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Nilpotent symmetries for QED in superfield formalism

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Abstract

In the framework of superfield approach, we derive the local, covariant, continuous and nilpotent (anti-)BRST and (anti-)co-BRST symmetry transformations on the $U(1)$ gauge field (A_μ) and the (anti-)ghost fields $((\bar{C})\ C)$ of the Lagrangian density of the two $(1+1)$-dimensional QED by exploiting the (dual-)horizontality conditions defined on the four $(2+2)$-dimensional supermanifold. The long-standing problem of the derivation of the above symmetry transformations for the matter (Dirac) fields $(\bar{\psi}, \psi)$ in the framework of superfield formulation is resolved by a new set of restrictions on the $(2+2)$-dimensional supermanifold. These new physically interesting restrictions on the supermanifold owe their origin to the invariance of conserved currents of the theory. The geometrical interpretation for all the above transformations is provided in the framework of superfield formalism.

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1. Introduction

One of the most attractive and intuitive geometrical approaches to gain an insight into the physics and mathematics behind the Becchi–Rouet–Stora–Tyutin (BRST) formalism is the superfield formulation [1–6]. In this scheme, a D-dimensional gauge theory (endowed with the first-class constraints in the language of Dirac [7,8]) is considered on a $(D+2)$-dimensional supermanifold parameterized by D-number of spacetime (even) co-ordinates x^μ $(\mu = 0, 1, 2, \ldots, D-1)$ and a couple of (odd) Grassmannian variables θ and $\bar{\theta}$ (with $\theta^2 = \bar{\theta}^2 = 0, \theta\bar{\theta} + \bar{\theta}\theta = 0$). In general, the $(p+1)$-form supercurvature $\tilde{F}$ constructed from the superexterior derivative $\tilde{d}$ (with $\tilde{d}^2 = 0$) and the super-p-form connection $\tilde{A}$ of a p-form $(p = 1, 2, 3, \ldots,)$ gauge theory through the Maurer–Cartan equation (i.e., $\tilde{d}\tilde{A} + \tilde{A} \wedge \tilde{A} = \tilde{F}$) is restricted to be flat along the Grassmannian directions of the $(D+2)$-dimensional supermanifold due to the so-called horizontality condition.[1] Mathematically, this condition implies $\tilde{F} = F$ where

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[1] Nakanishi and Ojima call it the "soul-flatness" condition which amounts to setting the Grassmannian components of a $(p+1)$-form supercurvature tensor (for a p-form gauge theory) equal to zero [9].

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$F = dA + A \wedge A$ is the $(p+1)$-form curvature defined on the ordinary D-dimensional spacetime manifold. The horizontality condition, where only one of the three de Rham cohomological operators[2] is exploited, leads to the derivation of the nilpotent (anti-)BRST symmetry transformations on the gauge- and (anti-)ghost fields of the (anti-)BRST invariant Lagrangian density of a given D-dimensional p-form gauge theory.

In a recent set of papers [15–17], all the three (super)de Rham cohomological operators have been exploited, in the generalized versions of the horizontality condition, to derive the (anti-)BRST, (anti-)co-BRST and a bosonic symmetry (which is equal to the anti-commutator(s) of the (anti-)BRST and (anti-)co-BRST symmetries) transformations for the free one-form Abelian gauge theory in two dimensions (2D) of spacetime. For the derivation of the above nilpotent symmetries, the super(co-)exterior derivatives $(\tilde{\delta})\tilde{d}$ have been exploited in the (dual-) horizontality conditions on the four $(2+2)$-dimensional supermanifold. The Lagrangian formulation of the above symmetries has also been carried out in a set of papers [18–20] where it has been shown that this theory presents (i) an example of a tractable field theoretical model for the Hodge theory, and (ii) an example of a new class of topological field theory where the Lagrangian density turns out to be like Witten type topological field theory but the symmetries of the theory are that of Schwarz type. Similar symmetries for the self-interacting 2D non-Abelian gauge theory have also been obtained in the framework of 2D Lagrangian formalism [21] as well as in the four $(2+2)$-dimensional superfield formulation [22]. Furthermore, the above type of symmetries have been shown to exist for the 4D 2-form free Abelian gauge theory in the Lagrangian formalism [23,24].

One of the most difficult and long-standing problems in the realm of superfield approach to BRST formalism has been to derive the (anti-)BRST symmetry transformations on the matter (e.g., Dirac, complex scalar, etc.) fields for a given interacting p-form gauge theory. The purpose of the present Letter is to demonstrate that an additional set of restrictions, besides the (dual-)horizontality conditions w.r.t. super(co-)exterior derivatives $(\tilde{\delta})\tilde{d}$, are required on the $(D+2)$-dimensional supermanifold for the derivation of the (anti-)BRST and (anti-)co-BRST transformations on the matter fields. For this purpose, as a prototype field theoretical model, we choose the two-dimensional interacting $U(1)$ gauge theory (i.e., QED[3]) and show that the (anti-)BRST and (anti-)co-BRST symmetry transformations on the matter fields, derived in our earlier works [25,26] in the framework of Lagrangian formalism, can be obtained by exploiting the invariance of the conserved (super)currents constructed by the (super)Dirac fields of the theory on a (super)manifold. In a more precise and sophisticated language, the *equality* of the supercurrents $\tilde{J}_\mu(x,\theta,\bar{\theta})$ and $\tilde{J}^{(5)}_\mu(x,\theta,\bar{\theta})$ constructed by the superfields (cf. Eqs. (4.2) and (4.9) below) on the four $(2+2)$-dimensional supermanifold *with* the conserved currents $J_\mu(x) = (\bar{\psi}\gamma_\mu\psi)(x)$ and $J^{(5)}_\mu((x) = (\bar{\psi}\gamma_\mu\gamma_5\psi)(x)$ constructed by the ordinary Dirac fields on the 2D ordinary manifold leads to the derivation of the (anti-)BRST and (anti-)co-BRST symmetry transformations on the Dirac fields, respectively. The above equality emerges automatically and is not imposed by hand. We also provide, in the present Letter, the geometrical interpretations for the nilpotent symmetries and the corresponding nilpotent generators.

The outline of our present Letter is as follows. In Section 2, we recapitulate the salient features of our earlier works [25,26] on the existence of the off-shell nilpotent (anti-)BRST- and (anti-)co-BRST symmetries in the Lagrangian formulation for the *interacting* $U(1)$ gauge theory in two dimensions of spacetime. Section 3 is devoted to the derivation of the above symmetry transformations on the gauge field A_μ and the (anti-)ghost fields $(\bar{C})\ C$ by exploiting the (dual-)horizontality conditions on the four $(2+2)$-dimensional supermanifold [17,22]. This exercise is carried out for the sake of this Letter to be self-contained. The central of our Letter is Section 4 where we derive the above symmetry transformations for the matter (Dirac) fields by invoking the *invariance* of the conserved

[2] On an ordinary manifold without a boundary, the three operators (d, δ, Δ) form a set of de Rham cohomological operators where $(\delta)d$ are the (co-)exterior derivatives with $d = dx^\mu \partial_\mu$, $\delta = \pm * d *$ and $d^2 = \delta^2 = 0$. Here $*$ is the Hodge duality operation on the manifold. The Laplacian operator $\Delta = (d+\delta)^2 = \{d, \delta\}$ turns out to be the Casimir operator for the full set of algebra: $\delta^2 = 0$, $d^2 = 0$, $\Delta = \{d, \delta\}$, $[\Delta, d] = 0$, $[\Delta, \delta] = 0$ obeyed by these cohomological operators belonging to the geometrical aspects of the subject of differential geometry (see, e.g., [10–14] for details).

[3] A dynamically closed and locally gauge invariant system of the photon and Dirac fields.

currents as the physical restriction on the supermanifold. Finally, we make some concluding remarks and pinpoint a few future directions in Section 5 for further investigations.

2. Preliminary: (anti-)BRST- and (anti-)co-BRST symmetries

To recapitulate the bare essentials of our earlier works [25,26] on QED in two dimensions, let us begin with the (anti-)BRST invariant Lagrangian density $\mathcal{L}_b$ for the *interacting* two $(1+1)$-dimensional (2D) $U(1)$ gauge theory in the Feynman gauge [27–29]

$$\begin{aligned}\mathcal{L}_b &= -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{\psi}\big(i\gamma^\mu D_\mu - m\big)\psi + B(\partial\cdot A) + \frac{1}{2}B^2 - i\partial_\mu\bar{C}\partial^\mu C \\ &\equiv \frac{1}{2}E^2 + \bar{\psi}\big(i\gamma^\mu D_\mu - m\big)\psi + B(\partial\cdot A) + \frac{1}{2}B^2 - i\partial_\mu\bar{C}\partial^\mu C,\end{aligned} \tag{2.1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength tensor for the $U(1)$ gauge theory that is derived from the 2-form $dA = (1/2)(dx^\mu \wedge dx^\nu)F_{\mu\nu}$.[4] As is evident, the latter is constructed by the application of the exterior derivative $d = dx^\mu\,\partial_\mu$ (with $d^2 = 0$) on the 1-form $A = dx^\mu A_\mu$ (which defines the vector potential A_μ). It will be noted that in 2D, $F_{\mu\nu}$ has only the electric component (i.e., $F_{01} = E$) and there is no magnetic component associated with it. The gauge-fixing term $(\partial\cdot A)$ is derived through the operation of the co-exterior derivative δ (with $\delta = -*d*, \delta^2 = 0$) on the one-form A (i.e., $\delta A = -*d*A = (\partial\cdot A)$), where $*$ is the Hodge duality operation. The fermionic Dirac fields $(\psi, \bar{\psi})$, with the mass m and charge e, couple to the $U(1)$ gauge field A_μ (i.e., $-e\bar{\psi}\gamma^\mu A_\mu\psi$) through the conserved current $J_\mu = \bar{\psi}\gamma_\mu\psi$. The anti-commuting ($C\bar{C} + \bar{C}C = 0$, $C^2 = \bar{C}^2 = 0$, $C\psi + \psi C = 0$, etc.) (anti-)ghost fields $(\bar{C})\ C$ are required to maintain the unitarity and "quantum" gauge (i.e., BRST) invariance together at any arbitrary order of perturbation theory.[5] The kinetic energy term $(E^2/2)$ of (2.1) can be linearized by invoking an auxiliary field $\mathcal{B}$

$$\mathcal{L}_B = \mathcal{B}E - \frac{1}{2}\mathcal{B}^2 + \bar{\psi}\big(i\gamma^\mu D_\mu - m\big)\psi + B(\partial\cdot A) + \frac{1}{2}B^2 - i\partial_\mu\bar{C}\partial^\mu C, \tag{2.2}$$

which is the analogue of the Nakanishi–Lautrup auxiliary field B that is required to linearize the gauge-fixing term $-(\partial\cdot A)^2/2$ in (2.1). The above Lagrangian density (2.2) respects the following off-shell nilpotent ($s_{(a)b}^2 = 0$, $s_{(a)d}^2 = 0$) (anti-)BRST ($s_{(a)b}$),[6] and (anti-)dual(co)-BRST ($s_{(a)d}$) symmetry transformations (with $s_b s_{ab} + s_{ab} s_b = 0$, $s_d s_{ad} + s_{ad} s_d = 0$) [25,26]

$$\begin{aligned}&s_b A_\mu = \partial_\mu C, \quad && s_b C = 0, \quad && s_b\bar{C} = iB, \quad && s_b\psi = -ieC\psi, \\ &s_b\bar{\psi} = -ie\bar{\psi}C, && s_b\mathcal{B} = 0, && s_b B = 0, && s_b E = 0, \quad s_b(\partial\cdot A) = \Box C, \\ &s_{ab} A_\mu = \partial_\mu\bar{C}, && s_{ab}\bar{C} = 0, && s_{ab} C = -iB, && s_{ab}\psi = -ie\bar{C}\psi, \\ &s_{ab}\bar{\psi} = -ie\bar{\psi}\bar{C}, && s_{ab}\mathcal{B} = 0, && s_{ab} B = 0, && s_{ab} E = 0, \quad s_{ab}(\partial\cdot A) = \Box\bar{C},\end{aligned} \tag{2.3}$$

[4] We adopt here the conventions and notations such that the 2D flat Minkowski metric is: $\eta_{\mu\nu} = \mathrm{diag}(+1,-1)$ and $\Box = \eta^{\mu\nu}\partial_\mu\partial_\nu = (\partial_0)^2 - (\partial_1)^2$, $\varepsilon_{\mu\nu} = -\varepsilon^{\mu\nu}$, $F_{01} = E = \partial_0 A_1 - \partial_1 A_0 = -\varepsilon^{\mu\nu}\partial_\mu A_\nu = F^{10}$, $\varepsilon_{01} = \varepsilon^{10} = +1$, $D_\mu\psi = \partial_\mu\psi + ieA_\mu\psi$. The Dirac γ matrices in two dimensions are chosen to be: $\gamma^0 = \sigma_2$, $\gamma^1 = i\sigma_1$, $\gamma_5 = \gamma^0\gamma^1 = \sigma_3$, $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$, $\gamma_\mu\gamma_5 = \varepsilon_{\mu\nu}\gamma^\nu$. Here σ's are the usual 2×2 Pauli matrices and the Greek indices: $\mu, \nu, \rho \ldots = 0, 1$ correspond to the spacetime directions on the manifold.

[5] The full strength of the (anti-)ghost fields turns up in the discussion of the unitarity and gauge invariance for the perturbative computations in the realm of non-Abelian gauge theory where the loop diagrams of the gauge (gluon) fields play a very important role (see, e.g., [30] for details).

[6] We adopt here the notations and conventions followed in [29]. In fact, in its full glory, a nilpotent ($\delta_B^2 = 0$) BRST transformation δ_B is equivalent to the product of an anti-commuting ($\eta C = -C\eta$, $\eta\bar{C} = -\bar{C}\eta$, $\eta\psi = -\psi\eta$, $\eta\bar{\psi} = -\bar{\psi}\eta$, etc.) spacetime independent parameter η and s_b (i.e., $\delta_B = \eta s_b$) where $s_b^2 = 0$.

$$
\begin{array}{lllll}
s_d A_\mu = -\varepsilon_{\mu\nu}\partial^\nu \bar{C}, & s_d B = 0, & s_d(\partial\cdot A) = 0, & s_d\bar{C} = 0, & s_d C = -i\mathcal{B}, \\
s_d\mathcal{B} = 0, & s_d\psi = -ie\bar{C}\gamma_5\psi, & s_d\bar{\psi} = +ie\bar{\psi}\bar{C}\gamma_5, & s_d E = \Box\bar{C}, & \\
s_{ad} A_\mu = -\varepsilon_{\mu\nu}\partial^\nu C, & s_{ad} B = 0, & s_{ad}(\partial\cdot A) = 0, & s_{ad} C = 0, & s_{ad}\bar{C} = +i\mathcal{B}, \\
s_{ad}\mathcal{B} = 0, & s_{ad}\psi = -ieC\gamma_5\psi, & s_{ad}\bar{\psi} = +ie\bar{\psi}C\gamma_5, & s_{ad} E = \Box C. &
\end{array} \tag{2.4}
$$

The noteworthy points, at this stage, are

(i) under the (anti-)BRST and (anti-)co-BRST transformations, it is the kinetic energy term (more precisely E itself) and the gauge-fixing term (more accurately $(\partial\cdot A)$ itself) that remain invariant, respectively.
(ii) The electric field E and $(\partial\cdot A)$ owe their origin to the operation of cohomological operators d and δ on the one-form $A = dx^\mu A_\mu$, respectively.
(iii) For the (anti-)co-BRST transformations to be the symmetry transformations for (2.2), there exists the restriction that $m = 0$ for the Dirac fields. There is no such restriction for the validity of the (anti-)BRST symmetry transformations.
(iv) The anti-commutator ($s_w = \{s_b s_d\} = \{s_{ab}, s_{ad}\}$) of the above nilpotent symmetries is a bosonic symmetry transformation s_w (with $s_w^2 \neq 0$) for the Lagrangian density (2.2) [26].
(v) The operator algebra among the above transformations is exactly identical to the algebra obeyed by the de Rham cohomological operators.
(vi) The symmetry transformations in (2.3) and (2.4) are generated by the local, conserved and nilpotent charges $Q_{(a)b}$ and $Q_{(a)d}$. This statement can be succinctly expressed in the mathematical form as

$$
s_r\Sigma(x) = -i\big[\Sigma(x), Q_r\big]_\pm, \quad r = b, ab, d, ad, \tag{2.5}
$$

where the local generic field $\Sigma = A_\mu, C, \bar{C}, \psi, \bar{\psi}, B, \mathcal{B}$ and the $(+)-$ signs, as the subscripts on the (anti-)commutator $[\,,\,]_\pm$, stand for Σ being (fermionic) bosonic in nature.

3. Nilpotent symmetries for the gauge- and (anti-)ghost fields

We begin here with a four $(2+2)$-dimensional supermanifold parametrized by the superspace coordinates $Z^M = (x^\mu, \theta, \bar{\theta})$ where $x^\mu (\mu = 0, 1)$ are a couple of even (bosonic) spacetime coordinates and θ and $\bar{\theta}$ are the two odd (Grassmannian) coordinates (with $\theta^2 = \bar{\theta}^2 = 0$, $\theta\bar{\theta} + \bar{\theta}\theta = 0$). On this supermanifold, one can define a supervector superfield $\tilde{A}_M$ (i.e., $\tilde{A}_M = (B_\mu(x,\theta,\bar{\theta}), \Phi(x,\theta,\bar{\theta}), \bar{\Phi}(x,\theta,\bar{\theta}))$ with $B_\mu, \Phi, \bar{\Phi}$ as the component multiplet superfields [4]. The superfields $B_\mu, \Phi, \bar{\Phi}$ can be expanded in terms of the basic fields $(A_\mu, C, \bar{C})$ and auxiliary fields $(B, \mathcal{B})$ of (2.2) and some extra secondary fields as follows

$$
\begin{aligned}
B_\mu(x,\theta,\bar{\theta}) &= A_\mu(x) + \theta\bar{R}_\mu(x) + \bar{\theta}R_\mu(x) + i\theta\bar{\theta}S_\mu(x), \\
\Phi(x,\theta,\bar{\theta}) &= C(x) + i\theta\bar{B}(x) - i\bar{\theta}\mathcal{B}(x) + i\theta\bar{\theta}s(x), \\
\bar{\Phi}(x,\theta,\bar{\theta}) &= \bar{C}(x) - i\theta\bar{\mathcal{B}}(x) + i\bar{\theta}B(x) + i\theta\bar{\theta}\bar{s}(x).
\end{aligned} \tag{3.1}
$$

It is straightforward to note that the local fields $R_\mu(x)$, $\bar{R}_\mu(x)$, $C(x)$, $\bar{C}(x)$, $s(x)$, $\bar{s}(x)$ are fermionic (anti-commuting) in nature and the bosonic (commuting) local fields in (3.1) are: $A_\mu(x)$, $S_\mu(x)$, $\mathcal{B}(x)$, $\bar{\mathcal{B}}(x)$, $B(x)$, $\bar{B}(x)$. It is unequivocally clear that, in the above expansion, the bosonic- and fermionic degrees of freedom match. This requirement is essential for the validity and sanctity of any arbitrary supersymmetric theory in the superfield formulation. In fact, all the secondary fields will be expressed in terms of basic fields due to the restrictions emerging from the application of horizontality condition (i.e., $\tilde{F} = F$), namely;

$$
\tilde{F} = \frac{1}{2}\big(dZ^M \wedge dZ^N\big)\tilde{F}_{MN} = \tilde{d}\tilde{A} \equiv dA = \frac{1}{2}\big(dx^\mu \wedge dx^\nu\big)F_{\mu\nu} = F, \tag{3.2}
$$

where the superexterior derivative $\tilde{d}$ and the connection superone-form $\tilde{A}$ are defined as

$$\begin{aligned}
\tilde{d} &= dZ^M\,\partial_M = dx^\mu\,\partial_\mu + d\theta\,\partial_\theta + d\bar{\theta}\,\partial_{\bar{\theta}},\\
\tilde{A} &= dZ^M\,\tilde{A}_M = dx^\mu\,B_\mu(x,\theta,\bar{\theta}) + d\theta\,\bar{\Phi}(x,\theta,\bar{\theta}) + d\bar{\theta}\,\Phi(x,\theta,\bar{\theta}).
\end{aligned}\tag{3.3}$$

In physical language, this requirement implies that the physical field E, derived from the curvature term $F_{\mu\nu}$, does not get any contribution from the Grassmannian variables. In other words, the physical electric field E for 2D QED remains intact in the superfield formulation. Mathematically, the condition (3.2) implies the "flatness" of all the components of the supercurvature (2-form) tensor $\tilde{F}_{MN}$ that are directed along the θ and/or $\bar{\theta}$ directions of the supermanifold. To this end in mind, first we expand $\tilde{d}\tilde{A}$ as

$$\begin{aligned}
\tilde{d}\tilde{A} &= \big(dx^\mu \wedge dx^\nu\big)(\partial_\mu B_\nu) - (d\theta \wedge d\theta)(\partial_\theta \bar{\Phi}) + \big(dx^\mu \wedge d\bar{\theta}\big)(\partial_\mu \Phi - \partial_{\bar{\theta}} B_\mu)\\
&\quad - (d\theta \wedge d\bar{\theta})(\partial_\theta \Phi + \partial_{\bar{\theta}} \bar{\Phi}) + \big(dx^\mu \wedge d\theta\big)(\partial_\mu \bar{\Phi} - \partial_\theta B_\mu) - (d\bar{\theta} \wedge d\bar{\theta})(\partial_{\bar{\theta}} \Phi).
\end{aligned}\tag{3.4}$$

Ultimately, the application of soul-flatness (horizontality) condition ($\tilde{d}\tilde{A} = dA$) yields [17]

$$\begin{aligned}
&R_\mu(x) = \partial_\mu C(x), \qquad \bar{R}_\mu(x) = \partial_\mu \bar{C}(x), \qquad s(x) = \bar{s}(x) = 0,\\
&S_\mu(x) = \partial_\mu B(x), \qquad B(x) + \bar{B}(x) = 0, \qquad \mathcal{B}(x) = \bar{\mathcal{B}}(x) = 0.
\end{aligned}\tag{3.5}$$

The insertion of all the above values in the expansion (3.1) leads to the derivation of the (anti-)BRST symmetries for the gauge- and (anti-)ghost fields of the Abelian gauge theory. In addition, this exercise provides the physical interpretation for the (anti-)BRST charges $Q_{(a)b}$ as the generators (cf. Eq. (2.5)) of translations (i.e., $\mathrm{Lim}_{\bar{\theta}\to 0}(\partial/\partial\theta)$, $\mathrm{Lim}_{\theta\to 0}(\partial/\partial\bar{\theta})$) along the Grassmannian directions of the supermanifold. Both these observations can be succinctly expressed, in a combined way, by re-writing the superexpansion (3.1) as

$$\begin{aligned}
B_\mu(x,\theta,\bar{\theta}) &= A_\mu(x) + \theta\big(s_{ab}A_\mu(x)\big) + \bar{\theta}\big(s_b A_\mu(x)\big) + \theta\bar{\theta}\big(s_b s_{ab} A_\mu(x)\big),\\
\Phi(x,\theta,\bar{\theta}) &= C(x) + \theta\big(s_{ab}C(x)\big) + \bar{\theta}\big(s_b C(x)\big) + \theta\bar{\theta}\big(s_b s_{ab} C(x)\big),\\
\bar{\Phi}\big(x,\theta,\bar{\theta}\big) &= \bar{C}(x) + \theta\big(s_{ab}\bar{C}(x)\big) + \bar{\theta}\big(s_b \bar{C}(x)\big) + \theta\bar{\theta}\big(s_b s_{ab} \bar{C}(x)\big).
\end{aligned}\tag{3.6}$$

To obtain the (anti-)co-BRST transformations on the gauge- and (anti-)ghost fields, we exploit the dual-horizontality condition $\tilde{\delta}\tilde{A} = \delta A$ on the $(2+2)$-dimensional supermanifold where $\tilde{\delta} = - \star \tilde{d} \star$ is the super-co-exterior derivative on the four $(2+2)$-dimensional supermanifold and $\delta = - * d *$ is the co-exterior derivative on the ordinary 2D manifold. The Hodge duality operations on the supermanifold and ordinary manifold are denoted by $\star$ and $*$, respectively. The $\star$ operations on the superdifferentials (dZ^M) and their wedge products ($dZ^M \wedge dZ^N$), etc., defined on the $(2+2)$-dimensional supermanifold, are [22,31]

$$\begin{aligned}
&\star\big(dx^\mu\big) = \varepsilon^{\mu\nu}(dx_\nu \wedge d\theta \wedge d\bar{\theta}), && \star(d\theta) = \frac{1}{2!}\varepsilon^{\mu\nu}(dx_\mu \wedge dx_\nu \wedge d\bar{\theta}),\\
&\star(d\bar{\theta}) = \frac{1}{2!}\varepsilon^{\mu\nu}(dx_\mu \wedge dx_\nu \wedge d\theta), && \star\big(dx^\mu \wedge dx^\nu\big) = \varepsilon^{\mu\nu}(d\theta \wedge d\bar{\theta}),\\
&\star\big(dx^\mu \wedge d\theta\big) = \varepsilon^{\mu\nu}(dx_\nu \wedge d\bar{\theta}), && \star\big(dx^\mu \wedge d\bar{\theta}\big) = \varepsilon^{\mu\nu}(dx_\nu \wedge d\theta),\\
&\star(d\theta \wedge d\theta) = \frac{1}{2!}s^{\theta\theta}\varepsilon^{\mu\nu}(dx_\mu \wedge dx_\nu), && \star(d\theta \wedge d\bar{\theta}) = \frac{1}{2!}s^{\theta\bar{\theta}}\varepsilon^{\mu\nu}(dx_\mu \wedge dx_\nu),\\
&\star(d\bar{\theta} \wedge d\bar{\theta}) = \frac{1}{2!}s^{\bar{\theta}\bar{\theta}}\varepsilon^{\mu\nu}(dx_\mu \wedge dx_\nu), && \star(dx_\mu \wedge d\theta \wedge d\bar{\theta}) = \varepsilon_{\mu\nu}\big(dx^\nu\big),\\
&\star(dx_\mu \wedge dx_\nu \wedge d\theta \wedge d\bar{\theta}) = \varepsilon_{\mu\nu}, && \star(dx_\mu \wedge dx_\nu \wedge d\theta) = \varepsilon_{\mu\nu}(d\bar{\theta}),\\
&\star(dx_\mu \wedge dx_\nu \wedge d\bar{\theta}) = \varepsilon_{\mu\nu}(d\theta), && \star(dx_\mu \wedge dx_\nu \wedge d\theta \wedge d\theta) = \varepsilon_{\mu\nu}s^{\theta\theta},
\end{aligned}\tag{3.7}$$

where ss are the symmetric (i.e., $s^{\theta\bar{\theta}} = s^{\bar{\theta}\theta}$) *constant* quantities on the Grassmannian submanifold of the four $(2+2)$-dimensional supermanifold. They are introduced to take care of the fact that two successive $\star$ operation on any differential should yield the same differential (see, [31] for detail discussions). With the above inputs, it can be checked that the superscalar superfield $\tilde{\delta}\tilde{A} = -\star\tilde{d}\star\tilde{A}$, turns out to be

$$\tilde{\delta}\tilde{A} = (\partial\cdot B) + s^{\theta\theta}(\partial_\theta\Phi) + s^{\bar{\theta}\bar{\theta}}(\partial_{\bar{\theta}}\bar{\Phi}) + s^{\theta\bar{\theta}}(\partial_\theta\bar{\Phi} + \partial_{\bar{\theta}}\Phi). \tag{3.8}$$

Ultimately, the dual-horizontality restriction $\tilde{\delta}\tilde{A} = \delta A$ produces the following restrictions on the component superfields (see, e.g., [31] for details)

$$\partial_\theta\bar{\Phi} + \partial_{\bar{\theta}}\Phi = 0, \quad \partial_\theta\Phi = 0, \quad \partial_{\bar{\theta}}\bar{\Phi} = 0, \quad (\partial\cdot B) = (\partial\cdot A), \tag{3.9}$$

where, as is evident, the r.h.s. of the last entry in the above equation is due to $\delta A = (\partial\cdot A)$. Exploiting the superexpansions of (3.1), we obtain

$$\begin{aligned} &(\partial\cdot R)(x) = (\partial\cdot\bar{R})(x) = (\partial\cdot S)(x) = 0, \quad s(x) = \bar{s}(x) = 0, \\ &B(x) = 0, \qquad \bar{B}(x) = 0, \qquad \mathcal{B}(x) + \bar{\mathcal{B}}(x) = 0. \end{aligned} \tag{3.10}$$

It is clear from the above that we cannot get a *unique* solution for R_μ, $\bar{R}_\mu$ and S_μ in terms of the basic fields of the Lagrangian density (2.2). This is why there are non-local and non-covariant solutions for these in the case of QED in 4D (see, e.g., [31]). It is interesting, however, to point out that for 2D QED, we have the local and covariant solutions as

$$R_\mu = -\varepsilon_{\mu\nu}\partial^\nu\bar{C}, \qquad \bar{R}_\mu = -\varepsilon_{\mu\nu}\partial^\nu C, \qquad S_\mu = +\varepsilon_{\mu\nu}\partial^\nu\mathcal{B}. \tag{3.11}$$

With the above insertions, it can be easily checked that the expansion (3.1) becomes

$$\begin{aligned} B_\mu(x,\theta,\bar{\theta}) &= A_\mu(x) + \theta\big(s_{ad}A_\mu(x)\big) + \bar{\theta}\big(s_d A_\mu(x)\big) + \theta\bar{\theta}\big(s_d s_{ad} A_\mu(x)\big), \\ \Phi(x,\theta,\bar{\theta}) &= C(x) + \theta\big(s_{ad}C(x)\big) + \bar{\theta}\big(s_d C(x)\big) + \theta\bar{\theta}\big(s_d s_{ad} C(x)\big), \\ \bar{\Phi}(x,\theta,\bar{\theta}) &= \bar{C}(x) + \theta\big(s_{ad}\bar{C}(x)\big) + \bar{\theta}\big(s_d\bar{C}(x)\big) + \theta\bar{\theta}\big(s_d s_{ad}\bar{C}(x)\big). \end{aligned} \tag{3.12}$$

Thus, the geometrical interpretation for the generators $Q_{(a)d}$ of the (anti-)co-BRST symmetries is identical to that of the (anti-)BRST charges $Q_{(a)b}$. However, there is a clear-cut distinction between $Q_{(a)d}$ and $Q_{(a)b}$ when the transformations on the (anti-)ghost fields are considered. For instance, the BRST charge Q_b generates a symmetry transformation such that the superfield $\bar{\Phi}(x,\theta,\bar{\theta})$ becomes *anti-chiral* and the superfield $\Phi(x,\theta,\bar{\theta})$ becomes an ordinary local field $C(x)$. In contrast, the co-BRST charge Q_d generates a symmetry transformation under which just the *opposite* of the above happens. Similarly, the distinction between Q_{ab} and Q_{ad} can be argued where one of the above superfields becomes *chiral*.

4. Nilpotent symmetries for the Dirac fields

In contrast to the (dual-)horizontality conditions that rely on the (super)co-exterior derivatives $(\tilde{\delta})\delta$, the (super) exterior derivative $(\tilde{d})d$ and the (super)one-form $(\tilde{A})A$ for the derivation of the (anti-)BRST and (anti-)co-BRST symmetry transformations on the gauge field A_μ and the (anti-)ghost fields $(\bar{C})$ C, the corresponding nilpotent symmetries for the matter (Dirac) fields $(\psi,\bar{\psi})$ are obtained due to the invariance of the conserved currents of the theory. To corroborate this assertion, first of all, we start off with the superexpansion of the superfields $(\Psi,\bar{\Psi})(x,\theta,\bar{\theta}))$, corresponding to the ordinary Dirac fields $(\psi,\bar{\psi})(x)$, as

$$\begin{aligned} \Psi(x,\theta,\bar{\theta}) &= \psi(x) + i\theta\bar{b}_1(x) + i\bar{\theta}b_2(x) + i\theta\bar{\theta}f(x), \\ \bar{\Psi}(x,\theta,\bar{\theta}) &= \bar{\psi}(x) + i\theta\bar{b}_2(x) + i\bar{\theta}b_1(x) + i\theta\bar{\theta}\bar{f}(x). \end{aligned} \tag{4.1}$$

It is clear and evident that, in the limit $(\theta, \bar{\theta}) \to 0$, we get back the Dirac fields $(\psi, \bar{\psi})$ of the Lagrangian density (2.1). Furthermore, the number of bosonic fields $(b_1, \bar{b}_1, b_2, \bar{b}_2)$ match with the fermionic fields $(\psi, \bar{\psi}, f, \bar{f})$ so that the above expansion is consistent with the basic tenets of supersymmetry. Now one can construct the supercurrent $\tilde{J}_\mu(x, \theta, \bar{\theta})$ from the above superfields with the following general superexpansion

$$\tilde{J}_\mu(x, \theta, \bar{\theta}) = \bar{\Psi}(x, \theta, \bar{\theta})\gamma_\mu \Psi(x, \theta, \bar{\theta}) = J_\mu(x) + \theta \bar{K}_\mu(x) + \bar{\theta} K_\mu(x) + i\theta\bar{\theta} L_\mu(x), \tag{4.2}$$

where the above components (i.e., $\bar{K}_\mu, K_\mu, L_\mu, J_\mu$), along the Grassmannian directions θ and $\bar{\theta}$ as well as the bosonic directions $\theta\bar{\theta}$ and identity $\hat{\mathbf{1}}$ of the supermanifold, can be expressed in terms of the components of the basic superexpansions (4.1), as

$$\begin{aligned} &\bar{K}_\mu(x) = i(\bar{b}_2\gamma_\mu\psi - \bar{\psi}\gamma_\mu\bar{b}_1), \qquad K_\mu(x) = i(b_1\gamma_\mu\psi - \bar{\psi}\gamma_\mu b_2), \\ &L_\mu(x) = \bar{f}\gamma_\mu\psi + \bar{\psi}\gamma_\mu f + i(\bar{b}_2\gamma_\mu b_2 - b_1\gamma_\mu\bar{b}_1), \qquad J_\mu(x) = \bar{\psi}\gamma_\mu\psi. \end{aligned} \tag{4.3}$$

To be consistent with our earlier observation that the (co-)BRST transformations $(s_{(d)b})$ are equivalent to the translations (i.e., $\mathrm{Lim}_{\theta\to 0}(\partial/\partial\bar{\theta})$) along the $\bar{\theta}$-direction and the anti-BRST (s_{ab}) and anti-co-BRST (s_{ad}) transformations are equivalent to the translations (i.e., $\mathrm{Lim}_{\bar{\theta}\to 0}(\partial/\partial\theta)$) along the θ-direction of the supermanifold, it is straightforward to re-express the expansion in (4.2) as follows

$$\tilde{J}_\mu(x, \theta, \bar{\theta}) = J_\mu(x) + \theta\big(s_{ab} J_\mu(x)\big) + \bar{\theta}\big(s_b J_\mu(x)\big) + \theta\bar{\theta}\big(s_b s_{ab} J_\mu(x)\big). \tag{4.4}$$

It can be checked explicitly that, under the (anti-)BRST transformations (2.3), the conserved current $J_\mu(x)$ remains invariant (i.e., $s_b J_\mu(x) = s_{ab} J_\mu(x) = 0$). This statement, with the help of (4.2) and (4.3), can be mathematically expressed as

$$b_1\gamma_\mu\psi = \bar{\psi}\gamma_\mu b_2, \qquad \bar{b}_2\gamma_\mu\psi = \bar{\psi}\gamma_\mu\bar{b}_1, \qquad \bar{f}\gamma_\mu\psi + \bar{\psi}\gamma_\mu f = i(b_1\gamma_\mu\bar{b}_1 - \bar{b}_2\gamma_\mu b_2). \tag{4.5}$$

One of the possible solutions of the above restrictions, in terms of the components of the basic expansions in (4.1) and the basic fields of the Lagrangian density (2.2), is

$$\begin{aligned} &b_1 = -e\bar{\psi}C, \qquad b_2 = -eC\psi, \qquad \bar{b}_1 = -e\bar{C}\psi, \qquad \bar{b}_2 = -e\bar{\psi}\bar{C}, \\ &f = -ie[B + e\bar{C}C]\psi, \qquad \bar{f} = +ie\bar{\psi}[B + eC\bar{C}]. \end{aligned} \tag{4.6}$$

At the moment, it appears to us that the above solutions are the *unique* solutions to all the restrictions in (4.5).[7] Ultimately, the restriction that emerges on the $(2+2)$-dimensional supermanifold is

$$\tilde{J}_\mu(x, \theta, \bar{\theta}) = J_\mu(x). \tag{4.7}$$

Physically, the above mathematical equation implies that there is no superspace contribution to the ordinary conserved current $J_\mu(x)$. In other words, the transformations on the Dirac fields ψ and $\bar{\psi}$ (cf. (2.3)) are such that the supercurrent $\tilde{J}_\mu(x, \theta, \bar{\theta})$ becomes a local composite field $J_\mu(x) = (\bar{\psi}\gamma_\mu\psi)(x)$ *vis-á-vis* equation (4.4) and there is no Grassmannian contribution to it. In a more sophisticated language, the conservation law $\partial \cdot J = 0$ remains intact despite our discussions connected with the superspace and supersymmetry. It is straightforward to check that the substitution of (4.6) into (4.1) leads to the following

$$\begin{aligned} &\Psi(x, \theta, \bar{\theta}) = \psi(x) + \theta\big(s_{ab}\psi(x)\big) + \bar{\theta}\big(s_b\psi(x)\big) + \theta\bar{\theta}\big(s_b s_{ab}\psi(x)\big), \\ &\bar{\Psi}\big(x, \theta, \bar{\theta}\big) = \bar{\psi}(x) + \theta\big(s_{ab}\bar{\psi}(x)\big) + \bar{\theta}\big(s_b\bar{\psi}(x)\big) + \theta\bar{\theta}\big(s_b s_{ab}\bar{\psi}(x)\big). \end{aligned} \tag{4.8}$$

[7] Let us focus on $b_1\gamma_\mu\psi = \bar{\psi}\gamma_\mu b_2$. It is evident that the pair of bosonic components b_1 and b_2 should be proportional to the pair of fermionic fields $\bar{\psi}$ and ψ, respectively. To make the latter pair bosonic in nature, we have to include the ghost field C of the Lagrangian density (2.2) to obtain: $b_1 \sim \bar{\psi}C$, $b_2 \sim C\psi$. Rest of the choices in (4.6) follow exactly similar kind of arguments.

This establishes the fact that the nilpotent (anti-)BRST charges $Q_{(a)b}$ are the translations generators $(\mathrm{Lim}_{\bar\theta\to 0}(\partial/\partial\theta))\,\mathrm{Lim}_{\theta\to 0}(\partial/\partial\bar\theta)$ along the $(\theta)\bar\theta$ directions of the supermanifold. The property of the nilpotency (i.e., $Q^2_{(a)b}=0$) is encoded in the two successive translations along the Grassmannian directions of the supermanifold (i.e., $(\partial/\partial\theta)^2=(\partial/\partial\bar\theta)^2=0$).

Now we shall concentrate on the derivation of the symmetry transformations (2.4) on the matter fields in the framework of superfield formulation. To this end in mind, we construct the superaxial-vector current $\tilde J^{(5)}_\mu(x,\theta,\bar\theta)$ and substitute (4.1) to obtain

$$\begin{aligned}\tilde J^{(5)}_\mu(x,\theta,\bar\theta) &= \bar\Psi(x,\theta,\bar\theta)\gamma_\mu\gamma_5\Psi(x,\theta,\bar\theta)\\ &= J^{(5)}_\mu(x)+\theta\bar K^{(5)}_\mu(x)+\bar\theta K^{(5)}_\mu(x)+i\theta\bar\theta L^{(5)}_\mu(x),\end{aligned}\tag{4.9}$$

where the above components on the r.h.s. can be expressed, in terms of the basic components of the expansion in (4.1), as

$$\begin{aligned}&\bar K^{(5)}_\mu(x)=i(\bar b_2\gamma_\mu\gamma_5\psi-\bar\psi\gamma_\mu\gamma_5\bar b_1),\qquad K^{(5)}_\mu(x)=i(b_1\gamma_\mu\gamma_5\psi-\bar\psi\gamma_\mu\gamma_5 b_2),\\ &L^{(5)}_\mu(x)=\bar f\gamma_\mu\gamma_5\psi+\bar\psi\gamma_\mu\gamma_5 f+i(\bar b_2\gamma_\mu\gamma_5 b_2-b_1\gamma_\mu\gamma_5\bar b_1),\qquad J^{(5)}_\mu(x)=\bar\psi\gamma_\mu\gamma_5\psi.\end{aligned}\tag{4.10}$$

Invoking the analogue of the condition (4.7) (i.e., $\tilde J^{(5)}_\mu(x,\theta,\bar\theta)=J^{(5)}_\mu(x)$), we obtain the following conditions on the components of the superexpansion in (4.9):

$$K^{(5)}_\mu(x)=0,\qquad \bar K^{(5)}_\mu(x)=0,\qquad L^{(5)}_\mu(x)=0.\tag{4.11}$$

Ultimately, these conditions lead to

$$\begin{aligned}&b_1=+e\bar\psi\bar C\gamma_5,\qquad b_2=-e\bar C\gamma_5\psi,\qquad \bar b_1=-eC\gamma_5\psi,\qquad \bar b_2=+e\bar\psi C\gamma_5,\\ &f=+ie[\mathcal B\gamma_5-eC\bar C]\psi,\qquad \bar f=+ie\bar\psi[\mathcal B\gamma_5+e\bar CC].\end{aligned}\tag{4.12}$$

The substitution of the above values in the superexpansion in (4.1) leads to the analogous expansion as in (4.8) with the replacements: $s_b\to s_d$, $s_{ab}\to s_{ad}$. Thus, we obtain

$$\begin{aligned}&\Psi(x,\theta,\bar\theta)=\psi(x)+\theta\big(s_{ad}\psi(x)\big)+\bar\theta\big(s_d\psi(x)\big)+\theta\bar\theta\big(s_ds_{ad}\psi(x)\big),\\ &\bar\Psi(x,\theta,\bar\theta)=\bar\psi(x)+\theta\big(s_{ad}\bar\psi(x)\big)+\bar\theta\big(s_d\bar\psi(x)\big)+\theta\bar\theta\big(s_ds_{ad}\bar\psi(x)\big).\end{aligned}\tag{4.13}$$

This provides the geometrical interpretation for the (anti-)co-BRST charges as the translation generators along the (θ) $\bar\theta$ directions of the supermanifold. This interpretation is exactly identical to the interpretation for the (anti-)BRST charges as the translation generators. The above statement for the (anti-)BRST- and (anti-)co-BRST charges can be succinctly expressed in the mathematical form, using (2.5), as

$$\begin{aligned}&s_r\Sigma(x)=\mathrm{Lim}_{\theta\to 0}\frac{\partial}{\partial\bar\theta}\tilde\Sigma(x,\theta,\bar\theta)\equiv -i\big\{\Sigma(x),Q_r\big\},\\ &s_t\Sigma(x)=\mathrm{Lim}_{\bar\theta\to 0}\frac{\partial}{\partial\theta}\tilde\Sigma(x,\theta,\bar\theta)\equiv -i\big\{\Sigma(x),Q_t\big\},\end{aligned}\tag{4.14}$$

where $r=b,d$, $t=ab,ad$ and $\Sigma(x)=\psi(x),\bar\psi(x)$, $\tilde\Sigma(x,\theta,\bar\theta)=\Psi(x,\theta,\bar\theta),\bar\Psi(x,\theta,\bar\theta)$. Thus, it is clear that the mapping that exists among the symmetry transformations, the conserved charges and the translation generators along the Grassmannian directions are

$$s_{b(d)}\leftrightarrow Q_{b(d)}\leftrightarrow \mathrm{Lim}_{\theta\to 0}\frac{\partial}{\partial\bar\theta},\qquad s_{ad}\leftrightarrow Q_{ad}\leftrightarrow \mathrm{Lim}_{\bar\theta\to 0}\frac{\partial}{\partial\theta},\qquad s_{ab}\leftrightarrow Q_{ab}\leftrightarrow \mathrm{Lim}_{\bar\theta\to 0}\frac{\partial}{\partial\theta}.\tag{4.15}$$

5. Conclusions

In the present investigation, we set out to derive the off-shell nilpotent (anti-)BRST and (anti-)co-BRST symmetries for the matter (Dirac) fields in the framework of geometrical superfield approach to BRST formalism. We chose the two-dimensional interacting $U(1)$ gauge theory (i.e., QED) for our discussion primarily for two reasons. First and foremost, this theory provides one of the simplest gauge theory and a *unique interacting* field theoretical model for the Hodge theory. Second, the Lagrangian density (2.2) of this theory is endowed with a local, covariant, continuous and nilpotent (anti-)co-BRST symmetries which is not the case for the four-dimensional QED where the (anti-)co-BRST transformations are non-local and non-covariant (see, e.g., [31] for details). We have been able to derive the off-shell nilpotent (anti-)BRST and (anti-)co-BRST symmetry transformations on the Dirac fields by invoking a couple of restrictions (i.e., $\tilde{J}_\mu(x,\theta,\bar{\theta}) = J_\mu(x)$ and $\tilde{J}^{(5)}_\mu(x,\theta,\bar{\theta}) = J^{(5)}_\mu(x)$) on the $(2+2)$-dimensional supermanifold. In contrast to the (dual-)horizontality conditions, these restrictions are not imposed by hand from the outside. Rather, they appear very naturally because of the fact that $s_{(a)b} J_\mu(x) = 0$, $s_{(a)d} J^{(5)}_\mu(x) = 0$ in the superexpansion of the supercurrents $\tilde{J}_\mu(x,\theta,\bar{\theta})$ and $\tilde{J}^{(5)}_\mu(x,\theta,\bar{\theta})$ (cf. Eqs. (4.4) and (4.9)). Physically, these conditions imply nothing but the conservation of the electric charge for the massive Dirac fields and the conservation of the spin (i.e., helicity in 2D spacetime) for the massless Dirac fields, respectively. These conservation laws persist even in the superfield formulation of the theory. This is why, automatically, we get the conditions $\tilde{J}_\mu(x,\theta,\bar{\theta}) = J_\mu(x)$ and $\tilde{J}^{(5)}_\mu(x,\theta,\bar{\theta}) = J^{(5)}_\mu(x)$. We would like to comment that our method of derivation of the (anti-)BRST transformations for the matter fields, in the framework of the superfield formalism, can be generalized to the physical 4D Abelian as well as non-Abelian gauge theories (see, e.g., [31,32] for transformations). It would be also interesting to obtain the on-shell nilpotent version of the above symmetries in the framework of the superfield formulation. These are some of the open problems which are under investigation and our results would be reported elsewhere [33].

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