Ain Shams Engineering Journal (2016) xxx, xxx–xxx

Ain Shams University
Ain Shams Engineering Journal
www.elsevier.com/locate/asej
www.sciencedirect.com

ENGINEERING PHYSICS AND MATHEMATICS

Heat and mass transfer analysis for the MHD flow of nanofluid with radiation absorption

P. Durga Prasad *, R.V.M.S.S. Kiran Kumar, S.V.K. Varma

Department of Mathematics, S.V. University, Tirupati 517502, A.P, India

Received 12 October 2015; revised 1 April 2016; accepted 29 April 2016

KEYWORDS

MHD;
Chemical reaction;
Dufour effect;
Nanofluid;
Porous medium

Abstract In this paper the effects of Diffusion thermo, radiation absorption and chemical reaction on MHD free convective heat and mass transfer flow of a nanofluid bounded by a semi-infinite flat plate are analyzed. The plate is moved with a constant velocity U_0, temperature and the concentration are assumed to be fluctuating with time harmonically from a constant mean at the plate. The analytical solutions of the boundary layer equations are assumed of oscillatory type and are solved by using the small perturbation technique. Two types of nanofluids namely Cu-water nanofluid and TiO_2-water nanofluid are used. The effects of various fluid flow parameters are discussed through graphs and tables. It is observed that the diffusion thermo parameter/radiation absorption parameter enhance the velocity, temperature and skin friction. This enhancement is very significant for copper nanoparticles. This is due to the high conductivity of the solid particles of Cu than those of TiO_2. Also it is noticed that the solutal boundary layer thickness decreases with an increase in chemical reaction parameter. It is because chemical molecular diffusivity reduces for higher values of *Kr*.

1. Introduction

It is well known that the convectional heat transfer fluids such as water, mineral oil and ethylene glycol have poor heat transfer properties compared to those of most solids in general. An innovative way of improving the heat transfer of fluids is to suspend small solid particles in the fluids. This new kind of fluid named as *nanofluid* was first introduced in 1995 by Choi [1]. The term nanofluid is used to describe a solid liquid mixture which consists of basic low volume fraction of high conductivity solid nanoparticles.

A nanofluid is a fluid in which nanometer-sized particles are suspended in a convectional heat transfer fluid to improve the heat transfer characteristics. Thus, nanofluids have many applications in industry such as coolants, lubricants, heat exchangers and micro-channel heat sinks. Therefore, numerous methods have been taken to improve the thermal conductivity of these fluids by suspending nano/micro sized particle materials in liquids. They reported breakthrough in substantially increasing the thermal conductivity of fluids by adding

* Corresponding author.
E-mail addresses: durga.prsd@gmail.com (P. Durga Prasad), kksaisiva@gmail.com (R.V.M.S.S. Kiran Kumar), svijayakumarvarma@yahoo.co.in (S.V.K. Varma).

Peer review under responsibility of Ain Shams University.

ELSEVIER Production and hosting by Elsevier

http://dx.doi.org/10.1016/j.asej.2016.04.016
2090-4479 © 2016 Faculty of Engineering, Ain Shams University. Production and hosting by Elsevier B.V.

Please cite this article in press as: Durga Prasad P et al., Heat and mass transfer analysis for the MHD flow of nanofluid with radiation absorption, Ain Shams Eng J (2016), http://dx.doi.org/10.1016/j.asej.2016.04.016

very small amounts of suspended metallic oxide nanoparticles (Cu, CuO and Al_2O_3) to the fluid [1–3].

In recent years, the natural convection flow of nanofluid has been studied by [4,5]. Kuznetsov and Nield [6] investigated the natural convective boundary-layer flow of a nanofluid past a vertical plate using Buongiorno model. Gorla and Chamkha [7] studied the natural convective boundary layer flow of nanofluid in a porous medium. Convective heat transfer and flow characteristics of nanofluids are given by [8,9].

Magnetohydrodynamic boundary-layer flow of nanofluid and heat transfer has received a lot of attention in the field of several industrial, scientific, and engineering applications in recent years. Nanofluids have many applications in industries, since materials of nanometer size with unique chemical and physical properties have sundry applications such as electronics cooling, and transformer cooling, and this study is more important in industries such as hot rolling, melt spinning, extrusion, glass fiber production, wire drawing, manufacture of plastic and rubber sheets, and polymer sheet and filaments. In view of the abovementioned applications of nanofluids, many researchers contributed in this area. Model and comparative study for peristaltic transport of water based nanofluids was given by Shehzad et al. [10]. Ghaly [11] considered the thermal radiation effects on a steady flow, whereas Raptis and Massalas [12] and El-Aziz [13] have analyzed the unsteady case. Mutuku-Njane and Makinde [14] investigated the hydromagnetic boundary layer flow of nanofluids over a permeable moving surface with Newtonian heating. Takhar et al. [15] have studied the radiation effects on the MHD free convection flow of a gas past a semi-infinite vertical plate. The natural convective boundary layer flows of a nanofluid past a vertical plate have been described by Kuznetsov and Nield [16,17]. In this model, the Brownian motion and thermophoresis are accounted with the simplest possible boundary conditions. Bachok et al. [18] have shown the steady boundary-layer flow of a nanofluid past a moving semi-infinite flat plate in a uniform free stream. It was assumed that the plate is moving in the same or opposite directions to the free stream to define the resulting system of non-linear ordinary differential equations. Raptis and Kafousis [19] have investigated steady hydrodynamic free convection flow through a porous medium bounded by an infinite vertical plate with constant suction velocity. Raptis [20] discussed unsteady two-dimensional natural convection flow of an electrically conducting, viscous and incompressible fluid along an infinite vertical plate embedded in a porous medium. A comprehensive survey of convective transport in nanofluids was made by Buongiorno [21]. Also he gave an explanation for the abnormal increases of the thermal conductivity of nanofluids after examining many mechanics in the absence of turbulence and he found that the Brownian diffusion and the thermophoresis effects are the most important and reported conservation laws for nanofluids in the presence of these two effects.

Magnetohydrodynamic stagnation-point flow of a power-law fluid toward a stretching surface in the presence of thermal radiation and suction/injection was studied by Mahapatra et al. [22]. MHD mixed convection in a nanofluid due to a stretching/shrinking surface with suction/injection and heat and mass transfer of nanofluid through an impulsively vertical stretching surface using the spectral relaxation method was studied by Haroun et al. [23,24]. The Radiation effects on an unsteady MHD axisymmetric stagnation-point flow over a shrinking sheet in the presence of temperature dependent thermal conductivity with Navier slip were considered by Mondal et al. [25]. Hamad and Pop [26] investigated an unsteady MHD free convective flow of nanofluid past a vertical permeable flat plate with constant heat source. MHD stagnation point flow and heat transfer problem from a stretching sheet in the presence of a heat source/sink and suction/injection in porous media was studied by Mondal et al. [27]. In their study, it is seen that spectral perturbation method can be used to find numerical solutions for complicated expansions encountered in perturbation schemes. Turkyilmazoglu [28,29] analyzed the heat and mass transfer effects in MHD flow of nanofluids.

Chemical reaction effects on heat and mass transfer are of considerable importance in hydrometallurgical industries and chemical technology. Several investigators have examined the effect of chemical reaction on the flow, heat and mass transfer past a vertical plate. Convective boundary-layer flow of MHD Nanofluid over a stretching surface with chemical reaction using the spectral relaxation method was studied by Haroun et al. [30]. Venkateswarlu and Satyanarayana [31] have studied the effects of chemical reaction and radiation absorption on the heat and mass transfer flow of nanofluid in a rotating system. Recently, Kiran Kumar et al. [32,33] analyzed the study of heat and mass transfer enhancement in free convection flow with chemical reaction and thermo-diffusion in nanofluids through porous medium in a rotating frame.

In the abovestated papers, the diffusion-thermo and thermal-diffusion terms were neglected from the energy and concentration equations respectively. But when heat and mass transfer occurs simultaneously in a moving fluid, the relation between the fluxes and the driving potentials is of intricate nature. It has been found that an energy flux can be generated not only by temperature gradient but also by concentration gradients. The energy flux caused by concentration gradient is called the Dufour or diffusion-thermo effect. The diffusion-thermo (Dufour) effect was found to be of considerable magnitude such that it cannot be ignored by Eckert and Darke [34]. In view of the importance of this diffusion-thermo effect, Jha and Singh [35] studied the free convection and mass transfer flow about an infinite vertical flat plate moving impulsively in its own plane.

Motivated by some of the researchers mentioned above and its applications in various fields of science and technology, it is of interest to discuss and analyze the Diffusion thermo and chemical reaction effects on the free convection heat and mass transfer flow of nanofluid over a vertical plate embedded in a porous medium in the presence of radiation absorption and constant heat source under fluctuating boundary conditions. Majority of the studies, reported in the literature, on free convective heat and mass transfer in nanofluid embedded in a porous medium deal with local similarity solutions and non-similarity solutions. But in the present study, the governing equations are solved using similarity transformations.

2. Formulation of the problem

An unsteady natural convectional flow of a nanofluid past a vertical permeable semi-infinite moving plate with constant heat source is considered. The physical model of the fluid flow is shown in Fig. 1. The flow is assumed to be in the x-direction which is taken along the plate and y-direction is normal to it.

A uniform external magnetic field of strength B_0 is taken to be acting along the y-direction. It is assumed that the induced magnetic field and the external electric field due to the polarization of charges are negligible. The plate and the fluid are at the same temperature T'_∞ and concentration C'_∞ in a stationary condition, when $t \geqslant 0$, the temperature and concentration at the plate fluctuate with time harmonically from a constant mean. The fluid is a water based nanofluid containing two types of nanoparticles either Cu (copper) or TiO_2 (Titanium oxide). The nanoparticles are assumed to have a uniform shape and size. Moreover, it is assumed that both the fluid phase nanoparticles are in thermal equilibrium state. Due to semi-infinite plate surface assumption, furthermore the flow variables are functions of y and time t only.

Under the above boundary layer approximations, the governing equations for the nanofluid flow are given by

$$\frac{\partial v'}{\partial y'} = 0 \tag{1}$$

$$\rho_{nf}\left(\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'}\right) = \mu_{nf}\frac{\partial^2 u'}{\partial y'^2} + (\rho\beta)_{nf}g(T' - T'_\infty) - \frac{\mu_{nf}u'}{K'} - \sigma B_0^2 u' \tag{2}$$

$$\left(\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'}\right) = \alpha_{nf}\frac{\partial^2 T'}{\partial y'^2} - \frac{Q'}{(\rho C_p)_{nf}}(T' - T'_\infty) + Q'_l(C' - C'_\infty) + \frac{D_m K_T}{C_s(\rho C_p)_{nf}}\frac{\partial^2 C'}{\partial y'^2} \tag{3}$$

$$\frac{\partial C'}{\partial t'} + v'\frac{\partial C'}{\partial y'} = D_B\frac{\partial^2 C'}{\partial y'^2} - K_l(C' - C'_\infty) \tag{4}$$

where u' and v' are the velocity components along x and y axes respectively. β_{nf} is the coefficient of thermal expansion of nanofluid, σ is the electric conductivity of the fluid, ρ_{nf} is the density of the nanofluid, μ_{nf} is the viscosity of the nanofluid, $(\rho C_p)_{nf}$ is the heat capacitance of the nanofluid fluid, g is the acceleration due to gravity, K' is the permeability porous medium, T' is the temperature of the nanofluid, Q is the temperature dependent volumetric rate of the heat source, and α_{nf} is the thermal diffusivity of the nanofluid, which are defined as follows [36], where ϕ is the solid volume fraction of the nanoparticles, K_{nf} and K_s are thermal conductivities of the base fluid and of the solid respectively. The thermo-physical properties of the pure fluid (water), copper and titanium which were used for code validation are given in Table 1.

The boundary conditions for the problem are given by

$$\begin{aligned} &t' < 0, \quad u'(y',t') = 0, \quad T' = T'_\infty, \quad C' = C'_\infty \\ &t' \geqslant 0, \quad u'(y',t') = U_0, \quad T' = T'_w + (T'_w - T'_\infty)\varepsilon e^{iw't'}, \quad C' = C'_w + (C'_w - C'_\infty)\varepsilon e^{iw't'} \ at\ y' = 0 \\ &u'(y',t') = 0, \quad T' = T'_\infty, \quad C' = C'_\infty as\, y' \to \infty \end{aligned} \tag{5}$$

where T' is the local temperature of the nanofluid and Q is the additional heat source. On the other hand, β_f and β_C are the coefficients of thermal expansion of the fluid and of the solid, respectively, ρ_f and ρ_C are the densities of the fluid and of the solid fractions, respectively, while ρ_{nf} is the viscosity of the nanofluid, α_{nf} is the thermal diffusivity of the nanofluid, and $(\rho C_p)_{nf}$ is the heat capacitance of the fluid, which are defined as (Abbasi [37,38])

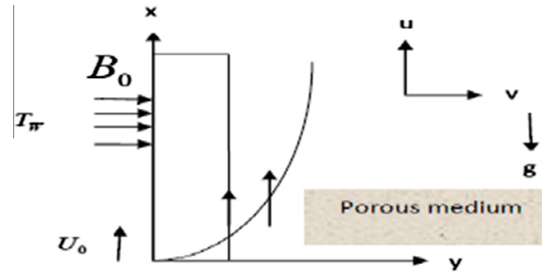


Figure 1 Schematic diagram of the physical problem.

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s, \quad (\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s,$$
$$(\rho\beta)_{nf} = (1-\phi)(\rho\beta)_f + \phi(\rho\beta)_s,$$

$$K_{nf} = K_f\left(\frac{K_s + 2K_f - 2\phi(K_f - K_s)}{K_s + 2K_f + 2\phi(K_f - K_s)}\right), \quad \mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}},$$
$$\alpha_{nf} = \frac{K_{nf}}{(\rho C_p)_{nf}} \tag{6}$$

$$v' = -V_0 \tag{7}$$

where the constant $-V_0$ represents the normal velocity at the plate which is positive suction $(V_0 > 0)$ and negative for blowing injection $(V_0 < 0)$.

Let us introduce the following dimensionless variables:

$$u = \frac{u'}{U_0}, \quad y = \frac{U_0 y'}{v_f}, \quad t = \frac{U_0^2 t'}{v_f}, \quad \omega = \frac{v_f\omega'}{U_0^2},$$
$$\theta = \frac{(T' - T'_\infty)}{(T'_w - T'_\infty)}, \quad S = \frac{V_0}{U_0}, \quad M = \frac{\sigma B_0^2 v_f}{\rho_f U_0^2}$$
$$Du = \frac{D_m K_T(C'_w - C'_\infty)}{k_f C_s(T'_w - T'_\infty)}, \quad Q_L = \frac{Q'_l(C'_w - C'_\infty)}{U_0^2(T'_w - T'_\infty)},$$
$$Kr = \frac{K_l v_f}{U_0^2}, \quad Sc = \frac{v_f}{D_B}$$
$$Q = \frac{Q' v_f^2}{K_f U_0^2}, \quad Pr = \frac{v_f}{\alpha_f}, \quad K = \frac{K'\rho_f U_0^2}{v_f^2},$$
$$Gr = \frac{(\rho\beta)_f g v_f(T'_w - T'_\infty)}{\rho_f U_0^3}, \quad \psi = \frac{(C' - C'_\infty)}{(C'_w - C'_\infty)} \tag{8}$$

Here $Pr = \frac{v_f}{\alpha_f}$ is the Prandtl number, S is the suction $(S > 0)$ or injection $(S < 0)$ parameter, M is the Magnetic parameter, and Q_L is the radiation absorption parameter, Kr is the chemical reaction parameter, Sc is the Schmidt number, Gr is the

Please cite this article in press as: Durga Prasad P et al., Heat and mass transfer analysis for the MHD flow of nanofluid with radiation absorption, Ain Shams Eng J (2016), http://dx.doi.org/10.1016/j.asej.2016.04.016

Table 1 Thermo-physical properties (see [36,10]).

Physical properties	Water	Copper (Cu)	Titanium oxide (TiO_2)
C_p (J/kg K)	4179	385	686.2
ρ (kg/m^3)	997.1	8933	4250
k (W/m K)	0.613	400	8.9538
$\beta \times 10^{-5}$ (1/K)	21	1.67	0.9

Grashof number, K is the permeability parameter, and Du is the Diffusion-thermo parameter.

In Eq. (2)–(4) with the boundary conditions (5), we get

$$A\left(\frac{\partial u}{\partial t} - S\frac{\partial u}{\partial y}\right) = D\frac{\partial^2 u}{\partial y^2} + BGr\theta - \left(M + \frac{1}{K}\right)u = 0 \qquad (9)$$

$$C\left(\frac{\partial \theta}{\partial t} - S\frac{\partial \theta}{\partial y} - Q_L\psi\right) = \frac{1}{Pr}\left(E\frac{\partial^2 \theta}{\partial y^2} - Q\theta\right) + \frac{Du}{Pr}\frac{\partial^2 \psi}{\partial y^2} \qquad (10)$$

$$\frac{\partial \psi}{\partial t} - S\frac{\partial \psi}{\partial y} = \frac{1}{Sc}\frac{\partial^2 \psi}{\partial y^2} - Kr\psi \qquad (11)$$

With the boundary conditions

$$\begin{aligned} &t < 0 : u = 0, \quad \theta = 0, \quad \psi = 0 \\ &t \geqslant 0 : u = 1, \quad \theta = 1 + \varepsilon e^{i\omega t}, \quad \psi = 1 + \varepsilon e^{i\omega t} \text{ at } y = 0 \\ &u = 0, \quad \theta = 0, \quad \psi = 0 \text{ as } y \to \infty \end{aligned} \qquad (12)$$

3. Solution of the problem

Eqs. (9)–(11) are coupled non-linear partial differential equations whose solutions in closed-form are difficult to obtain. To solve these equations by converting into ordinary differential equations, the unsteady flow is superimposed on the mean steady flow, so that in the neighborhood of the plate, the expressions for velocity, temperature and concentration are assumed as

$$u(y,t) = u_0 + \varepsilon u_1 e^{i\omega t}$$

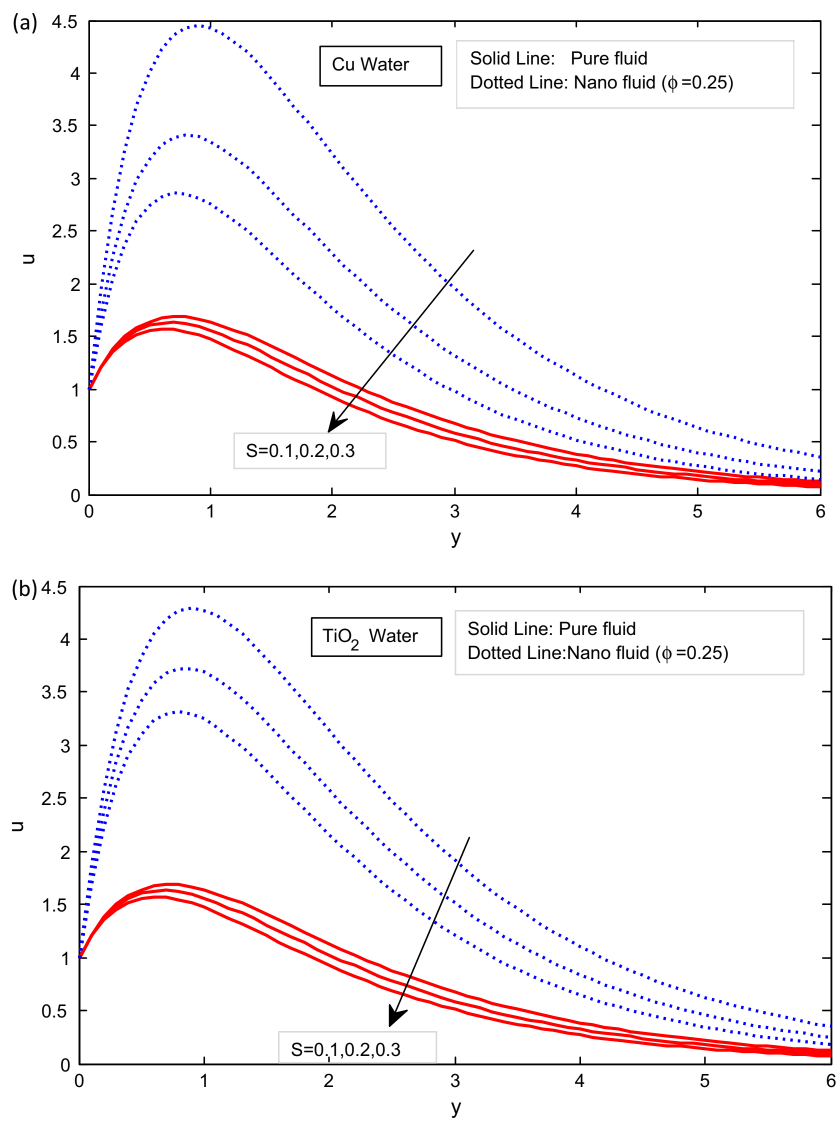


Figure 2 (a) and (b) Velocity profiles for S with $Sc = 0.60, Kr = 0.5, Q = 2, Du = 0.5, K = 4, \phi = 0.25, M = 0.5, Q_L = 2, Gr = 2$.

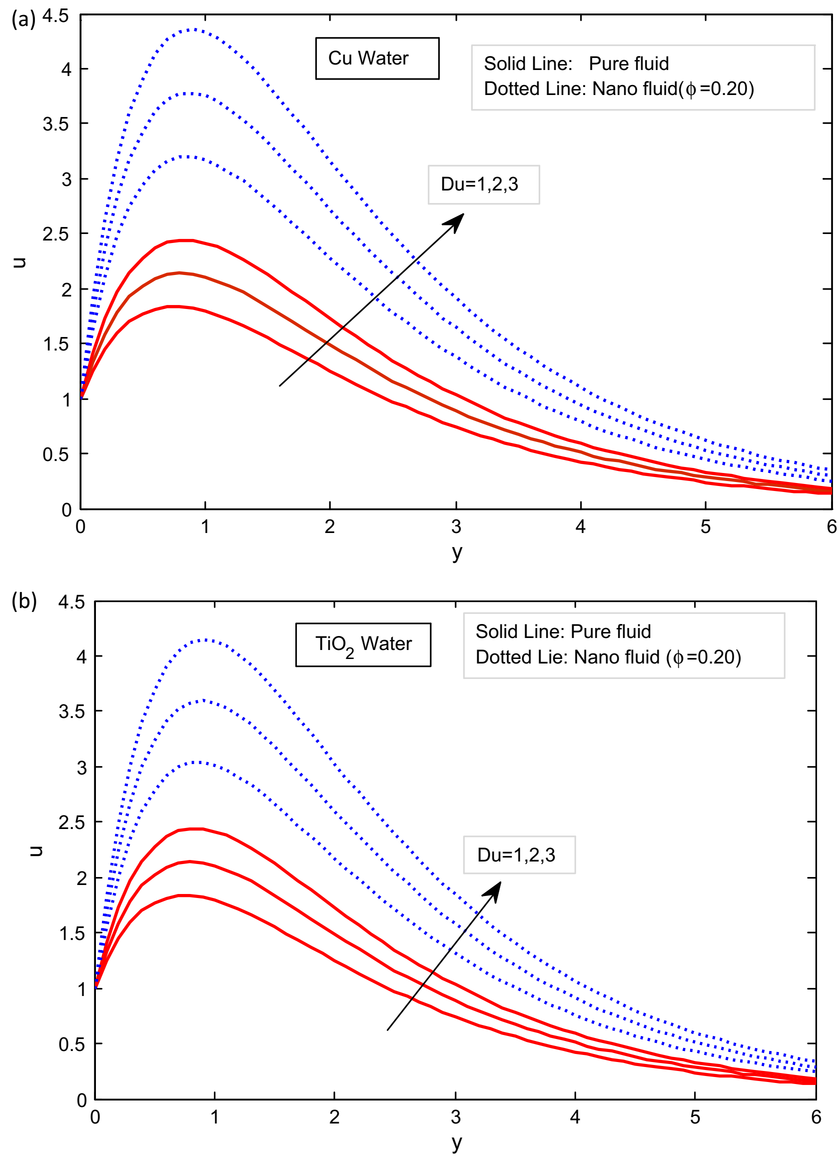


Figure 3 (a) and (b) Velocity profiles for Du with $Sc = 0.60, S = 0.1, Kr = 0.5, Q = 2, K = 4, \phi = 0.20, M = 0.5, Q_L = 2, Gr = 2$.

$$\theta(y,t) = \theta_0 + \varepsilon\theta_1 e^{i\omega t}$$

$$\psi(y,t) = \psi_0 + \varepsilon\psi_1 e^{i\omega t} \tag{13}$$

where $\varepsilon(\ll 1)$ is a parameter.

Eqs. (9)–(11) are reduced to

$$Du_0'' + ASu_0' - \left(M + \frac{1}{K}\right)u_0 = -BGr\theta_0 \tag{14}$$

$$Du_1'' + ASu_1' - \left(\left(M + \frac{1}{K}\right) + Ai\omega\right)u_1 = -BGr\theta_1 \tag{15}$$

$$E\theta_0'' + PrCS\theta_0' - Q\theta_0 = -Du\psi_0'' - PrCQ_L\psi_0 \tag{16}$$

$$E\theta_1'' + PrCS\theta_1' - (Q + PrCi\omega)\theta_1 = -Du\psi_1'' - PrCQ_L\psi_1 \tag{17}$$

$$\psi_0'' + SSc\psi_0' - KrSc\psi_0 = 0 \tag{18}$$

$$\psi_1'' + SSc\psi_1' - (i\omega + Kr)Sc\psi_1 = 0 \tag{19}$$

The boundary conditions (12) become

$$\begin{aligned} &u_0 = 1, \quad u_1 = 0, \quad \theta_0 = 1, \quad \theta_1 = 1, \quad \psi_0 = 1, \quad \psi_1 = 1 \; at \; y = 0 \\ &u_0 = 0, \quad u_1 = 0, \quad \theta_0 = 0, \quad \theta_1 = 0, \quad \psi_0 = 0, \quad \psi_1 = 0 \; as \; y \to \infty \end{aligned} \tag{20}$$

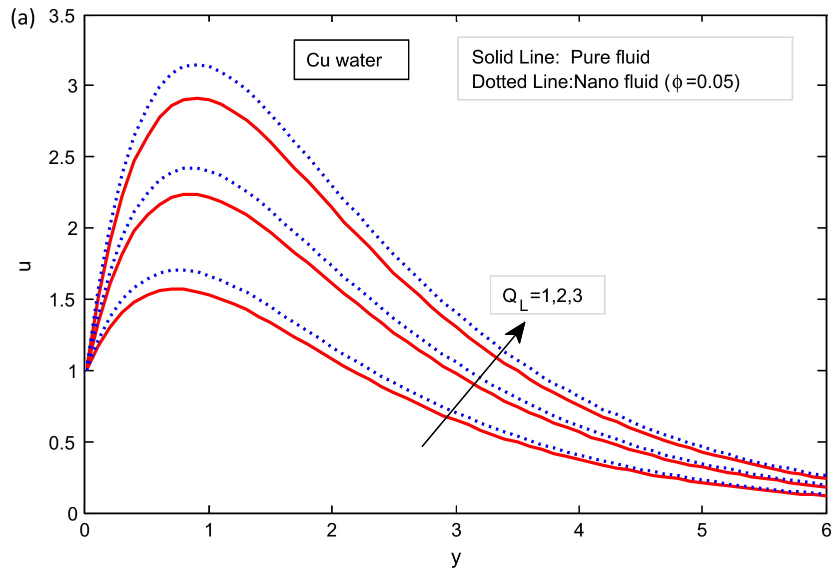
By solving Eqs. (14)–(19) using the boundary conditions (20), we get

$$\begin{aligned} u(y,t) = {} & (B_5 e^{-m_5 y} + B_3 e^{-m_3 y} + B_4 e^{-m_1 y}) \\ & + \varepsilon(B_8 e^{-m_6 y} + B_6 e^{-m_4 y} + B_7 e^{-m_2 y})e^{i\omega t} \end{aligned} \tag{21}$$

$$\theta(y,t) = (B_1 e^{-m_3 y} + A_1 e^{-m_1 y}) + \varepsilon(B_2 e^{-m_4 y} + A_2 e^{-m_2 y})e^{i\omega t} \tag{22}$$

$$\psi(y,t) = (e^{-m_1 y}) + \varepsilon(e^{-m_2 y})e^{i\omega t} \tag{23}$$

The shearing stress at the plate in dimensional form is given by



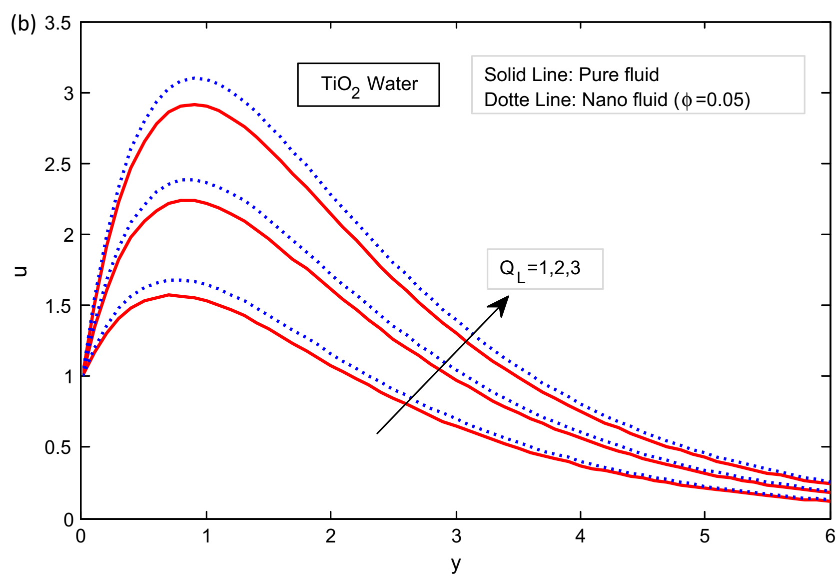


Figure 4 (a) and (b) Velocity profiles for Q_L with $Sc = 0.60, S = 0.1, Kr = 0.1, Q = 2, Du = 2, K = 5, \phi = 0.05, M = 0.5, Gr = 2$.

$$\tau = \left(\frac{\partial u}{\partial t}\right)_{y=0} = (-B_5m_5 - B_3m_3 - B_4m_1) + \varepsilon(-B_8m_6 - B_6m_4 - B_7m_2)e^{i\omega t} \quad (24)$$

Similarly, the rate of heat transfer at the plate/Nusselt number is given by

$$Nu = -\left(\frac{\partial \theta}{\partial t}\right)_{y=0} = (B_1m_3 + A_1m_1) + \varepsilon(B_2m_4 + A_2m_2)e^{i\omega t} \quad (25)$$

The rate of mass transfer at the plate/Sherwood number is given by

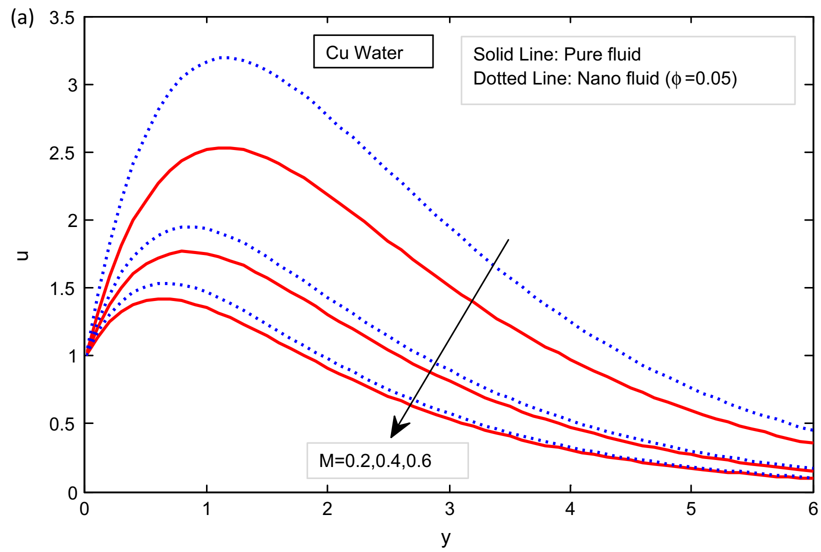
$$Sh = -\left(\frac{\partial \psi}{\partial t}\right)_{y=0} = m_1 + \varepsilon m_2 e^{i\omega t} \quad (26)$$

4. Results and discussions

In order to bring out the salient features of the flow, heat and mass transfer characteristics with nanoparticles, the results are presented in Figs. 2–10 and in Tables 2 and 3. The effects of nanoparticles on the velocity, the temperature and the concentration distributions as well as on the skin friction and the rate of heat and transfer coefficients are discussed numerically. We have chosen here $\varepsilon = 0.02,\ t = 1,\ \omega = 1$, and $Pr = 0.71$, while the remaining parameters are varied over a range, which are listed in figures.

4.1. Effect of suction parameter (*S*)

Fig. 2(a) and (b) demonstrates the effect of suction parameter S on fluid velocity u for both regular ($\phi = 0$) and nanofluid ($\phi \neq 0$). As an output of figures, it is seen that the velocity of



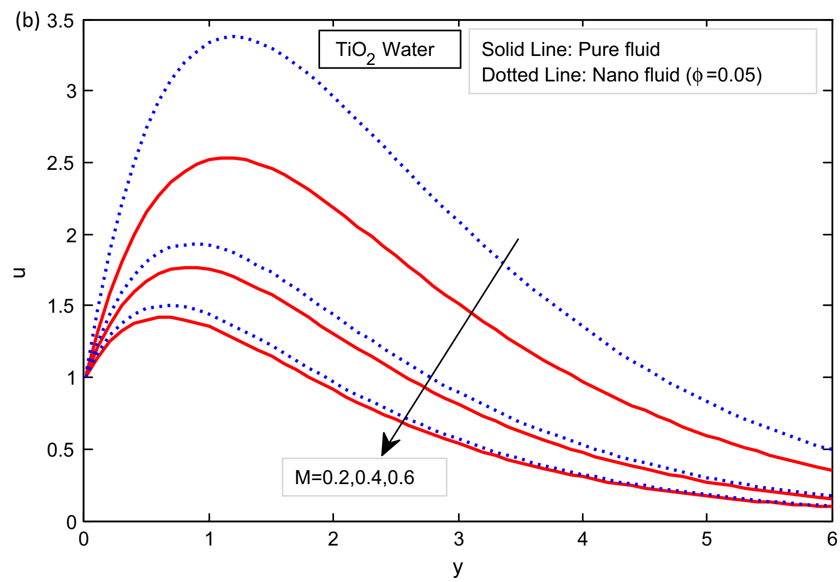


Figure 5 (a) and (b) Velocity profiles for M with $Sc = 0.60, S = 0.1, Kr = 0.5, Q = 2, Du = 2, K = 5, \phi = 0.05, Q_L = 1, Gr = 2$.

the fluid across the boundary layer decreases by increasing the suction parameter S for both regular fluid and nanofluid with nanoparticles Cu and TiO_2. It is worth mentioned here that the influence of the suction parameter S on the fluid velocity is more effective for nanofluid with the nanoparticles Cu and TiO_2. It is also observed that the maximum velocity of Cu–water nanofluid is higher than that of the TiO_2–water nanofluid attains in the neighborhood of $y = 1$. Fig. 9 displays the effects of the suction parameter (S) on the species concentration profiles. As the suction parameter increases the species concentration, the solutal boundary layer thickness decreases. This is due to the usual fact that the suction stabilizes the boundary growth. These consequences are obviously supported from the physical point of view.

4.2. Effect of Dufour number (Du)

The influence of Diffusion-thermo parameter (Du) on the velocity distribution for Cu–water and TiO_2–water nanofluids is shown in Fig. 3(a) and (b) respectively. It is noticed that the boundary layer thickness increases with Du for both regular and nanofluids. Thus the hydrodynamic boundary layer thickness increases as the Dufour number increases. Fig. 6(a) and (b) depicts the effects of Du on the temperature profiles within the boundary layer. With the increasing values of Du, the temperature of nanofluid is found to increase for both Cu and TiO_2 nanofluids, i.e., Du causes to increase the thermal boundary layer thickness. Also the thermal boundary layer decreases faster for lower values of Dufour number for both regular fluid and nanofluids. Physically, Decreasing Du clearly reduces the influence of species gradients on the temperature field, so that temperature function values are clearly lowered and the boundary layer regime is cooled. On other hand concentration function in the boundary layer regime is increased as Du is decreased. Mass diffusion is evidently enhanced in the domain as a result of the contribution of temperature gradients.

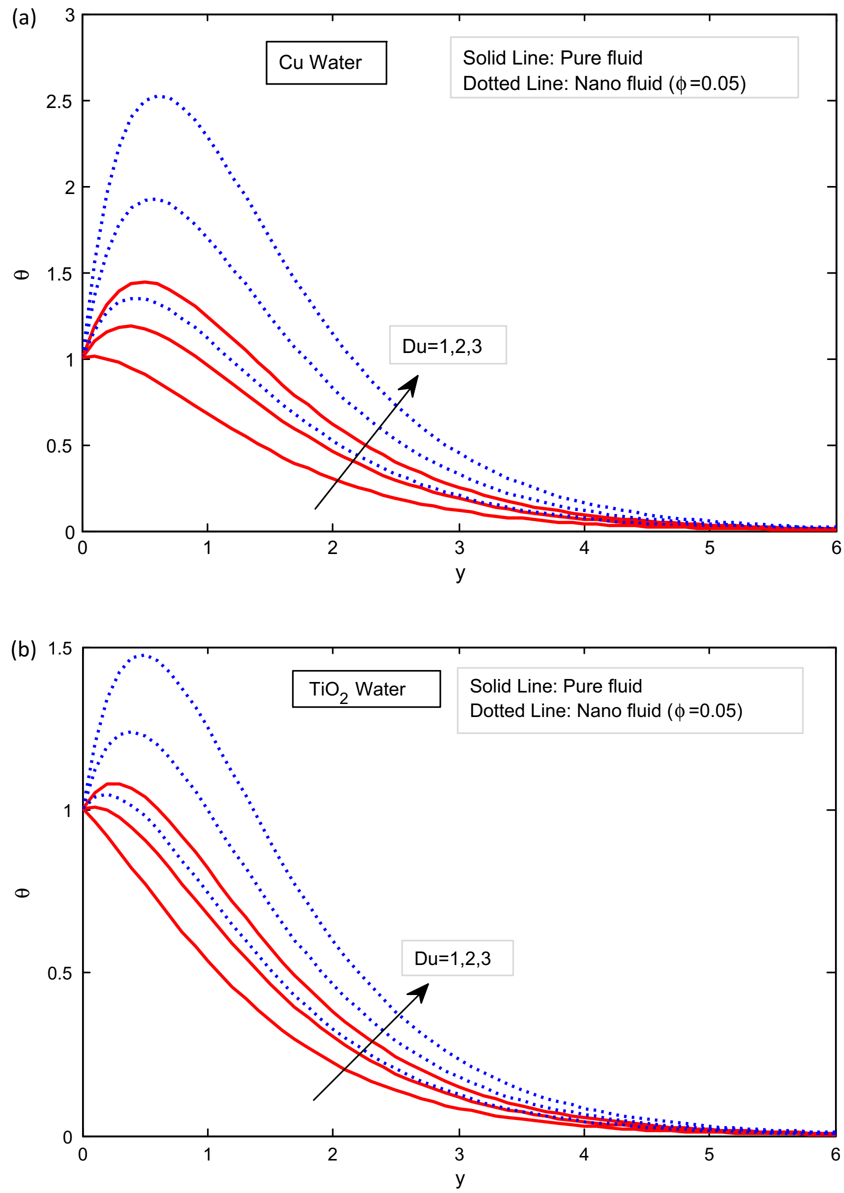


Figure 6 (a) and (b) Temperature profiles for Du with $Sc = 0.60, S = 0.1, Kr = 2, Q = 2, \phi = 0.05, Q_L = 2, Gr = 2$.

4.3. Effect of radiation absorption parameter ($\boldsymbol{Q_L}$)

Figs. 4(a), (b) and 7(a), (b) are graphical representation of the velocity and temperature profiles for different values of Q_L with *Cu* and TiO_2 nanoparticles. It is clear from these figures that the velocity and temperature profiles increase with increase of Q_L. This is due to the fact that, when heat is absorbed, the buoyancy force accelerates the flow. Also, it is observed that in the case of Cu-nanoparticles thermal boundary layer is very thicker than that of TiO_2-nanoparticles. The large Q_L values correspond to an increased dominance of conduction over absorption radiation thereby increasing buoyancy force and thickness of the thermal and momentum boundary layers.

4.4. Effect of magnetic field parameter ($\boldsymbol{M}$)

Fig. 5(a) and (b) presents the typical nanofluid velocity profiles for various values of magnetic field parameter (M) for the nanoparticles Cu and TiO_2. From these graphs, it is obvious that the nanofluid velocity of the fluid decelerates with an increase in the strength of magnetic field. The effects of a transverse magnetic field on an electrically, conducting fluid give rise to a resistive-type force called the Lorentz force. This force has the tendency to slow down the motion of the fluid in the boundary layer. These results quantitatively agree with the expectations, since magnetic field exerts retarding force on natural convection flow. Also, it is clear that the nanofluid velocity is lower for the regular fluid and the velocity reaches the maximum peak value for the nanoparticle TiO_2 in the comparison of Cu nanoparticles.

4.5. Effect of Schmidt number ($\boldsymbol{Sc}$)

The variation in the concentration boundary layer of the flow field is shown in Fig. 8 for H_2, H_2O vapor and NH_3. This figure depicts the concentration distribution in the presence of the

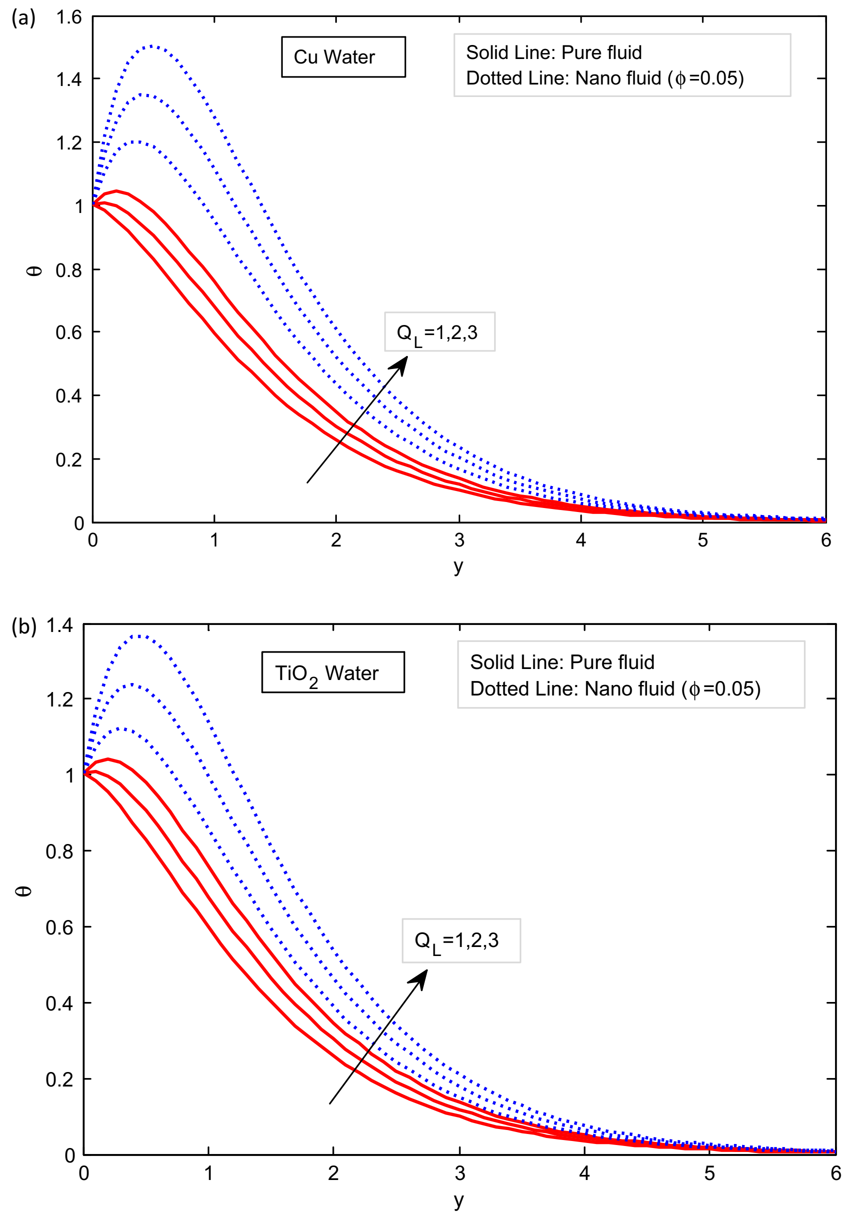


Figure 7 (a) and (b) Temperature profiles for Q_L with $Sc = 0.60, S = 0.1, Kr = 2, Q = 2, \phi = 0.05, Gr = 2, Du = 2$.

flow field. Comparing the curves of the said figure, it is observed that the growing Schmidt number decreases the concentration boundary layer thickness of the flow field at all points. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid flow. The reductions in the concentration profiles are accompanied by simultaneous reductions in the concentration boundary layer.

4.6. *Effect of chemical reaction parameter* (***Kr***)

For different values of destructive chemical reaction parameter $Kr(>0)$, the concentration profiles are plotted in Fig. 10. An increase in chemical reaction parameter will suppress the concentration of the fluid. Higher values of Kr amount to a fall in the chemical molecular diffusivity, i.e., less diffusion. Therefore, they are obtained by species transfer. An increase in Kr will suppress species concentration. The concentration distribution decreases at all points of the flow field with the increase in the reaction parameter.

The numerical values of the Skin-friction coefficient and Nusselt number for the nanoparticles Cu and TiO_2 are presented in Tables 2 and 3. From Table 2, it is seen that the local Nusselt number decreases with increasing values of suction parameter S, Dufour number Du, and volume fraction parameter ϕ, while it increases with increasing values of radiation absorption parameter Q_L and heat source parameter Q for both the nanoparticles Cu and TiO_2. It is also observed that the rate of heat transfer is higher in the

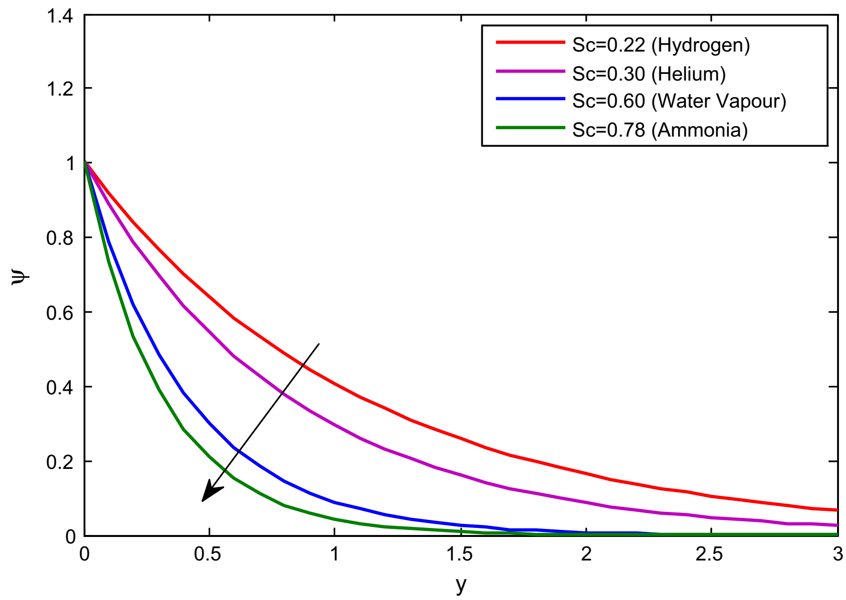


Figure 8 Concentration profiles for Sc with $S = 2, Kr = 0.1$.

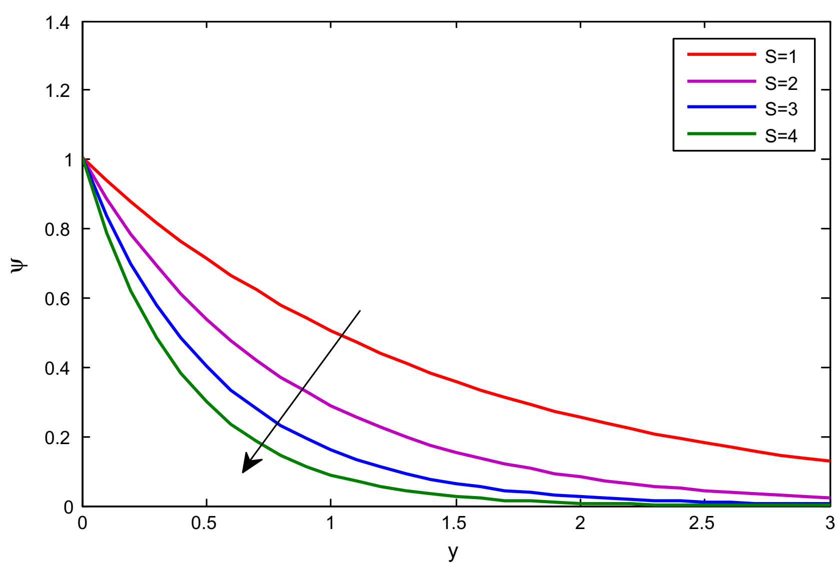


Figure 9 Concentration profiles for S with $Sc = 0.60, Kr = 0.1$.

Cu–water nanofluid than in TiO_2–water nanofluid. This is due to the high conductivity of the solid particles Cu than those of TiO_2. From Table 3 it is clear that the local skin friction coefficient increases with the increasing values of Gr, K, Q_L, Du, Kr and ϕ, as magnetic field creates Lorentz force which decreases the value of skin friction for both the nanoparticles Cu and TiO_2.

5. Conclusions

In the present study, we have theoretically studied the effects of the metallic nanoparticles on the unsteady MHD free convective flow of an incompressible fluid past a moving infinite vertical porous plate. The set of governing equations are solved analytically by using perturbation technique. The effects of various fluid flow parameters on velocity, temperature and species concentration, Skin-friction and the rate of heat transfer coefficient are derived and discussed through graphs and tables. The following conclusions are made from the present investigation:

1. In the boundary layer region, fluid velocity decreases with the increasing values of magnetic field parameter and suction parameter for both the nanoparticles Cu and TiO_2, while it increases with the increasing values of Dufour number and radiation absorption parameter.

Please cite this article in press as: Durga Prasad P et al., Heat and mass transfer analysis for the MHD flow of nanofluid with radiation absorption, Ain Shams Eng J (2016), http://dx.doi.org/10.1016/j.asej.2016.04.016

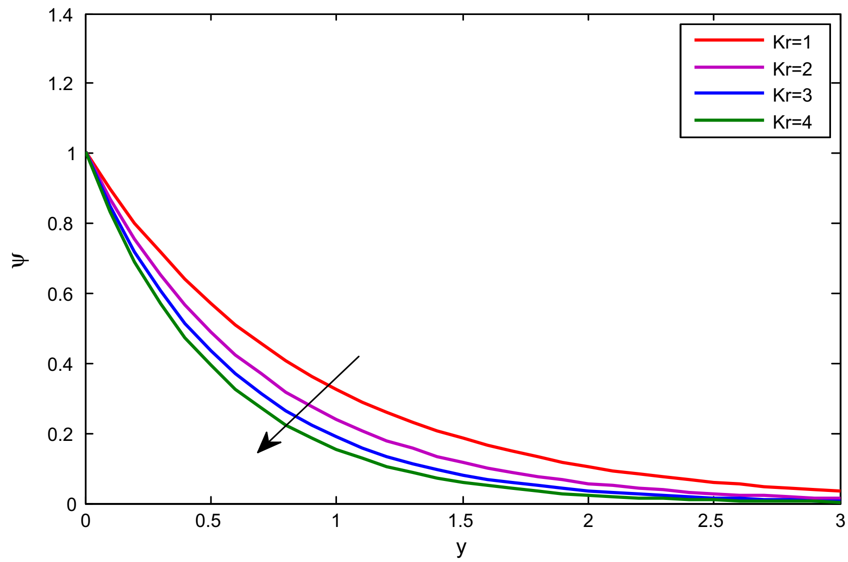


Figure 10 Concentration profiles for S with $Sc = 0.60, S = 1$.

Table 2 Numerical values of Skin-friction coefficient (Cf) with $Sc = 0.60, S = 0.1, t = 1, Q = 2, Pr = 0.71$.

Gr	M	K	Q_L	Du	Kr	ϕ	Skin friction for copper water fluid	Skin friction for titanium oxide water fluid
1	0.5	2	1.5	0.5	0.5	0.15	0.1161	0.3000
2	0.5	2	1.5	0.5	0.5	0.15	1.9215	1.5062
3	0.5	2	1.5	0.5	0.5	0.15	3.9550	3.3080
2	**0.1**	2	1.5	0.5	0.5	0.15	4.2564	4.2403
2	**0.5**	2	1.5	0.5	0.5	0.15	1.9215	1.5062
2	**1.0**	2	1.5	0.5	0.5	0.15	0.4719	0.0533
2	0.5	**2**	1.5	0.5	0.5	0.15	1.9215	1.5062
2	0.5	**4**	1.5	0.5	0.5	0.15	3.0118	2.6953
2	0.5	**6**	1.5	0.5	0.5	0.15	3.5858	3.3745
2	0.5	2	**1**	0.5	0.5	0.15	0.7440	0.4441
2	0.5	2	**3**	0.5	0.5	0.15	5.4544	4.6951
2	0.5	2	**5**	0.5	0.5	0.15	10.1650	8.9472
2	0.5	2	1.5	**0.1**	0.5	0.15	1.4828	1.1090
2	0.5	2	1.5	**0.2**	0.5	0.15	1.5925	1.2083
2	0.5	2	1.5	**0.3**	0.5	0.15	1.7021	1.3076
2	0.5	2	1.5	0.5	**0.1**	0.15	0.0279	0.2332
2	0.5	2	1.5	0.5	**5**	0.15	2.0418	2.1227
2	0.5	2	1.5	0.5	**10**	0.15	2.2124	2.2715
2	0.5	2	1.5	0.5	0.5	**0.05**	0.9921	0.8832
2	0.5	2	1.5	0.5	0.5	**0.15**	1.9215	1.5062
2	0.5	2	1.5	0.5	0.5	**0.2**	2.8177	2.0827

2. An increase in the Dufour number and radiation absorption parameter leads to increase the thermal boundary layer thickness.
3. The species concentration decreases with the increasing values of suction parameter, chemical reaction parameter and Schmidt number.
4. There is a significant effect of M and Q_L on the skin friction coefficient.

Table 3 Numerical values of Nusselt number $(Nu)Sc = 0.60, t = 1, Pr = 0.71$.

S	Du	ϕ	Q_L	Q	Nu for copper water fluid	Nu for titanium oxide water fluid
0.1	0.5	0.15	1.5	2	0.7928	0.8416
0.5	0.5	0.15	1.5	2	0.7224	0.8307
1.0	0.5	0.15	1.5	2	0.4004	0.6923
0.1	**0.1**	0.15	1.5	2	0.9221	0.9555
0.1	**0.2**	0.15	1.5	2	0.8898	0.9270
0.1	**0.3**	0.15	1.5	2	0.8574	0.8986
0.1	0.5	**0.05**	1.5	2	0.8381	0.8415
0.1	0.5	**0.15**	1.5	2	0.7928	0.8416
0.1	0.5	**0.2**	1.5	2	0.6688	0.8023
0.1	0.5	0.15	**2**	2	0.4558	0.5502
0.1	0.5	0.15	**4**	2	0.8920	0.6157
0.1	0.5	0.15	**6**	2	2.2399	1.7815
0.1	0.5	0.15	1.5	**3**	0.9468	1.4675
0.1	0.5	0.15	1.5	**4**	1.4974	1.9287
0.1	0.5	0.15	1.5	**5**	2.4122	2.3096

Appendix A

$$A = \left((1-\phi) + \phi\left(\frac{\rho_s}{\rho_f}\right)\right), \quad B = \left((1-\phi) + \phi\left(\frac{(\rho\beta)_s}{(\rho\beta)_f}\right)\right),$$

$$C = \left((1-\phi) + \phi\left(\frac{(\rho C_p)_s}{(\rho C_p)_f}\right)\right),$$

$$E = \frac{k_{nf}}{k_f} = \left(\frac{(1+2\phi) + (2-2\phi)\left(\frac{K_f}{K_s}\right)}{(1-2\phi) + (2+2\phi)\left(\frac{K_f}{K_s}\right)}\right), \quad D = \frac{1}{(1-\phi)^{2.5}},$$

$$A_1 = -\left(\frac{m_1^2 Du + PrQ_L C}{Em_1^2 - PrCSm_1 - Q}\right),$$

$$A_2 = -\left(\frac{m_2^2 Du + PrQ_L C}{Em_2^2 - \mathrm{Pr}CSm_2 - (Q + i\omega PrC)}\right),$$
$$B_1 = (1 - A_1), \quad B_2 = (1 - A_2),$$

$$B_3 = -\left(\frac{BB_1 Gr}{Dm_3^2 - ASm_3 - (M + \frac{1}{K})}\right),$$
$$B_4 = -\left(\frac{BA_1 Gr}{Dm_1^2 - ASm_1 - (M + \frac{1}{K})}\right), \quad B_5 = (1 - B_3 - B_4),$$

$$B_6 = -\left(\frac{BB_2 Gr}{Dm_4^2 - ASm_4 - (Ai\omega + M + \frac{1}{K})}\right),$$
$$B_7 = -\left(\frac{BA_2 Gr}{Dm_2^2 - ASm_2 - (Ai\omega + M + \frac{1}{K})}\right),$$

$$B_8 = -(B_6 + B_7), m_1 = \left(\frac{SSc + \sqrt{(SSc)^2 + 4KrSc}}{2}\right),$$
$$m_2 = \left(\frac{SSc + \sqrt{(SSc)^2 + 4Sc(i\omega + Kr)}}{2}\right),$$

$$m_3 = \left(\frac{PrCS + \sqrt{(PrCS)^2 + 4EQ}}{2}\right),$$
$$m_4 = \left(\frac{PrCS + \sqrt{(PrCS)^2 + 4E(Q + i\omega PrC)}}{2E}\right),$$

$$m_5 = \left(\frac{AS + \sqrt{(AS)^2 + 4D(M + \frac{1}{K})}}{2D}\right),$$
$$m_6 = \left(\frac{AS + \sqrt{(AS)^2 + 4D(Ai\omega + (M + \frac{1}{K}))}}{2D}\right)$$

References

[1] Choi S. Enhancing thermal conductivity of fluids with nanoparticle. In: Siginer DA, Wang HP, editors, Developments and applications of non-newtonian flows, vol. 66, American Society of Mathematical Engineers, New York; 1995. p. 99–105.
[2] Lee S, Choi SUS, Eastman JA. Measuring thermal conductivity of fluids containing oxide nanoparticles. J Heat Transfer 1999;121:280–9.
[3] Das SK, Choi SUS, Yu W, Pradeep T. Nanofluids: science and technology. New Jersey: Wiley; 2007.
[4] Makinde OD, Aziz A. Boundary layer flow of a nanofluid past a stretching sheet with a convective boundary condition. Int J Therm Sci 2011;50:1326–32.
[5] Ellahi R, Aziz S, Zeeshan A. Non Newtonian nanofluids flow through a porous medium between two coaxial cylinders with heat transfer and variable viscosity. J Porous Media 2013;16:205–16.
[6] Kuznetsov AV, Nield DA. Natural convective boundary layer flow of a nanofluid past a vertical plate. Int J Therm Sci 2010;49:243–7.
[7] Gorla RSR, Chamkha A. Natural convective boundary layer flow over a horizontal plate embedded in a porous medium saturated with a nanofluid. J Modern Phys 2011;2:62–71.
[8] LI Qiang, Xuan Yimin. Convective heat transfer and flow characteristics of Cu-water nanofluid. Sci China (Series E) 2002;45:408–16.
[9] Das Sarit Kumar, Choi Stephen US, Patel Hrishikesh E. Heat transfer in nanofluids: a review. Heat Transfer Eng 2006;27:3–19.
[10] Shehzad SA, Abbasi FM, Hayat T, Alsaadi F. Model and comparative study for peristaltic transport of water based nanofluids. J Mol Liq 2015;209:723–8.
[11] Ghaly AY. Radiation effect on a certain MHD free convection flow. Chaos Solitons Fractal 2002;13:1843–50.
[12] Raptis A, Massalas CV. Magnetohydrodynamic flow past a plate by the presence of radiation. Heat Mass Transfer 1998;34:107–9.
[13] El-Aziz MA. Radiation effect on the flow and heat transfer over an unsteady stretching surface. Int Commun Heat Mass Trans 2009;36:521–4.
[14] Mutuku-Njane WN, Makinde OD. On hydromagnetic boundary layer flow of nanofluids over a permeable moving surface with Newtonian heating. Latin Am Appl Res 2014;44:57–62.
[15] Takhar HS, Gorla RSR, Soundalgekar VM. Nano-linear one-step method for initial value problems. Int Num Meth Heat Fluid Flow 1996;6:22–83.
[16] Kuzenstov AV, Nield DA. Natural convective boundary-layer flow of a nanofluid past a vertical plate. Int J Thermal Sci 2010;49:243–7.
[17] Kuzenstov AV, Nield DA. The Cheng–Minkowycz problem for natural convective boundary layer flow in a porous medium saturated by a nanofluid. Int J Mass Transf 2009;52:5792–5.
[18] Bachok N, Ishak A, Pop I. Boundary-layer flow of a nanofluid over a moving surface in a flowing fluid. Int J Thermal Sci 2010;49:1663–8.
[19] Raptis A, Kafousis N. Heat transfer in flow through porous medium bounded by an infinite vertical plate under the action of a magnetic field. Int J Energy Res 1982;6:241–5.
[20] Raptis A. Flow through a porous medium in the presence of a magnetic field. Int J Energy Res 1986;10:97–100.
[21] Buongiorno J. Convective transport in nanofluids. J Heat Transfer 2006;128:240–50.
[22] Mahapatra Tapas Ray, Mondal Sabyasachi, Pal Dulal. Heat transfer due to magnetohydrodynamic stagnation-point flow of a power-law fluid towards a stretching surface in the presence of thermal radiation and suction/injection. Int Schol Res Netw ISRN Thermodyn 2012:1–9.
[23] Haroun Nageeb AH, Precious Sibanda, Mondal Sabyasachi, Motsa Sandile S. On unsteady MHD mixed convection in a nanofluid due to a stretching/shrinking surface with suction/injection using the spectral relaxation method. Bound Value Probl 2015;24:1–17.
[24] Haroun Nageeb AH, Sibanda Precious, Mondal Sabyasachi, Motsa Sandile S, Rashidi Mohammad M. Heat and mass transfer of nanofluid through an impulsively vertical stretching surface using the spectral relaxation method. Boundary Value Prob 2015;161:1–16.
[25] Mondal S, Haroun NAH, Sibanda P. The effects of thermal radiation on an unsteady MHD axisymmetric stagnation-point flow over a shrinking sheet in presence of temperature dependent thermal conductivity with Navier slip. PLoS one 2015;10:1–23.
[26] Hamad MAA, Pop I. Unsteady MHD free convection flow past a vertical permeable flat plate in a rotating frame of reference with constant heat source in a nanofluid. Heat Mass Transfer 2011;47:1517–24.
[27] Agbaje TM, Mondal S, Makukula ZG, Motsa SS, Sibanda P. A new numerical approach to MHD stagnation point flow and heat transfer towards a stretching sheet. Ain Shams Eng J 2015.
[28] Turkyilmazoglu M. Exact analytical solutions for heat and mass transfer of MHD slip flow in nanofluids. Chem Eng Sci 2012;84:182–7.
[29] Turkyilmazoglu M. Nanofluid flow and heat transfer due to a rotating disk. Comput Fluids 2014;94:139–46.

Please cite this article in press as: Durga Prasad P et al., Heat and mass transfer analysis for the MHD flow of nanofluid with radiation absorption, Ain Shams Eng J (2016), http://dx.doi.org/10.1016/j.asej.2016.04.016

[30] Haroun Nageeb AH, Mondal S, Sibanda Precious. Unsteady natural convective boundary-layer flow of MHD nanofluid over a stretching surfaces with chemical reaction using the spectral relaxation method a revised model. Procedia Eng 2015;127:18–24.
[31] Venkateswarlu B, Satyanarayana PV. Chemical reaction and radiation absorption effects on the flow and heat transfer of a nanofluid in a rotating system. J Appl Nano Sci 2015;5:351–60.
[32] Kiran Kumar RVMSS, Durga Prasad P, Varma SVK. Analytical study of heat and mass transfer enhancement in free convection flow with chemical reaction and constant heat source in nanofluids. Procedia Eng 2015;127:978–85.
[33] Kiran Kumar RVMSS, Prasad P Durga, Varma SVK. Thermo-diffusion and chemical reaction effects on free convective heat and mass transfer flow of conducting nanofluid through porous medium in a rotating frame. Global J Pure Appl Math 2016;12:342–51.
[34] Eckert ERG, Drake RM. Analysis of heat and mass transfer. New-York: Mc. Graw-Hill; 1972.
[35] Jha BK, Singh AK. Soret effect on free convection and mass transfer flow in the stokes problem for an infinite vertical plate. Astrophys Space Sci 1990;173:251–5.
[36] Oztop HF, Abu-Nada E. Numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids. Int J Heat Mass Transf 2008;29:1326–36.
[37] Abbasi FM, Hayat T, Alsaadi F. Peristaltic transport of magneto-nanoparticles submerged in water: model for drug delivery system. Physica E 2015;68:123–32.
[38] Abbasi FM, Hayat T, Alsaadi F. Hydromagnetic peristaltic transport of water-based nanofluids with slip effects through an asymmetric channel. Int J Mod Phys B 2015;29:1–17.

P. Durga Prasad is currently doing M.Phil. in Sri Venkatewswara University, Tirupati, Andhra Pradesh-517502, India. His area of interest is FLUID DYNAMICS. He has 2 years research experience in research and 3 years of teaching experience at various levels. He presented/attended several International/National conferences/seminars/work-shops.

R.V.M.S.S. Kiran Kumar is currently doing Ph.D in Srivenketeswara University, Tirupati, A.P.-517502, India. He is doing his Ph.D in the area of FLUID DYNAMICS, and he has 2 years research experience and 2 years of teaching experience at various levels. He is qualified CSIR-NET With AIR-44, GATE-2014 with AIR-281, GATE-2013 with AIR-527 and awarded UGC MERITORIOUS BSR-FELLOWSHIP. He is a life member of Andhra Pradesh Mathematical Society.

Professor S.V.K. Varma, a senior professor in the Department of Mathematics, Sri Venkateswara University, Tirupati, Andhra Pradesh, India. He has vast experience in teaching and administration and also in research. His area of research is Fluid Dynamics, Magneto hydrodynamics, and Heat and Mass transfer. He published 160 papers in national and international journals. He collaborated with foreign researchers such as A.J. Chamkha of Kuwait, and J. Prakash of Botswana. He presented several papers in various conferences at national and international levels. He also visited abroad as a resource person for an international conference at University of Botswana, Gaborone, in 2013. He has organized several national conferences and now is going to organize an international conference with the collaboration of University of Botswana in June 2014. He guided 15 students for Ph.D. and 18 for M.Phil. He is the life member of various bodies. He is the member of Board of Studies of various universities and Autonomous Institutions.

Please cite this article in press as: Durga Prasad P et al., Heat and mass transfer analysis for the MHD flow of nanofluid with radiation absorption, Ain Shams Eng J (2016), http://dx.doi.org/10.1016/j.asej.2016.04.016