0}{1+\rho} = W_0 - bC_0 + \frac{W_1}{1+r} - \frac{b}{1+\rho}C_0 \tag{23}$$

† It can be shown (Potuzak 2014) that the endowment point $A^2$ is below the old budget line when (for a constant stream of wages) the interest rate is lower than the subjective discount rate (i.e. $r < \rho$) and vice versa. Furthermore, it can be shown that for constant wages all budget lines cross the 45 degree line at the same point W/(1+b).

‡ However, one possible (and most probably correct) interpretation is as follows: a reduction in the interest rate shifts the point which represents the optimum intertemporal allocation of labour and the resulting income endowment (along a hypothetical PPF) to the left (i.e. the growth rate in income rises). At the same time, a decline in the interest rate results in a decrease in the optimal growth rate of consumption. Thus, the equilibrium interest rate can be found where these two tendencies offset each other. On the PPF, an increasing income stream is consistent with a lower interest rate. In case of the (consumption) indifference curve, an increasing consumption stream is associated with a higher interest rate. Thus, it can be said that the lower interest rate decreases the supply of present goods (due to the reduction in the supply of present labour) and raises the demand for present goods (due to higher consumption demand). An increase in the interest rate has the opposite diverging effects. Thus, the interest rate must adjust to equilibrate these two tendencies.

$$\frac{(1+\rho)C_0 + C_0 + b(1+\rho)C_0 + bC_0}{1+\rho} = \mathrm{W}_0 + \frac{\mathrm{W}_1}{1+r} \tag{24}$$

$$C_0(2+\rho+2b+b\rho) = \frac{\mathrm{W}_0(1+r)+\mathrm{W}_1}{(1+r)}(1+\rho) \tag{25}$$

$$C_0(2+\rho)(1+b) = \frac{\mathrm{W}_0(1+r)+\mathrm{W}_1}{(1+r)}(1+\rho) \tag{26}$$

$$C_0(2+\rho)(1+b) = \frac{\mathrm{W}_0(1+r)+\mathrm{W}_1}{(1+r)}(1+\rho) \tag{27}$$

$$C_0^* = \frac{\mathrm{W}_0(1+r)+\mathrm{W}_1}{(1+r)} \frac{(1+\rho)}{(2+\rho)(1+b)} \tag{28}$$

Optimum future consumption is derived when (28) is substituted into (9):

$$C_1^* = \frac{\mathrm{W}_0(1+r)+\mathrm{W}_1}{(2+\rho)(1+b)} \tag{29}$$

Equations (28) and (29) imply that parameter *b* (preference for leisure time) decreases consumption in both periods. Furthermore, the comparison of $C_0^*$ with $W_0L_0^*$ can determine whether the given individual is a lender or a borrower for the given *r*. Figure 1 shows a consumer, whose subjective discount rate is higher than the real interest rate. The stream of wages is assumed constant. According to (12), her endowment point is above the 45º line because she works relatively more in the future. Thus, she also earns relatively more in the future ($Y_1=WL_1^*>Y_0=WL_0^*$). Using (9), her optimal consumption stream is decreasing ($C_1^*< C_0^*$), and this particular consumer is a borrower because $C_0^*> Y_0=WL_0^*$.

Consider a reduction in the interest rate ($r_2 < r_1$). As can be seen in Figure 1, it will raise present consumption from $C_0^{1*}$ to $C_0^{2*}$ and reduce present labour supply, which will consequently decrease present labour income from $Y_0^1$ to $Y_0^2$. The endowment point moves along the given concave "income possibility curve". Moreover, a reduction in the interest rate decreases the amount of saving to a greater extent when the intertemporal substitution of labour exists compared with its absence. The reason lies in a decline in present income, which drives up the difference between present income ($Y_0$) and present optimal consumption ($C_0$*).

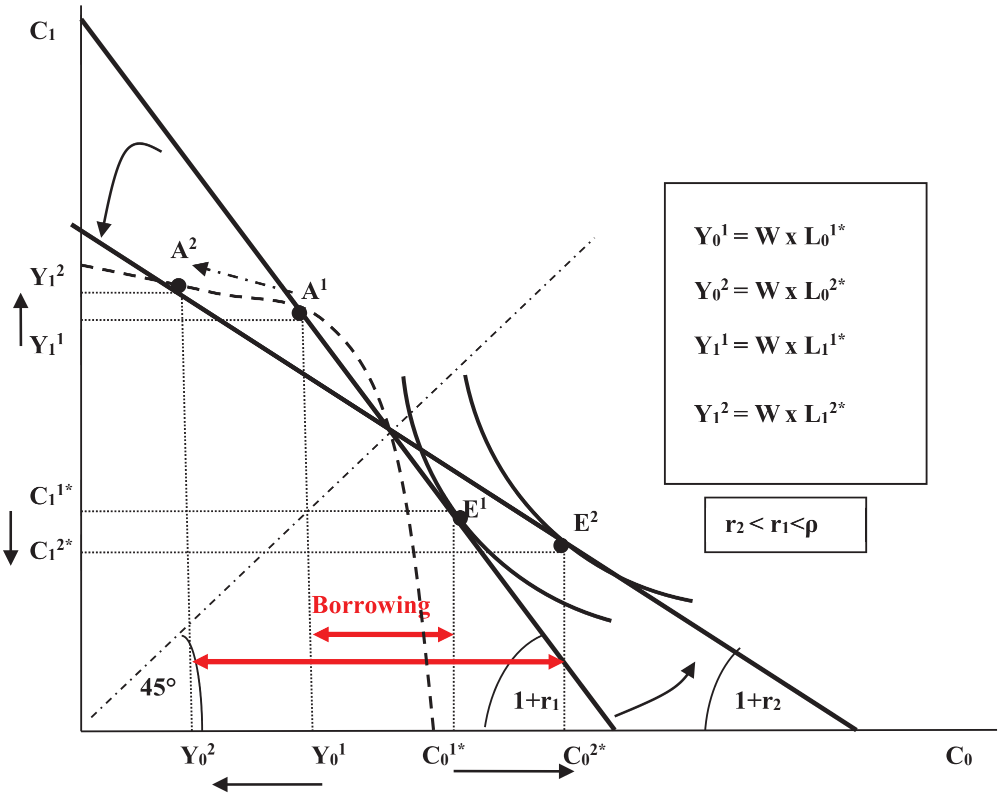


Fig. 1. An impact of a decrease in the interest rate on consumption and on income endowment when the intertemporal substitution of labour is effective

Thus, the saving curve is more elastic when the intertemporal substitution in labour (ISL) is included in the model. The reason lies in the fact that the reduction in the interest rate decreases present labour supply and hence the present labour income and raises future labour supply and future labour income. Both changes in income shift the traditional saving curve to the left. As a result, the saving curve that includes both the intertemporal substitution in consumption (ISC) and in labour might be constructed as follows (Figure 2): a drop in the interest rate moves the optimum saving along the traditional saving curve, which neglects ISL, from point $E^1$ to point B. The second round effect on the income stream shifts the entire traditional saving curve to the left. The new point of optimum can be found at point $E^2$. Connecting points $E^1$ and $E^2$, the more general saving curve can be found ($S^{ISL}$). This curve reflects both the ISC and the ISL. As can be seen, the representative consumer makes negative saving since present consumption exceeds present income.[§]

[§] A linear saving curve is constructed for simplicity. As can be seen from (19) and (28), the relationship between optimum saving and real interest rate must be non-linear. Furthermore, logarithmic utility function and the presence of future labour income lead to an upward sloping saving curve. The response of present consumption to the change in the interest rate is negative (see equation 28): $\partial C_0^*/\partial r = -K.W_1/(1+r)^2$, where $K = (1+\rho)/[(2+\rho)(1+b)]$. Thus, with a lower interest rate, optimum saving declines. If there was no future wage ($W_1 = 0$), the saving curve would be vertical (neither the optimum present consumption nor the optimum present leisure would depend on the interest rate).

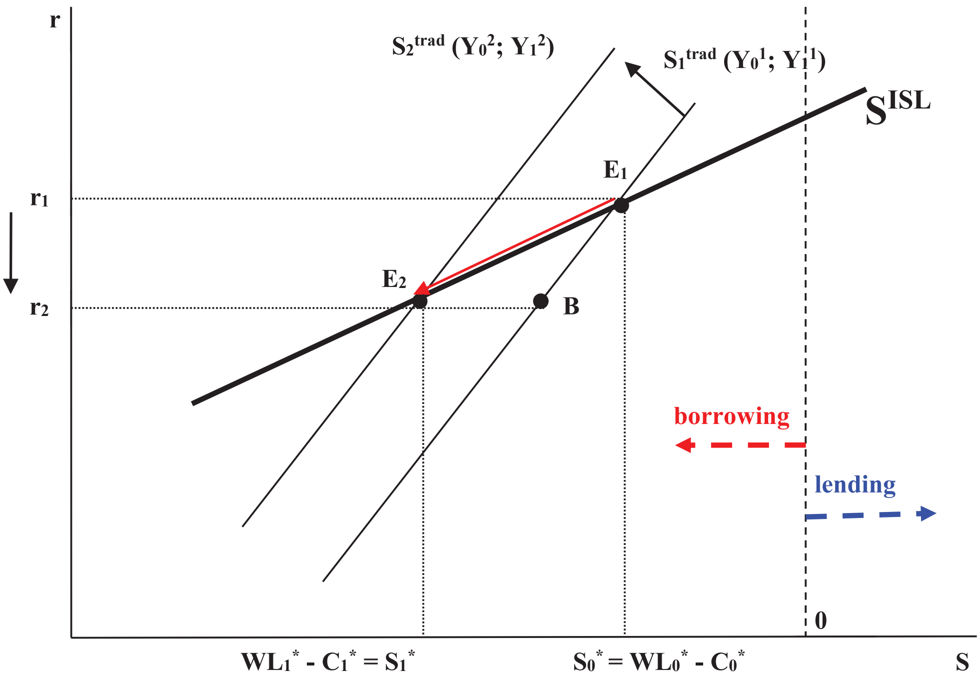


Fig. 2. Construction of the saving curve, which includes both the intertemporal substitution in consumption and in labour

The equilibrium real interest rate for two different (groups of) agents can be found by using the aggregate constraints for the whole economy (30) and (31), which require that aggregate consumption may not exceed aggregate income.

$$C_0^{A*} + C_0^{B*} = \mathrm{W}_0^{\mathrm{A}}\mathrm{L}_0^{\mathrm{A}*} + \mathrm{W}_0^{\mathrm{B}}\mathrm{L}_0^{\mathrm{B}*} \tag{30}$$

$$C_1^{A*} + C_1^{B*} = \mathrm{W}_1^{\mathrm{A}}\mathrm{L}_1^{\mathrm{A}*} + \mathrm{W}_1^{\mathrm{B}}\mathrm{L}_1^{\mathrm{B}*} \tag{31}$$

However, such an analysis would be too complicated compared with the results acquired. Thus, let us assume that all individuals have an identical subjective discount rate and exogenous stream of wages. Such homogeneity implies that individual saving is zero.

As a result, neither the interest rate $r_1$ nor $r_2$ in Figure 1 is an equilibrium rate of interest. Both are too low since they result in the excess of demand for present goods over their available supply ($C_0^* > Y_0$). The condition for an equilibrium rate of interest is given by (32):

$$C_0^* = \mathrm{W}_0\mathrm{L}_0^* = Y_0 \tag{32}$$

From (28), (19) and (3) we get:

$$\frac{W_0(1+r)+W_1}{(1+r)}\frac{(1+\rho)}{(2+\rho)(1+b)} = W_0(1-\frac{1+\frac{1}{1+r}\frac{W_1}{W_0}}{1+\frac{1}{b}+\frac{1}{1+\rho}+\frac{1}{b(1+\rho)}}) \tag{33}$$

The only unknown is the real interest rate *r*. However, instead of solving (33) we can directly substitute (32) into (13) and a similar constraint $C_1^* = W_1 L_1^*$ into (15), which yields:

$$L_0 = \frac{1}{1+b} \tag{34}$$

$$L_1 = \frac{1}{1+b} \tag{35}$$

Thus, the labour supply will be the same in both periods. As a result, the equilibrium interest rate will depend only on the flow of wages and the subjective discount rate. Substitution of (34) and (35) into (12) yields:

$$(1+r) = \frac{W_1}{W_0}(1+\rho) \tag{36}$$

Equation (36) shows that if the flow of wages is constant, the equilibrium real interest rate is equal to the subjective discount rate—the interest phenomenon will exist only due to the second Böhm-Bawerkian cause. If the stream of wages is increasing, the interest rate will be greater than the subjective discount rate, and the interest phenomenon will emerge due to the first and the second cause.

In this particular case, the equilibrium income endowment is not affected by the intertemporal substitution of labour. The reason is as follows: an increase in the average intertemporal wage ($W_1/W_0$) benefits present leisure time. However, higher $W_1/W_0$ accordingly increases the real interest rate, which perfectly offsets the original tendency. The endowment point and the resulting equilibrium interest rate therefore depend only on the time shape of wages, not on the allocation of labour over time, which is constant in the general equilibrium of this model. Parameter *b* (the relative importance of leisure in the utility function) does not affect the equilibrium interest rate either.

In other words, in this homogenous-agent model the intertemporal allocation of labour will not be affected by the time shape of the stream of wages, because any shape will accordingly modify the equilibrium interest rate, which will eventually leave the optimal intertemporal allocation of labour at the previous level that is characterised by $L=1/(1+b)$ in every period.

## 3. Underestimation of future wants

A similar analysis can be pursued for the subjective discount rate. In the new general equilibrium, its rise will increase the real interest rate by the same amount (see 36), keeping the equilibrium intertemporal allocation of labour unaffected. The only outcome will be a steeper indifference curve and a steeper budget line. There will be no impact on the representative endowment point. However, the transition period may uncover how the second reason for interest, in the presence of the intertemporal substitution in labour, reinforces the first reason and the corresponding rise in the interest rate.

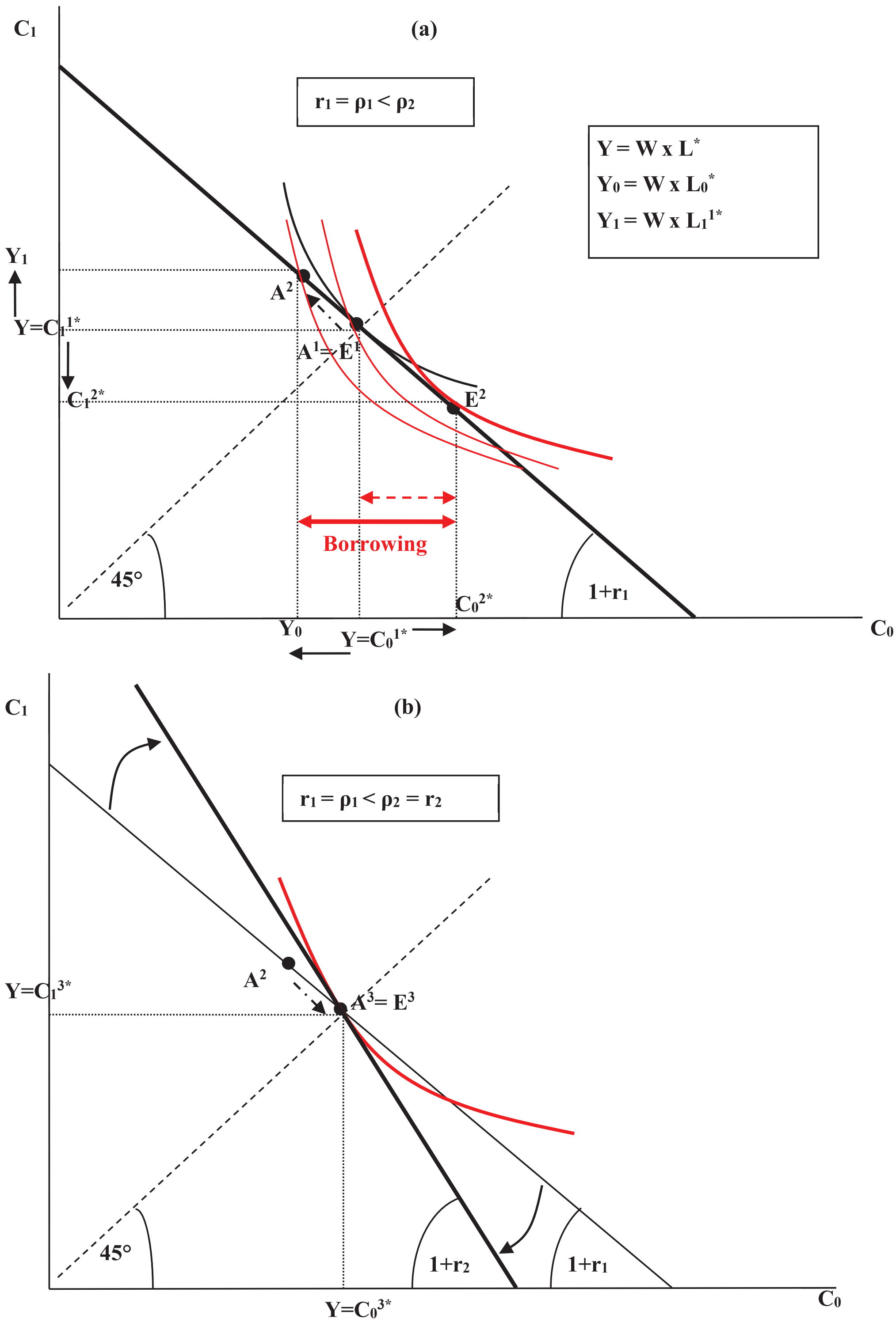
(a)
$C_1$
$r_1 = \rho_1 < \rho_2$
$Y = W \times L^*$
$Y_0 = W \times L_0^*$
$Y_1 = W \times L_1^{1*}$
$Y_1$
$Y=C_1^{1*}$
$A^2$
$A^1 = E^1$
$C_1^{2*}$
$E^2$
Borrowing
45°
$1+r_1$
$C_0^{2*}$
$Y_0$
$Y=C_0^{1*}$
$C_0$
(b)
$C_1$
$r_1 = \rho_1 < \rho_2 = r_2$
$Y=C_1^{3*}$
$A^2$
$A^3 = E^3$
45°
$1+r_2$
$1+r_1$
$Y=C_0^{3*}$
$C_0$

Fig. 3. The impact of an increase in the subjective discount rate on consumption, income endowment, and eventually on the interest rate

This analysis is presented in Figure 3. Assume a constant stream of wages ($W_1=W_0=W$). The labour supply in both periods is the same $L^*=1/(1+b)$. As a result, the time shape of the income stream is constant ($Y_0=Y_1=Y=WL^*$). According to equation (36), the interest rate must be equal to the subjective discount rate. Thus, consumption is also smoothed over time (see equation 9).

Consider an increase in the subjective discount rate—a stronger underestimation of future wants in the Böhm-Bawerk theory. The (consumption) indifference curves become steeper at the 45 degree line, which reflects the fact that even for the constant stream of income the consumer is willing to substitute more future goods for the given amount of present goods. However, the intensification of the second ground has one more effect. The presence of the intertemporal substitution of labour induces that the increase in the subjective discount rate moves more labour to the future at the expense of the present. In other words, higher $\rho$ benefits present leisure time at the expense of future leisure time. As a result, present income decreases from Y to $Y_0=WL_0^*$ and consequently the income endowment point moves from $A^1$ to $A^2$ (see panel a).[**] This creates an increasing shape of the income stream and activates the first Böhm-Bawerkian ground for interest. At point $A^2$, the willingness to substitute future goods for present goods is very strong, as can be seen by a very high slope of the indifference curve reflecting a great marginal rate of substitution between future and present consumption.

However, panel (a) is not a stable situation since at the aggregate level, the total income falls short of the desired total consumption. The resulting excess of demand for present goods over their supply ($C_0^{2*}>Y_0=WL_0^*$) is even greater compared with the situation if leisure is not included in the utility function (compare the size of borrowing represented by the red solid line and the dashed line). The reason is that the intertemporal substitution of labour moves the income endowment point to the top left. To equilibrate the demand for present goods ($C_0^*$) with their available supply ($Y=W_0L_0^*$) the interest rate must go up. In the end, the interest rate is equal to the new subjective discount rate ($r_2 = \rho_2$). Furthermore, labour supply is the same in both periods. The same holds for income and consumption. Thus, the endowment point is eventually in the same position as it was in the beginning (see panel b).

However, the individual intertemporal substitution of labour might play an important role in equilibrium if there is heterogeneity across individuals. Equation (33) for heterogeneous agents would be modified to (see equation 30):

$$\frac{W_0^A(1+r)+W_1^A}{(1+r)}\frac{(1+\rho_A)}{(2+\rho_A)(1+b)}+\frac{W_0^B(1+r)+W_1^B}{(1+r)}\frac{(1+\rho_B)}{(2+\rho_B)(1+b)}=$$
$$=W_0^A\left(1-\frac{1+\frac{1}{1+r}\frac{W_1^A}{W_0^A}}{1+\frac{1}{b}+\frac{1}{1+\rho_A}+\frac{1}{b(1+\rho_A)}}\right)+W_0^B\left(1-\frac{1+\frac{1}{1+r}\frac{W_1^B}{W_0^B}}{1+\frac{1}{b}+\frac{1}{1+\rho_B}+\frac{1}{b(1+\rho_B)}}\right) \qquad (37)$$

We do not have the ambition to solve this complicated equation for the equilibrium interest rate r. Yet, it is obvious that it will depend negatively on present wages $W_{0i}$ and positively on future wages $W_{1i}$ and subjective discount rates $\rho_i$. Furthermore, more patient people (low $\rho_i$) will consume less and work more in the present. As a result, their net lending position will be positive. It will be more positive than if the intertemporal substitution of labour does not exist. The opposite result would hold for less patient individuals (high $\rho_i$). An increasing stream of wages will lead to a lower present labour supply and therefore even to a lower present income. Thus, this channel will further raise borrowing of people with an increasing time shape of wages.

[**] It can be shown (Potuzak 2014) that the new endowment point must lie on the original budget line because a change in the subjective discount rate leaves the present value of the income stream unaffected. Alternatively, it can be demonstrated that the relative shift of the endowment point is in the direction of (1+r), which perfectly coincides with the slope (in absolute value) of the intertemporal budget constraint.

As can be seen, the inclusion of leisure into the utility function and the resulting intertemporal substitution of labour reinforce the Böhm-Bawerkian theory. The reason lies in the fact that, in the first place, the subjective discount rate influences the position of the individual endowment point (provided that *r* does not move one-for-one with $\rho_i$ ). In the second place, the shape of the exogenous stream of wages affects the position of the endowment point ($Y_{0i} = W_{0i} \times L_{0i}^*; Y_{1i} = W_{1i} \times L_{1i}^*$) not only directly due to the magnitude of $W_{1i}/W_{0i}$, but also due the impact on the optimum allocation of labour ($L_{1i}^*$; $L_{0i}^*$). Thus, both the subjective discount rate and the exogenous flow of wages will in turn affect the individual net borrowing/lending position and maybe the resulting equilibrium interest rate. In other words, each individual exogenous parameter might have a stronger impact on the equilibrium interest rate when the intertemporal substitution of labour exists.

The followers of Böhm-Bawerk in the Austrian School, developing the Pure Time Preference Theory (Mises 1949; Rothbard 1962; Garrison 1979; Kirzner 1993) did not discuss the possibility of the intertemporal substitution of labour. However, this channel might amplify the link between the time preference (represented by the subjective discount rate) and the natural rate of interest. The reason is that time preference favours not only present satisfaction from consumption goods, but it also favours present leisure. As a result, a relatively greater leisure time in the present (and lower in the future) reduces the provision of present goods and improves their future provision. This phenomenon therefore supports the first Böhm-Bawerkian ground for interest. It can be said that owing to the preference for present leisure (and the resulting intertemporal substitution of labour) the second cause for interest reinforces the first cause for interest due to the impact on the relative provision of goods over time.

## 4. Concluding remarks

In this paper, we assumed that people enjoy also their leisure time, not only consumption goods. The phenomenon of the intertemporal substitution of labour generated a kind of the PPF curve. However, this curve depended just on utility (or rather disutility), not technical productivity. We also demonstrated that the subjective discount rate affected not only the shape of the (intertemporal consumption) indifference curves but also the position of the endowment point(s). Thus, all important outcomes in this sub-model of the theory of interest depended on subjective phenomena. An increase in the underestimation of future wants (second Böhm-Bawerkian ground for interest) resulted in a more increasing shape of the income stream (first Böhm-Bawerkian ground). Even in the environment of a constant wage rate over time, the presence of the second cause was sufficient to generate the first cause for interest.

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